# MEGALITHIC SITES IN BRITAIN 

## BY

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Chapter 1. Introduction<br>Chapter 2. Statistical Ideas<br>Chapter 3. Astronomical Background<br>Chapter 4. Mathematical Background<br>Chapter 5. The Megalithic Yard<br>Chapter 13. The Extinction Angle Chapter 14. Conclusions<br>List of Figures, Tables and Sites (added)

## OXFORD

 AT THE CLARENDON PRESS
worked example.

1. INTRODUCTION-Selected from hundreds of small-scale copies of surveys made by Professor Thom over the past thirty years, examples are chosen to illustrate some of the conclusions that can be drawn regarding the knowledge possessed by Megalithic builders. Attention is concentrated almost entirely on circles, rings, outliers and their alignments. A standard unit of length - the Megalithic Yard $(M Y)=2.72$ feet used through-out Megalithic Britain during the period 2000-1600 BC is established. The study embraces Megalithic astronomy \& mathematics including geometry, circles, ellipses \& pythagorean triangles. 450 sites were visited, 300 surveyed.
2. STATISTICAL IDEAS-Probable error and standard deviation; standard deviation of the mean; quantum hypotheses; variance of the quantum; Figure 2.1: Probability Levels; use of Broadbent's Criterion; Figure 2.2: Test of a quantum hypothesis (after Broadbent) and a
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15. Circles, Rings, Megalithic Astronomy
16. The Calendar, Indications of Lunar Declinations4. The Outer Hebrides, Variety of Sites


| $A 8=75 \mathrm{My}$ | $A C=100 \mathrm{My}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $C B=125$ | $35=260$ | $C S=140$ |  |  |  |
| $C D=60$ | $D E=D S=D K=D L=200$ |  |  |  |  |
| ED in paraltel to $B A$ and $H$ is on BC produced |  |  |  |  |  |
| Ars MEF | has centre | an ED | and | radius | 750 |
| Arc. FG | ${ }^{\prime}$ | at $A$ | $\stackrel{-}{*}$ | - | 260 |
| Arg GSH | " | B | $*$ | $\stackrel{ }{ }$ | 260 |
| Arc LM | 4 | $C$ | I' | * | 260 |
| Ars HJ | $\cdots$ | on CB | I' | * | 750 |
| Art $/ \mathrm{K}$ | * | at $P$ | $n$ | * | PK |
| $\angle C O O$ is $90^{\circ}$ and $P Q=D Q$ |  |  |  |  |  |



In main ring concrete markers are shown by a cross +
Burning pits are shown by a dotted ring is
Diameter of inner circles $=125 \mathrm{My}=3.40 .0 \mathrm{ft}$
Distance between centres $=145 \mathrm{My}$

Geometrical construction superimposed in red on an accurate survey of the site (see pages 89-91)

## PREFACE

THIS work is restricted to a study of Megalithic circles, alignments, and isolated standing stones; it does not contain any systematic investigation into chambered tombs, tumuli, barps, or other similar structures. The information on which it is based was obtained almost exclusively by an examination of some 600 sites in Britain.

While I am fully capable of making surveys of any required accuracy, I do not consider myself qualified to dig an archaeological site. I restricted the measurements to what shows on the surface, augmented occasionally by prodding with a bayonet. Where trained archaeologists had already cleared the site of vegetation and loose surface accretion much information was available that would otherwise have remained hidden. It must, however, be remarked that where 're-erection' has been done by unqualified people the result is a lowering of the value of the site. I must make a plea for every stone to be left where it lies until a survey has been completed-and by 'survey' I do not mean the kind of plan that appears in many reports.

As long walks, sometimes unaccompanied, were often necessary I reduced by about one-half the weight of the theodolite that was normally used. The accuracy obtained was sufficient for most purposes, but, as the investigation proceeded, it became apparent that the precision with which some of the larger monuments had been set out demanded surveys of a high accuracy such as could be obtained only by a qualified team using high-class equipment. It is to be hoped that this will soon be appreciated and large-scale precise surveys made of all sites. One of the objects of this book is to show that many sites are worthy of the greatest care in their excavation and survey.

All the surveys except two were made by me, but some have been published before, and thanks are due to the Pergamon Press for permission to use those which appeared in Vistas in Astronomy, vol. 7. Acknowledgement is also made to the Mathematical Gazette, Antiquity, and the Royal Statistical Society.

It is hoped that the very many friends who assisted with the surveys will accept an over-all acknowledgement. But in this connexion I must mention specifically my wife and other members of my family. I also wish to thank the many farmes, crofters, shepherds, and foresters who helped to find many of the out-of-the-way sites.

My thanks are also due to Dr. A. E. Roy of the Department of Astronomy, University of Glasgow, For much helpful criticism and advice when the astronomical chapters were being prepared.\{vi\}

I am particularly indebted to the staff of the Map Room of the Bodleian Library for the tireless manner in which over the years they helped me by making available hundreds of surveys of various kinds. Thanks are also due to the Ordnance Survey for their courteous assistance in various ways.

NOTE TO THE 1971 REPRINT
Since this book originally appeared much further work has been done in Britain and in Brittany, some of which is described in my book Megalithic Lunar Observatories (Clarendon Press, 1971).
A. T.

Dunlop, Ayrshire, June 1971.

## INTRODUCTION

(Thom 1971:1-5)
SCATTERED throughout Britain there are thousands of Megalithic sites. A few of these are well known but the great majority lie off the beaten track in the fields and on the moors. Many are not even recognized (or obviously recognizable) as being Megalithic at all. Circles seem to attract most attention, but of many circles little or nothing now remains. The destruction is mostly of recent years and is still proceeding apace. Nevertheless many hundreds are still in such a condition that much can be learned from a careful examination and analysis of accurately made surveys. Sketch plans such as many journals carry are, for the purposes we have in view here, of little use. The surveys must be made with the same accuracy as was used in the original setting out and it will be shown that some sites, for example Avebury, were set out with an accuracy approaching 1 in 1000 . Only an experienced surveyor with good equipment is likely to attain this kind of accuracy. The differences in tension applied to an ordinary measuring tape by different individuals can produce variations in length of this amount or even more. The necessity for this kind of accuracy has not in the past been appreciated and has in fact only become apparent as the work recorded here progressed.

In this monograph will be found small-scale copies of a number of surveys selected from hundreds made by the author in the past thirty years. The examples have been chosen to illustrate some of the conclusions which can be drawn regarding the knowledge possessed by the Megalithic builders. Attention has been concentrated almost entirely on circles, rings, outliers, and alignments. The geometrical patterns to which the builders worked were outlined on the ground by stones of all shapes varying in size from 1 to 500 cubic feet. The features studied fall under two headings, geometrical and astronomical, but information of a wider scope can obviously be inferred.

Under the first heading we make a study of the units of measurement employed by the builders and of the geometrical shapes used for the rings, i.e. circles, flattened circles, egg shapes, ellipses, and other more complicated designs. Astronomically it has long been recognized that many of the sites contain indicators showing rising or setting points of the sun at the solstices. But the present work shows that there is a probability amounting to a certainty that other equally-spaced dates throughout the year are indicated. It also shows that the moon was carefully observed and that the first-magnitude stars may also come into the picture. An argument which has been raised against the use of the stars is $\{$ p. $2>\}$ that there are so many stars that almost any line is certain to show the rising or setting point of one or another.

But this argument is quite untenable because we can in general only speak of the rising points of first-magnitude stars. We cannot see, for example, a third-magnitude star rise-except on an elevated horizon. This is because such a star does not become visible even in clear weather until it has attained an altitude of some three degrees. Restricting
ourselves to first-magnitude stars, i.e. stars brighter than magnitude $1 \cdot 5$, we find that in Britain at the period in which we are interested, say 2000 to 1600 B.C., only some ten or twelve stars, depending on the latitude, etc., could rise or set. The others were either circumpolar or were too far south to be seen in these latitudes.

If we think of the long winter nights, if anything longer then than now, it is evident that throughout the greater part of the twenty-four hours the stars were the only indicators of time available. The hour would be indicated by the rising or setting of certain stars or by their transit over the meridian. There are many indications that both these methods were in use, or, to be more exact, there remain many indicators of rising and setting points of first-magnitude stars and many slabs and alignments still standing accurately in the meridian.

We can, I think, assume that in highly organized communities such as must have existed it would often be necessary to know the time of night. Much speculation has been directed to the necessity of accurate time-keeping for 'ritualistic purposes but certainly more practical reasons also existed. A civilization which could carry a unit of length from one end of Britain to the other, and perhaps much further afield, with an accuracy of $0 \cdot 1$ per cent and could call for the erection of 5000 to 10,000 megaliths must have made demands on its engineers. It is difficult to think of these responding without making use of time-keeping. One has only to think of the tremendous organizing effort which would be necessary to transport and erect numbers of stones some weighing up to 30 tons. Swampy ground might make it necessary to operate in winter when the ground was frozen. Think of feeding hundreds of men and the necessity of starting before dawn in the short winter day. The hour was important. Thus methods of obtaining time from the stars must have been well understood, To obtain time from the stars the date must be known and this as we shall see came from the sun at the calendar sites. Initially the necessary indicators would almost certainly have been of wood but it appears that in many places stone was substituted.

It is fortunate for us that Megalithic man liked, for some reason or another, to get as many as possible of the dimensions of his constructions to be multiples of his basic unit. \{p. $3>\}$ We are thereby enabled to determine unequivocally the exact size of this unit. In fact probably no linear unit of antiquity is at present known with a precision approaching our knowledge of the Megalithic yard. The reason for his obsession with integers is not entirely clear, but undoubtedly the unit was universally used, perhaps universally sacred.

It may have been that in the absence of paper and pen he found it necessary to record in stone his geometrical and perhaps also his arithmetical discoveries. Such of these as are known to us are of no mean order and there is no reason to suppose that our knowledge of what he knew is by any means complete. When it is recalled that our knowledge of his achievements in this field is only a decade or so old it is obvious that we have no right to imagine that it is complete. This mistake has indeed been made too often.

It is remarkable that 1000 years before the earliest mathematicians of classical Greece, people in these islands not only had a practical knowledge of geometry and were capable of setting out elaborate geometrical designs but could also set out ellipses based on Pythagorean triangles. We need not be surprised to find that their calendar was a highly
developed arrangement involving an exact knowledge of the length of the year or that they had set up many stations for observing the eighteen-year cycle of the revolution of the lunar nodes.

It is important to do everything we can to protect the fast-vanishing sites until we am sure that we really understand them all. Places like Stonehenge and Avebury are presumably for the time being fairly safe but it should be noted how much of our knowledge has come from the humble circle on the hillside. We cannot judge by an inspection on the ground what secrets a site may yield. It must be accurately surveyed, prodded, and eventually excavated before we can assess its value. The clues which eventually led the author to the unravelling of the geometry of Avebury did not come from Stonehenge or Stanton Drew but from small unimpressive circles on the Scottish moors and the hills of Wales.

## The surveys

In all some 450 sites have been visited and about 300 surveyed. A sufficient number of these surveys are reproduced here on a small scale to give an idea of what exists throughout the country.

The surveys were made usually by theodolite and tape, the orientation being determined by at least two time/azimuth observations of the sun. In overcast weather angles were measured to one or more distant points, for example mountain peaks, which could be identified on the Ordnance Survey. The azimuths of these marks were afterwards determined by a geodetic type of calculation from their geographical coordinates. If the weather or the type of country made it necessary to use marks near at hand then the azimuths might be determined by a large protractor. An accuracy of $0^{\circ} \cdot 1$ is usually sufficient but for some important sites single minutes of arc were wanted.

At one or two sites the inaccessibility and the long distance to be walked precluded the use of the theodolite and the surveys were made by prismatic compass. In some places the $\{$ p. $4>\}$ local attraction is severe but this need not affect the accuracy provided that at every compass station a distant mark is included in the round of angles, the azimuth of the mark being computed later from the O.S. An error of $0^{\circ} \cdot 2$ will produce an error of only about $0^{\circ} \cdot 1$ in the declination. Orientation of the survey by compass alone is not reliable but if there is no alternative the variation appropriate to the place and date must be applied and checks made to detect local anomalies.

Ideally at every site of any importance the horizon altitudes ought to be measured round the whole horizon. Time after time sites have had to be revisited to measure the horizon altitude on a line which only became apparent when the survey was plotted. Many of the lines tabulated are uncertain because a second visit was impossible and the ground was such that estimates made from the O.S. contours were unsatisfactory. Photography can be a help here. If the coordinates, azimuth and altitude, of two points included in a picture have been measured then the whole horizon shown can be measured with sufficient accuracy from an enlargement.

During the surveys a bayonet was often used to prod the ground. In this way many buried stones were discovered and many broken stumps found. For example, at Strathaird
in Skye (H7/9) only three stones were upright but five more were felt below the peat. No attempt has been made to contour the surveys simply because this could not be done properly in the time available.

The azimuth of anything which looked like a sight line was noted together with the horizon altitude. The latter quantity is designated by $h$, but no symbol is used in the surveys for the azimuths. Thus where an angle is shown numerically without designation it is either the azimuth of a line drawn on the plan or of some point or object shown. In some cases the derived declination ( $\delta$ ) is added but it does not follow that the line was considered worthy of being included in the tables.

In recording a stone measurements were made to its base and the shaded, hatched, or blackened part on the surveys shows the plan section at or near ground level. A great many stones originally vertical are now leaning at all sorts of angles and in all directions making it impossible to be sure as to where the bases had been. A line of small v's across the plan of a stone shows where a sloping surface, for example the upper side of an inclined stone, runs into the ground. The remainder of the stone below ground may have been estimated by prodding, in which case it will be shown dotted with the other end, the top, shown in full line. One is thereby able to make a guess as to how much the shaded area has to be displaced in estimating afterwards the" original position.

Much use was made of the Ordnance Survey, especially the 6 -in, both the $1^{\text {st }}$ and $2^{\text {nd }}$ editions being sometimes consulted. Unfortunately the 21 -in which has contours at $25-\mathrm{ft}$ intervals does not cover the whole country. \{p. 5>\} The 6-in has contours at this interval for the island of Lewis but most sheets are not contoured. Information regarding the orientation of the sheet edges for the 25 -in can be obtained so that sometimes a reliable estimate of an azimuth can be obtained from this survey. Unfortunately many sites lie in districts which are not covered on this scale.

Chapter 1: Introduction. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford 1971:1-5.

## 2

## STATISTICALIDEAS

(Thom 1971:6-14)
IT will be necessary to make extensive use of statistical theory if any reliable conclusions are to be drawn from the mass of material presented in later chapters. Consequently it seems desirable to devote a chapter to some of the ideas and formulae of which extensive use will be made. This is the more necessary since some of the theory used has so far appeared only in scientific journals.

Throughout we shall use the quantity 'standard deviation' $(\sigma)$ as an indication of the precision of a measurement or of a derived constant. The older method and that still used in some branches of science is to give the 'probable error', which is related to the standard deviation by the formula

$$
\text { probable error }=0.67 \mathrm{x} \text { standard deviation. }
$$

When we write a length as $L \pm \sigma$ we understand that the chance of an error of $2 \sigma$ or more in $L$ is about 1 in 20 or 5 per cent.

The simplest case is where we have found the arithmetic mean of a number of measurements of a single quantity. The deviation of each measurement from the mean may be called $\epsilon$ and the 'variance' is the mean of the squares of all values of $\epsilon$. The square root of the variance is then $\sigma$, the standard deviation, so we have

$$
\sigma^{2}=\sum \epsilon^{2} / n,
$$

where $n$ is the number of measurements.

So long as we go on taking measurements of the same quantity by the same method we should expect to get approximately the same value for $\sigma$. It is a measure of the kind of deviation we should expect to get in any future measurement of the same kind. But when we form the mean of a group of our measurements we have a quantity of a much more precise nature and this is expressed by the formula

$$
\sigma_{\text {mean }}=\sigma / \sqrt{n}
$$

This is sometimes called the 'standard error of the mean' or sometimes the 'standard deviation of the mean'.

When we write $6 \cdot 23 \pm 0 \cdot 02$ it is understood that $6 \cdot 23$ has been determined as the best or most likely value from the observations considered. If $6 \cdot 23$ has been found by taking an arithmetic mean as above then 0.02 is understood to be the standard error of the mean, but if it has been determined indirectly by a more complicated method such as a
'least squares' solution of equations based on other related measured quantities, then 0.02 still refers to the final result ( $6 \cdot 23$ ) and not to any individual measurement.

In many statistical investigations it is necessary to attach a 'probability level' to a quantity. This has a different meaning and refers to the probability that the quantity is real and is not a spurious result obtained by accident. The probability level is in fact the probability (usually expressed as a percentage) of the result occurring by accident.

As an illustration consider a simple application of Bernoulli's theorem. Suppose we have three dice and suspect that they have been loaded to make them tend to show vi on being thrown. We throw them once and obtain one vi. There would be nothing suspicious in this. By elementary algebra we find the probability of at least one vi to be $91 / 216$ or $0 \cdot 42$. But suppose that at the first throw we get three vi's, then there is some ground for suspicion because with perfect dice the probability of three vi's in the first throw is $(1 / 6)^{3}$ or 0.5 per cent. This is not proof that the dice are loaded but we might say that we can accept the hypothesis that they are loaded at a probability level of 0.5 per cent. The natural thing to do is to throw again. If we again get three vi's we feel that our suspicion is justified because theory indicates that the probability is 1 in 46,656. In other words the probability level is about 0.002 per cent.

The value of the probability level at which we accept the hypothesis we are examining (in the above example that the dice were loaded) is a matter which depends on circumstances and in fact on the individual or group of individuals concerned. Where one man will accept a risk another will not. If the probability of a flying accident were 5 per cent very few people would fly, but for many purposes the 5 per cent level may be accepted.

In the simple example given above, elementary algebra gives a definite method of calculating the probability level, but in many cases the analysis is much more complicated. As an example, of which much use will be made later, consider the proof of the existence of a 'quantum' in a set of measurements. Suppose we have made a note of the times of the occurrence of a recurring event and we think there is a periodicity so that the event tends to happen at more or less regular intervals. How is a probability level to be assigned to our hypothesis? This is a most important type of problem which crops up in many branches of science but it is only in recent years that a solution (albeit empirical) has been available.

We shall set out the problem or rather problems in mathematical form and later, to simplify matters, give a full example.

In general we have a set of measurements $Y_{1}, Y_{2}, Y_{3}, \ldots, Y_{\mathrm{n}}$. We suggest that these can be represented by a 'quantum hypothesis' of the form:

$$
Y_{i}=\beta+2 m_{i} \delta+\epsilon_{i}
$$

where $i$ takes the values $1,2,3, \ldots, n$.
$m$ is zero or an integer. The values of $y$ are grouped round regularly spaced nodes, the groups being numbered $m=0, m=1$, etc.; $2 \delta$ and $\beta$ are constants, $2 \delta$ being the quantum or uniform spacing between the groups whose existent we seek to prove or disprove. $\beta$ allows for the possibility that the zero of our measurements may not agree with a node. If it does then $\beta$ is zero. $\varepsilon_{i}$ is the inevitable error or discrepancy of the $i$ th measurement. If $\varepsilon$ is everywhere zero then all the measurements fall exactly at nodes and it will be obvious without calculation that the hypothesis is true.

It is essential to recognize two distinct classes of problem.
Case I. In the first case we have come to the problem with an a priori knowledge that a quantum may exist. We have an idea of its magnitude an we wish to test the hypothesis that its existence is demonstrated by the measurements $Y_{1}, Y_{2}$, etc.
Case II. In the second and much more difficult case the quantum has come from the data themselves. We had in fact no idea beforehand that such quantum existed nor with hindsight can we say, 'Ah, but we ought to have expected such a quantum because

The logical approach to the two cases is quite different. Both have been dealt with by Broadbent, who has given in a first paper (1955) a rigid method of handling the first case and in a subsequent paper (1956) a Monte Carlo solution leading to a method of handling the second case.

There are two subdivisions of each case:
(a) when we know definitely from the nature of the problem that $\beta$ is zero.
(b) when $\beta$ may not be zero but must be determined from the data.

A further subdivision may be necessary. The standard deviation ( $\sigma$ ) of the measurements may be the same for all the groups, that is for all values of $m$, or alternatively $\sigma$ may increase with increasing $m$. To illustrate this point suppose that $Y_{1}, Y_{2}$, etc. have been measured with an accurate tape but suppose that the tape could not be brought into dose contact with the objects being measured, then $\sigma$ would be of the same order for all the groups and so the larger measures would not necessarily be less accurate than the smaller. If, on the other hand, the measuring appliance were in itself crude, then $\sigma$ might be proportional to $m$ or would at least increase with $m$. This would happen if distances were obtained by pacing. It will be shown that in the application of the theory used in this monograph we need only consider the formula 1 constant $\sigma$. Reference to Broadbent's first paper may be made if the formula for $\sigma$ proportional to $m$ are required.

We shall now consider the two subdivisions of Case I.
Case I (a). We have obtained an approximate value of the quantum and we know that the constant $\beta$ must be zero. The formula for estimating the revised value of the quantum is

$$
2 \delta=\Sigma m y / \Sigma m^{2},
$$

The variance of the quantum may be estimated according to Broadbent by

$$
\sigma^{2}=s_{1}^{2} /(n-1) \sum m^{2},
$$

where $s_{1}^{2}=\sum y^{2}-\left(\sum m y^{2}\right) / \sum m^{2}$
and $n$ is the total number of observations.
This formula is not always suitable for desk computations as it depends on the (small) difference of two large numbers but it can easily be modified.

Case $I(b)$. We have obtained an approximate value of the quantum but the constant $\beta$ is not necessarily zero. The formulae for obtaining the revised values of $2 \delta$ and $\beta$ are

$$
\begin{array}{r}
2 \delta=(n \Sigma m y-\Sigma m \Sigma y) / \Delta, \\
\beta=\left(\Sigma m^{2} \Sigma y-\Sigma m \Sigma m y\right) / \Delta,
\end{array}
$$

where

$$
\Delta=n \Sigma m^{2}-(\Sigma m)^{2} .
$$

The variance of the quantum may be estimated according to Broadbent by

$$
\sigma^{2}=s_{2}^{2} / \Delta
$$

where $s_{2}^{2} \quad$ is obtained $\operatorname{fr} m(n-2) s_{2}^{2}=\Delta^{\prime}-(2 \delta)^{2} \Delta \quad \Delta^{\prime}=n \sum y^{2}-\left(\sum y\right)^{2}, \quad$ and
Similarly the variance of $\beta$ is estimated by

$$
n \sigma^{2}=s_{2}^{2}\left[1+\left(\sum m\right)^{2} / n \Delta\right],
$$

It will be noticed that in applying the above formulae it is necessary to have an approximate value of $2 \delta$ initially in order to decide on the value of $m$ to associate with each value of $y$ or, in other words, to decide to which group each $y$ is to be associated. If the calculated quantum turns out to be much different from the assumed value it may be necessary to repeat the calculation.

The above expressions give a value for the quantum $2 \delta$ but we must now obtain the probability level at which we can accept the result. This is done by finding for each measurement, i.e., for each $y$, its deviation from the nearest node, i.e.,

$$
\epsilon=y-\beta-2 m \delta .
$$

Values of $\epsilon$ may already have been formed for the calculation of the variance of $2 \delta$. Having found $\epsilon$ for each observation we calculate what Broadbent calls the 'lumped variance' $\left(s^{2}\right)$ from

$$
n s^{2}=\Sigma \epsilon^{2} .
$$

We already have $2 \delta$ and so we can find $s^{2} / \delta^{2}$ with which to enter Fig. 2.1.
This figure shows at the top the required value of the probability level for any pair of the values of $n$ and $s^{2} / \delta^{2}$. The probability level so obtained is, it must be remembered, only valid if we had, before we began the investigation, an idea that a quantum existed with a value close to that obtained in the end.

Case II. We have now to consider the case where there is no a priori reason for expecting or adopting a particular quantum. We have merely inspected the data and have noticed that the measurements seem to group themselves round more or less evenly spaced nodes. We first determine the values of $2 \delta$ and $\beta$ by using the formulae given for Case I, but when we come to consider the probability level the greatest care is necessary


Fig. 2.1. Probability level.
because experience has shown that almost any set of random numbers scattered between, say, 0 and 200 will show a rough periodicity of some sort and we may have by accident obtained a particularly 'good' set.

There is at present no rigid mathematical approach to the problem of assigning a probability level but Broadbent (1956), by a Monte Carlo method, has produced a criterion which is easy to apply. A reading of his paper shows that we have here a reliable method of detecting a spurious quantum. There will of course be borderline cases and for these we must either bring other considerations into the argument or obtain further measurements.

Suppose that we have a large number $n$ of observations with no periodicity, scattered more or less uniformly but randomly along the range. If we test these by the method given for Case I for some suspected quantum (which is of course non-existent) then we ought to find $s^{2} / \delta^{2}$ very close to $\frac{1}{3}$.

Since we are in effect finding the second moment of a rectangle this is fairly clear and in fact Broadbent calls this a rectangular distribution, each rectangle having a width equal to the quantum. The more the measurements, in an actual case, cluster round the nodes the further is the distribution from rectangular, the further the calculated value of $s^{2} / \delta^{2}$ will fall below $\frac{1}{3}$, and the greater becomes the likelihood that the quantum is real.

This is qualitatively obvious but it is possible to be more definite. Broadbent's criterion is

$$
C=\sqrt{ } n\left(\frac{1}{3}-s^{2} / \delta^{2}\right)
$$



Fig. 2.2. Test of a quantum hypothesis (after Broadbent).
Roughly it may be said that $C$ should be greater than unity. If it falls much below unity then the data lend no support to the idea that a quantum exists. A slightly more accurate idea may be obtained from Fig. 2.2, which is taken from Broadbent's second paper and shows approximate values for the probability level. For values of $C$ definitely above unity we can accept the hypothesis with confidence. For values near the line marked 'mean' the data do not support the hypothesis, neither do they indicate that the hypothesis is definitely wrong.

## Example

To illustrate the use of the above methods we shall apply them to examine the entirely imaginary data presented in the first column of Table 2.1. Here we have a set of twenty measurements ( $y$ ) which we wish to examine to see if they support a quantum of about $1 \frac{1}{2}$. We might, for example, think of these figures as being the measured lengths of the sides of bricks taken from an old building. The bricks were perhaps damaged or broken and had to be repaired before being measured. We might further assume that other similar buildings some distance away contained bricks having sides which were definitely
multiples of $1 \frac{1}{2}$. Our immediate problem is then to decide if the measurements in Table 2.1 show that our bricks belong to the same culture.

Table 2.1

| $y$ | $m$ | $\epsilon$ | $y$ | m | $\epsilon$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1 \cdot 4$ | 1 | $-0 \cdot 1$ | $7 \cdot 6$ | 5 | $+0 \cdot 1$ |
| $1 \cdot 5$ | 1 | 0.0 | 8.0 | 5 | +0.5 |
| $1 \cdot 7$ | 1 | $+0.2$ | $8 \cdot 6$ | 6 | -0.4 |
| $2 \cdot 8$ | 2 | $-0.2$ | 9.0 | 6 | 0.0 |
| $4 \cdot 5$ | 3 | 0.0 | $9 \cdot 1$ | 6 | +0.1 |
| $4 \cdot 7$ | 3 | $+0.2$ | $9 \cdot 5$ | 6 | +0.5 |
| 5.9 | 4 | $-0 \cdot 1$ | 9.9 | 7 | -0.6 |
| $6 \cdot 1$ | 4 | $+0 \cdot 1$ | $10 \cdot 4$ | 7 | -0.1 |
| $6 \cdot 4$ | 4 | $+0.4$ | $11 \cdot 8$ | 8 | -0.2 |
| $7 \cdot 4$ | 5 | $-0 \cdot 1$ | $12 \cdot 3$ | 8 | +0.3 |

$$
\begin{aligned}
n & =20 & \Sigma y & =138.6 \\
\sum m=92 & \Sigma y^{2} & =1171.5 & \sum_{\epsilon}=+0.6 \\
\Sigma m^{2} & =518 & \Sigma m y & =778.5
\end{aligned}
$$

From which

$$
\begin{array}{rlrlrl}
\Delta & =n \Sigma m^{2}-\left(\sum m\right)^{2} & & =1896 \\
2 \delta & =\left(n \Sigma m y-\sum m \sum y\right) / \Delta & & =1.487, & \sigma=0.007 \\
\beta & =\left(\sum m^{2} \Sigma y-\sum m \sum m y\right) / \Delta & & =+0.091 & \\
\text { Or with } \beta=0 \quad 2 \delta & =\Sigma m y / \Sigma m^{2} & & =1.503, & \sigma=0.012
\end{array}
$$

First we find the multiple of the assumed quantum ( $1 \frac{1}{2}$ ) which is nearest to each $y$. The necessary multipliers are $m$ and we enter these in column 2 . Then we find the deviations $(\epsilon)$ from the multiples, namely

$$
\epsilon=y-m \times 1 \frac{1}{2}
$$

The mean of the squares of $\epsilon$ is

$$
s^{2}=\Sigma \epsilon^{2} / n \text { or } 0 \cdot 075,
$$

where $n$ is the number of observations, namely 20 , so

$$
s^{2} / \delta^{2} \text { is } 0 \cdot 133 \text { where } \delta=\text { half quantum }=0.75
$$

Figure 2.1does not extend as low as $s^{2} / \delta^{2}=0.133$ but we see that with $n=20$ the probability level is less than 1 per cent, being perhaps about 0.2 per cent. So the hypothesis that there is a quantum of $1 \frac{1}{2}$ can be accepted at a probability level of this amount. In fact we can be reasonably certain that the bricks belong to the same culture as those of the other buildings.

We have in the above assumed that there is no constant $\beta$. Such a constant could only arise if somehow a constant amount had been added to or taken from each dimension.

A shrinkage of the brick, say on firing, would not produce such a constant since the STATISTICAL IDEAS
amount of shrinkage would be proportional to the size of the brick and so would affect the quantum $2 \delta$ but not $\beta$.

Let us now take the analysis further by finding the values of $2 \delta$ (the quantum) and $\beta$ which fit the measurements best.

The values of the various sums $\Sigma y, \Sigma y^{2}$, etc., are shown and from these we find as shown $2 \delta=1 \cdot 487$ and $\beta=+0 \cdot 091$. We can find the standard deviation of $2 \delta$ and $\beta$. So we obtain

$$
\begin{aligned}
2 \delta & =1 \cdot 487 \pm 0.007, \\
\beta & =+0.09 \pm 0.07 .
\end{aligned}
$$

Thus the measurements are represented by

$$
y=1 \cdot 487 m+0 \cdot 09,
$$

but since the standard deviation of $\beta$ is practically as large as $\beta$ itself we have no real justification for assuming $\beta$ to be anything else than zero.

We have assumed above that we had reason to expect a quantum of $1 \frac{1}{2}$ before we began the investigation. Now suppose we had no such prior information. Can we say from the twenty measurements that a quantum exists? We find Broadbent's criterion $C$ from

$$
C=\sqrt{ } n\left(\frac{1}{3}-s^{2} / \delta^{2}\right)
$$

to be about $0 \cdot 89$; and so Fig. 2.2 shows that the hypothesis may be accepted at a probability level of about 2 per cent. If we care to take the trouble we can form a new set of residuals from

$$
\epsilon=y-\beta-2 m \delta .
$$

Using the values of $2 \delta$ and $\beta$ found above leads to $\Sigma \epsilon^{2}=1.457$, from which $C=0.91$, a value which is only marginally different from that found above.

Chapter 2: Statistical Ideas. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford 1971:6-14.

## 3

## ASTRONOMICALBACKGROUND

(Thom 1971:15-26)
OMAR KHAYAM spoke of 'that inverted bowl we call the sky' and thereby suggested the best method of thinking of the over-all appearance of the heavens - stars painted on the inside of a transparent bowl. But we cannot draw on paper a representation of the inside of a bowl or hemisphere as seen from its centre. Accordingly astronomers prefer to draw the sphere from the outside and then to apply the trigonometry of the sphere to the necessary calculations of the positions of the stars relative to one another or to the horizon.

This approach will be found fully discussed in text-books on spherical astronomy. To understand what might be called descriptive spherical astronomy it is necessary to have a grasp of the astronomers' approach and also to study the actual appearance of the night sky and its movements.

Before going further it is necessary to clear up our ideas about what is meant by terms like declination, azimuth, altitude, etc.

The declination of a star can be thought of as the latitude of a point on the Earth immediately under the star, i.e. it is the latitude of an observer who finds the star once a night passing through his zenith. In Fig. 3.1 let $S$ be the star in the zenith of the observer at $A_{1}$. Then the star's declination is $\delta$. Twelve hours later the observer is at $A_{2}$ and the star (for the values depicted) is below the observer's horizon because the line joining the observer and the star passes through the Earth. For another observer at $B$ with a latitude of $\left(90^{\circ}-\delta\right)$ the star would be on the horizon. For an observer still further north the star would never set at all. It would be circumpolar.

The altitude of a star or of a terrestrial object is the angle of elevation to the star or object measured from the horizontal. The true altitude is the altitude of the straight line joining the observer and the star. But the light ray from the star is bent as it passes into and through the atmosphere and so the star appears higher than it really is. The amount of the bending is known as refraction, so that true altitude = apparent altitude-refraction.

The angle of refraction to a star is a function of its altitude and is at a maximum of about $0^{\circ} .6$ when the altitude is zero. The ray of light from a distant terrestrial point is also subject to refraction, known as terrestrial refraction, so if we wish to calculate the apparent altitude of an object such as a mountain top we must not only know its height above us and its distance but we must allow for the curvature of the Earth and for terrestrial refraction (see p.25).

The lowest position which the apparent horizon, as viewed from a given place, can occupy is that of the apparent sea horizon. The amount by which the sea horizon appears below the horizontal is called the dip and is given by

$$
\text { dip of sea horizon }=0.98 \sqrt{ } H,
$$

where the dip is in minutes of arc and $H$ is the observer's height in feet above sea level. It follows that if the altitude of any land horizon is calculated, or measured in poor visibility, and is found to have a maximum value which is lower than the dip the latter must be substituted with the sign changed and used in any calculation of, say, the moon's declination setting over the point concerned. In other words, in clear weather the sea would appear above the land.


Fig. 3.1.

The extinction angle of a star is the smallest apparent altitude at which, in perfectly clear weather, it can be seen. Below this altitude its light is always absorbed by the atmosphere, however clear. The value of the extinction angle in degrees is roughly equal to the magnitude of the star, so that a third-magnitude star cannot be seen below $3^{\circ}$ altitude. Only two stars, Sirius and Canopus, are bright enough to be seen down to zero altitude and of these only Sirius is visible in Britain.

The coordinates of a celestial body are normally referred to the Earth's centre. Looking at Fig. 3.1 it is evident that unless the body $S$ is infinitely distant the direction in which it is seen will only be correct if the body is in the observer's zenith, that is if it appears directly overhead and its altitude is $90^{\circ}$. The error called parallax makes the altitude appear too small and is a maximum when the observer is at B. The body is then on the
observer's horizon and the error is known as the horizontal parallax. For purposes of this book parallax can always be neglected except in the case of the moon, when it becomes very important.

The azimuth of a star or terrestrial object defines the direction in the horizontal plane in which we have to look to see the star or object. It is measured in the convention used in this book in degrees from north through east up to $360^{\circ}$.
Thus

|  | east becomes | $90^{\circ}$, |  |
| :--- | :--- | :--- | ---: |
| and | south | $"$ | $180^{\circ}$, |
| west | $"$ | $270^{\circ}$. |  |

It is important to remember that azimuth is measured clockwise from the true geographical north and not from magnetic north. The latter is quite useless for our present purpose as it changes from year to year, but often surveys of archaeological sites show a north point which is really magnetic without any indication that it is not the true north and without a date from which the variation or difference between the two can be deduced.

There is a definite trigonometrical relation between the four angles-azimuth, declination, latitude, and altitude. So knowing any three the other can be found. In this way the declination of a star seen to rise at a given point can be calculated provided we can find the azimuth and altitude of the point (see p. 17). The azimuth of a point is best found by observing by theodolite the difference in azimuth between the point and the sun at a noted time. Later the azimuth of the sun at the noted time can be calculated, again by the spherical triangle, and so the azimuth of the point can be found. Once the azimuth of one point is obtained that of any other observed from the same position is easily determined.

For anyone who does not want to follow through the foregoing in detail it is perhaps easier to look at Fig. 3.2. This is an imaginary picture of the western horizon as seen from these latitudes. An attempt has been made to show the apparent movements of the stars as the night progresses. Each star will be seen to move along one of the lines according to its declination. A vertical erected in the north ( N ) on the inside of the bowl will pass through the pole to the zenith and so to the south point at $S$. This line or rather great circle is the meridian and shows the position where each star reaches its greatest altitude if it is in the south or its east altitude if it is a circumpolar star crossing the meridian in the north below the pole. A star of zero declination will be seen to travel down the thick line to set in the west and it will set in the west in any latitude. Stars with negative declinations will set between west and south and stars with positive declinations will set, if they set at all, between west and north. Since the diagram is prepared for latitude $55^{\circ}$ N. a star with a declination greater than $\left(90^{\circ}-55^{\circ}\right)$ or $35^{\circ}$ will be circumpolar and will never set.

At midsummer the sun has its maximum declination and will appear to move along the upper dotted line at declination, $-24^{\circ}$ to set in the north-west. Similarly at midwinter it moves along the lower dotted line at declination - $24^{\circ}$ to set in the south-west.


Fig. 3.2. Aspect of western sky -2000 b.c.-lat. $55^{\circ} \mathrm{N}$.
Looking at the apparent movements of a close circumpolar star we see that once a day it reaches a position when its azimuth is a minimum. It is then said to be at its western elongation. Roughly twelve hours later it is on the other side of the pole and when its azimuth is a maximum it is said to be at eastern elongation. An azimuth midway between these positions is of course due north.

The figure is drawn to represent roughly the state of affairs at 2000 B.C. Orion, seen setting, was further south than it is now and consequently it was a shorter time above the horizon. The same remark applies to Sirius but the change in declination for Sirius has not been so great. These changes are not primarily due to movements of the stars themselves but are due to the precession of the equinoxes, a phenomenon which will be discussed later. The drawing shows that when the declination of a star is known its setting point can be found. Conversely, if the setting point is known the declination can be found. The setting point is defined by azimuth and altitude. The star Aldebaran with declination almost exactly zero sets in the west at azimuth $270^{\circ}$ on a low horizon, but had the mountain shown been a little further to the right the setting point might have been as low as $265^{\circ}$

The relation between declination, azimuth, and altitude which we require in this book is

$$
\sin \delta=\sin \lambda \sin h+\cos \lambda \cos A \cos h \cos A
$$

where

$$
\delta=\text { declination, } \quad h=\text { horizon altitude (true), }
$$

$$
\lambda=\text { latitude, } A=\text { azimuth. }
$$

Since in most cases an accuracy of $\pm 0^{\circ} \cdot 1$ is sufficient; values of the declination can be taken from Table 3.1 by interpolation in the section for the latitude.
Table 3.1. Declination in terms of azimuth, altitude, and latitude

| Latitude $=60^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Amp. | Decl. | Add for $+1^{\circ}$ Lat. | Add for $+1^{\circ} \mathrm{Alt}$. |
| $0^{\circ}$ | $0.00^{\circ}$ | $0.00^{\circ}$ | $+0.87^{\circ}$ |
| 5 | 2.50 | -0.08 | 0.87 |
| 10 | 4.98 | -0.15 | 0.87 |
| 15 | $7 \cdot 44$ | -0.23 | 0.87 |
| 20 | $9 \cdot 85$ | -0.30 | 0.88 |
| 25 | 12.20 | $-0.38$ | 0.89 |
| 30 | 14.48 | -0.45 | 0.89 |
| 35 | 16.67 | -0.52 | 0.90 |
| 40 | 18.75 | -0.59 | 0.91 |
| 45 | 20.70 | -0.66 | 0.92 |
| 50 | 22.52 | -0.72 | 0.94 |
| 55 | $24 \cdot 18$ | -0.78 | 0.95 |
| 60 | 25.66 | -0.83 | 0.96 |
| 65 | 26.95 | -0.88 | 0.97 |
| 70 | 28.02 | -0.92 | 0.98 |
| 75 | 28.88 | -0.96 | 0.99 |
| 80 | 29.50 | -0.98 | 0.99 |
| 85 | 29.87 | $-1.00$ | $1 \cdot 00$ |
| 90 | 30.00 | $-1.00$ | 1.00 |


| Latitude $=50^{\circ}$ |  |  |  | Latitude $=55^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Amp. | Decl. | Add for $+1^{\circ}$ Lat. | Add for $+1^{\circ} \mathrm{Alt}$ | Amp. | Decl. | Add for $+1^{\circ}$ Lat. | Add for $+1^{\circ} \mathrm{Alt}$. |
| $0^{\circ}$ | $0.00^{\circ}$ | $0.00^{\circ}$ | $+0.77^{\circ}$ | $0^{\circ}$ | $0.00^{\circ}$ | $0.00^{\circ}$ | $+0.82^{\circ}$ |
| 5 | 3.21 | -0.07 | 0.77 | 5 | 2.87 | -0.07 | 0.82 |
| 10 | 6.41 | -0.14 | 0.77 | 10 | $5 \cdot 72$ | -0.14 | 0.82 |
| 15 | $9 \cdot 58$ | $-0.20$ | 0.78 | 15 | 8.54 | -0.22 | 0.83 |
| 20 | 12.70 | -0.27 | 0.78 | 20 | 11.31 | -0.29 | 0.84 |
| 25 | 15.76 | -0.34 | 0.79 | 25 | 14.03 | -0.36 | 0.84 |
| 30 | 18.75 | -0.41 | 0.81 | 30 | 16.67 | -0.43 | 0.85 |
| 35 | 21.63 | -0.48 | 0.82 | 35 | 19.21 | $-0.50$ | 0.87 |
| 40 | 24.40 | -0.55 | 0.84 | 40 | 21.63 | -0.57 | 0.88 |
| 45 | 27.03 | -0.61 | 0.86 | 45 | 23.93 | -0.64 | 0.89 |
| 50 | 29.50 | -0.68 | 0.88 | 50 | 26.06 | -0.70 | 0.91 |
| 55 | 31.77 | -0.74 | 0.90 | 55 | 28.02 | -0.76 | 0.93 |
| 60 | $33 \cdot 83$ | -0.80 | 0.92 | 60 | 29.78 | -0.82 | 0.94 |
| 65 | $35 \cdot 63$ | -0.86 | 0.94 | 65 | 31.32 | -0.87 | 0.96 |
| 70 | 37-16 | -0.91 | 0.96 | 70 | $32 \cdot 61$ | -0.92 | 0.97 |
| 75 | $38 \cdot 38$ | -0.95 | 0.98 | 75 | 33.64 | -0.95 | 0.98 |
| 80 | $39 \cdot 27$ | -0.98 | 0.99 | 80 | $34 \cdot 39$ | -0.98 | 0.99 |
| 85 | 39:82 | -0.99 | 1.00 | 85 | 34.85 | -0.99 | 1.00 |
| 90 | $40 \cdot 00$ | $-1.00$ | 1.00 | 90 | 35.00 | $-1.00$ | 1.00 |

[^0]nearest to that of the site. The value must then be corrected for the latitude difference and for the true altitude according to the rules given on the table.

## The position of the Earth's orbit

The orbit which the Earth describes about the sun is an ellipse with the sun in one of the foci but as far as appearances go we can say that it is the sun which describes an elliptical orbit about us. In this section we shall look at the changes which take place in the position of the plane which contains the orbit. Later it will be necessary to consider the changes which take place in the ellipse itself and in its position in the plane.


Fig. 3.3.

Let $A A A A$ (Fig. 3.3) be the plane of the Earth's orbit. It is thus the plane in which the sun and the Earth lie throughout the year and so when we look at the sun our line of sight lies in this plane, which is called the ecliptic. The plane of the equator is $B B B B$ intersecting the ecliptic in the line PQ, which consequently lies in both planes. It is much easier to draw these planes on a sphere-the celestial sphere. This sphere is centred at $O$, the observer's position on the line $P Q$. The ecliptic and the equator then become great circles on the sphere, these circles being the circles in which the planes cut the sphere. The sun at the vernal equinox will be seen along the line $O P$, which is simply designated by oe, the first point of Aries. As the spring advances the sun appears to move along the ecliptic in an anticlockwise direction till at midsummer it is at $M$. Its declination is then a maximum and equal to the angle between the planes-the so-called obliquity of the ecliptic (e).

The Earth turns on its axis $O N$, which is at right angles to the equator. $N$ is the north pole of the celestial sphere, near to the present-day position of the pole-star. The axis $O N$ precesses like the axis of a spinning top and so describes a cone. Thus $N$ moves clockwise round the small circle $N N^{\prime}$ taking about 25,000 years to go round once. Since N is the pole of the equator the equator moves with it causing $\gamma$ to move slowly along the ecliptic.

After a few centuries $\Upsilon$ will have moved to $\Upsilon^{\prime}$ and the equator will have moved into the dotted position. Since the ecliptic remains fixed in space the equator is continuously changing its position relative to the stars. But we measure declination from the equator so the declinations of the stars are steadily changing. We have seen that a star's rising point is fixed by its declination. This means that the rising point of a given star slowly moves along the horizon through the ages. The change at a given time is more rapid for some stars than for others depending as it does on the star's position on the celestial sphere. It will be evident that for a star near $M$ the change will be slower than for a star near $\Upsilon$.

The obliquity of the ecliptic has been decreasing slowly for a very long time. The decrease is so slow that in 10000 years it only amounts to about a degree. Probably the best modern determination of the obliquity is by de Sitter (1938) and his formula yields

| 2000 B.C. | $23^{\circ} .9292$ |
| :--- | :--- |
| 1700 B.C. | $23^{\circ} .8969$ |
| 1000 B.C. | $23^{\circ} .8175$ |
| A.D. 1900 | $23^{\circ} .4523$ |

## The moon's orbit

The moon describes an orbit round the Earth inclined to the ecliptic at an angle which varies periodically by a small amount (about $0^{\circ} .15$ ) from the mean value of $I=5^{\circ} \cdot 15$. Astronomers believe that this mean value has remained constant for many thousands of years. If we substitute 'moon's orbit' for 'equator' in Fig. 3.3 it can be used to explain the terms. The line $P Q$ is now called the line of nodes. This line, like the equinoctial line, is also moving round in the ecliptic but much more rapidly. It completes a circuit in 18.6 years. This rotation of the line of nodes has an important effect on the position of the full moon in the sky. When we face the full moon the sun is at our back below the horizon, lighting the side of the moon at which we are looking. So the full moon is always diametrically opposite the sun. It follows that if the inclination of the moon's orbit to the ecliptic were zero the full moon would always be eclipsed since the Earth would be directly between the sun and the moon. As things arc, the moon can only be eclipsed when it is near the ecliptic, i.e. when it is near the line of nodes. It also follows that at midwinter when the sun is at its lowest declination the full moon is at its highest declination and so is giving us the greatest and longest illumination. Just how high it then is depends on the position of the line of nodes. We shall consider only the extreme conditions which occur at the solstices when the line of nodes is along the equinoctial line, i.e. when one end or the other is at the first point of Aries. The two cases are shown in Fig. 3.4, where we are supposed to be looking along the line of intersection of the three planes - the ecliptic, the equator, and the moon's orbit. Each plane then becomes a line.

The lunar orbit can be either at $L L$ or at $K K, 5^{\circ} .15$ on either side of the ecliptic. When the orbit is at $L L$ the moon as it goes round in this plane can attain a maximum declination of $\angle C O L$, i.e. the sum of the obliquity of the ecliptic and the inclination of the lunar orbit, or about $29^{\circ}$. On the other hand $9 \cdot 3$ years later the orbit is at $K K$ and the maximum declination is the difference of the two angles, about $19^{\circ}$. So, for a midwinter full moon, the extremes of declination are $+19^{\circ}$ and $+29^{\circ}$. Similarly at a midsummer full moon the declination lies between $-19^{\circ}$ and $-29^{\circ}$.

One may ask in what way these changes in the position of the moon's orbit throughout the nineteen-year cycle would make themselves apparent. For a community whose only effective illumination during the long winter nights was the moon perhaps the most important apparent change would be that the midwinter full moon's altitude on the meridian varied from about $57^{\circ}$ to $67^{\circ}$ ( latitude $52^{\circ}$ ) with a lengthening of the time the full moon was above the horizon of some $2 \frac{1}{2}$ hours. A difference which is evident to all but the most unobservant is that of the maximum altitude of the midsummer full moon, which in latitude $55^{\circ} \mathrm{N}$. varies from $16^{\circ}$ to $6^{\circ}$. Even in the south of England the change from $20^{\circ}$ to $10^{\circ}$ is very obvious.

But transcending these phenomena in importance lay the challenge of the eclipse. To early man the eclipse of the sun or of the moon must have been an impressive spectacle and a desire to master eclipse prediction probably motivated Megalithic man's preoccupation with lunar phenomena. Since eclipses happen only when the moon is at a node it would soon have become apparent that no eclipse occurred near the solstices when the full moon was in either of its extreme positions but only in the years which lay midway between these.


Fig. 3.4.
To understand some of the lunar sites dealt with in Chapters 11 and 12 it is necessary to examine the moon's motion in greater detail and to illustrate it by showing the actual changes of declination near the maximum.


Fig. 3.5. Behaviour of moon's declination.
In Fig 3.5 (a) the limiting values of the declination are shown throughout one revolution of the nodes. The declination rises every lunar month to the upper line and falls to the lower. This is illustrated by plotting the actual declination during the summer of A.D. 1950, for which of course full particulars were available (Fig. 3.5 (b)). In Fig 3.5 (a) these oscillations are crowded so closely together that they could not be plotted. But neither

Fig. 3.5 (a) nor Fig. 3.5 (b) could show the small oscillation of $\pm 9$ which is brought out in Fig. 3.5 (c) by greatly increasing the declination scale and using a time scale which allows several years to be included. In this figure each dot represents one of the upper peaks of Fig. 3.5(b). The mean dotted line is a small portion of the top of the upper line in Fig. 3.5(a) and so has a period of 18.6 years. Superimposed on this we see the small $\pm 9$ ' oscillation already mentioned.

Every $346 \cdot 62$ days the sun comes round to the ascending node of the lunar orbit. This is consequently known as the length of the 'eclipse year' because eclipses can only happen when the sun is near one of the nodes. But there are two nodes, the ascending and the descending, and so the time taken from one to the other is half an eclipse year or $173 \cdot 31$ days. This is the period of the small oscillation seen in Fig. 3.5 (c).

The lunar declination reached one of its maxima in 1950 but for several months the mean value shown by the dotted line changed very little. Consequently two or three waves of the small oscillation would be clearly observable. These would show up in the movement of the setting moon along the horizon, especially in northern latitudes, where the path of the setting moon at its lowest declination makes a very small angle with the horizon (e.g. see Fig. 11.4). We shall see that Megalithic man understood very clearly the advantage in sensitivity of observing a glancing phenomenon of this kind and so it was quite possible for him to have observed these two or three oscillations around the maximum or minimum positions of the moon. Evidence will be given that he did observe this phenomenon, but to be able to assess this evidence it is necessary to understand clearly what happens when the moon is in one of the limiting positions. This is the reason why we have gone into the matter in some detail.

## Earth's orbit

Just as on the Earth's surface a point can be located by giving its latitude and longitude so a point in the heavens can be specified by two coordinates called by the same names. But when an astronomer speaks of (celestial) latitude and longitude he is thinking of coordinates which while similar in conception to terrestrial latitude and longitude refer to entirely different planes or axes. Celestial longitude is measured along the ecliptic anticlockwise from the first point of Aries $(\Upsilon)$ and latitude is measured from the ecliptic towards its poles. When the Earth is at the point in its elliptic orbit nearest to the sun it is said to be at perihelion. Today the longitude of perihelion is about $102^{\circ} \cdot 3$. So the sun's longitude at perigee (i.e. when it is nearest the Earth) is this increased by $180^{\circ}$, or $282^{\circ} \cdot 3$. This occurs in the first week in January so that the Earth's speed in its orbit, obviously a maximum at perihelion, is greater in the winter months than in the summer. It follows that the interval between the autumnal equinox $\left(I=180^{\circ}\right)$ and the vernal the vernal equinox ( $I=0^{\circ}$ ) is shorter by some $7 \frac{1}{2}$ days than the summer half of the year. This has not always been so. In 4040 B.C. perihelion occurred at the autumnal equinox so that the winter and summer halves of the year were equal.

Astronomers speak of the longitude of the dynamic mean sun, meaning thereby the longitude of an imaginary body which moves round the ecliptic with a speed uniform and equal to the mean speed of the actual sun in a year. Due to the varying speed of the Earth in its orbit the sun is sometimes ahead and sometimes behind the mean sun. This is expressed mathematically by

$$
\odot=1+2 \operatorname{esin}(l-\pi),
$$

where

$$
\begin{aligned}
& \odot=\text { longitude of sun }, \\
& l=\text { longitude of the dynamic mean sun }, \\
& \pi=\text { longitude of sun at perigee }, \\
& e=\text { eccentricity. }
\end{aligned}
$$

Knowing the longitude of the sun we can calculate its declination from:

$$
\sin \delta=\sin \odot \sin e,
$$

where

$$
e=\text { obliquity of ecliptic. }
$$

## Azimuth and altitude from Ordnance Survey maps

When an Ordnance Survey map has the national grid superimposed it shows in the margin the angle between grid north and true north for definite positions on the sheet. Hence if the azimuth of a line is calculated, or measured, with respect to the grid it can be reduced to true north by applying the correction obtaining at the observer's end of the line. Alternatively one can obtain formulae and tables for finding the azimuth of a line joining two points when the grid coordinates of the points are known.

The seventh series of the 1 -in O.S. maps have, in addition to the national grid, the intersection points of latitude and longitude marked by a cross at 5 ' intervals. Using these or otherwise it is possible to obtain the latitude and longitude to about one second of arc. Using the $2 \frac{1}{2}$-in or the 6 -in maps the coordinates can be obtained to a fraction of a second. Given then two points probably on different sheets the following formulae will give the required azimuth and distance. The distance will be necessary in the calculation of angles of altitude.

Let $\lambda_{c}, L_{c}$ be the latitude and longitude of the observer at C and $\lambda_{c}, L_{c}$ be the same coordinates for the observed point D .

$$
\begin{aligned}
& \Delta \lambda_{\mathrm{c}}=\lambda_{\mathrm{d}}-\lambda_{\mathrm{c}} \\
& \Delta L=L_{\mathrm{a}}-L_{\mathrm{e}} \text { (east longitude reckoned positive), } \\
& \lambda_{\mathrm{m}}=\frac{1}{2}\left(\lambda_{\mathrm{d}}+\lambda_{\mathrm{c}}\right)=\text { mean latitude }, \\
& A=\text { required azimuth measured clockwise from north. }
\end{aligned}
$$

Then find $\tan B$ from

$$
\tan B=K \cos \lambda_{\mathrm{m}} \Delta L / \Delta \lambda,
$$

which gives $B$. Find $\Delta A$ from $\quad \Delta A=\Delta L \sin \lambda_{\mathrm{m}}$
and the required azimuth of $D$ from $C$ is $A=B-\frac{1}{2} \Delta A$.
If the Earth were a sphere $K$ would be unity. To allow for the fact that the Earth is not a sphere but approximately an oblate spheroid $K$ can be taken as varying from 1.0028 in latitude $50^{\circ}$ to 1.0017 in latitude $60^{\circ}$.

The distance CD in statute miles is obtained with sufficient accuracy from:

$$
c=C D=0.01922 \Delta \lambda / \cos B \text { or } 0.01926 \Delta L \cos \lambda_{\mathrm{m}} / \sin B .
$$

It is often necessary to calculate the apparent angle of altitude of one point $D$ as seen from another point $C$ in terms of the distance $c$ between the points and the amount by which the height of $D$ exceeds that of $C$. It is necessary to take account of the curvature of the Earth and of the refraction which bends the ray between $D$ and $C$. Both of these effects are taken into account with sufficient accuracy for most purposes in the formula

$$
h=H / c-c(1-2 k) / 2 R,
$$

where

$$
\begin{aligned}
& H=\text { height of } D \text { above } C, \\
& c=\text { distance of } D \text { from } C, \\
& R=\text { radius of curvature of the spheroid, } \\
& k=\text { coefficient of refraction. }
\end{aligned}
$$

Commonly used values for $k$ are 0.075 for rays passing over land and 0.081 over the sea.
If $H$ is expressed in feet, $c$ in statute miles, and $h$ in minutes of arc then the above formula becomes approximately:

$$
h=0.65 H / c-0.37 c .
$$

It must be remembered that the refraction of a ray near to a land or water surface is liable to be considerably affected by the steep temperature gradients (with height) which may exist.

It is as well to be quite clear about the difference between astronomical refraction and terrestrial refraction. The ray of light from a star is refracted along the whole length of its track through the atmosphere. The total effect on the altitude of the star is the astronomical refraction. Suppose now that the ray passes close to a mountain top before it reaches the observer, then the deflexion which it suffers after the mountain top produces terrestrial refraction. The effect is to make the mountain top appear too high by an angle of $k c / R$, while the curvature of the earth makes it appear too low by $c / 2 R$. Hence the above formula.

The astronomical refraction used in this work is indicated by the values:
Apparent altitude: $\begin{array}{cccccccc}-\frac{1}{2} & 0^{\circ} & 1^{\circ} & 2^{\circ} & 3^{\circ} & 5^{\circ} & 10^{\circ}\end{array}$
Refraction: $\quad 40^{\prime} \quad 33^{\prime} \quad 24^{\prime} 18^{\prime} 14^{\prime} 10^{\prime} \quad 5^{\prime}$

Normally the apparent altitude of the mountain top would be measured and the altitude of a star on the mountain top would be found by deducting the astronomical refraction. But if the altitude of the mountain top is to be found by calculation what we require is the apparent altitude and this is found by the above formula.

Chapter 3: Astronomical Background. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford, 1971:14-26.

## 4

## MATHEMATICAL BACKGROUND

(Thom 1971:27-33)
As we shall see later the builders of the circles, rings, alignments, etc., had a remarkable knowledge of practical geometry. In this chapter we shall set out in modern terminology some of the ideas which they developed and show how their constructions can be analyzed. They were intensely interested in measurements and attained a proficiency which as we shall see is only equaled today by a trained surveyor. They concentrated on geometrical figures which had as many dimensions as possible arranged to be integral multiples of their units of length. They abhorred 'incommensurable' lengths. This is fortunate for us because once we have established their unit of length we can very often unravel designs which would otherwise be meaningless. These people also measured along curves and so it is necessary to devote some space to the methods of calculating the perimeters of the various rings which they developed.

The basic figure of their geometry, as of ours, is the triangle. Today everyone knows the Pythagorean theorem which states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. We do not know if Megalithic man knew the theorem. Perhaps not, but he was feeling his way towards it. One can almost say that he was obsessed by the desire to discover and record in stone as many triangles as possible which were right-angled and yet had all three sides integers. The most famous of the so-called Pythagorean triangles is the 3, 4, 5 -right-angled because $3^{2}+4^{2}=5^{2}$. He used this triangle so often that he may well have noticed the relation. Limiting the hypotenuse to 40 there are six true Pythagorean triangles. These are:
(1) $3,4,5$
(4) $7,24,25$
(2) $5,12,13$
(5) $20,21,29$
(3) $8,15,17$
(6) $12,35,37$

Megalithic man knew at least three of these. He may have known all six and we simply have not yet found the sites where they were used, but we shall see later that there were other conditions to be fulfilled and these certainly restricted the use of some of these triangles. The remarkable thing is that the largest, the $12,35,37$, was known and exploited more than any other with the exception of the $3,4,5$.

But Megalithic man used many close approximations to Pythagorean triangles. For
example, he used the triangle $8,9,12$, but $8^{2}+9^{2}$ is 145 and $12^{2}$ is 144 . The error in the hypotenuse is only 1 in 300 , which he probably accepted because the triangle as he used it gave as we shall see a suitable perimeter to the ring which he based on it. Some of his approximations were worse than this and some very much better. When he used a poor value it was not because he believed it to be perfect but because other conditions had to be fulfilled.

## Flattened circles



Fig. 4.1. Flattened circle. Type A


Fig. 4.2. Flattened circle. Type B.

In many places flattened circles were used of two very definite types. So far thirty or so have been found but there were probably many more, some of which may yet be located.
The construction and geometry of these rings is shown in Figs. 4.1 and 4.2. To draw a Type A ring set out a circular arc of $240^{\circ} \mathrm{CMANG}$. The angle COA is easily constructed by making two equilateral triangles as shown. This makes the required $120^{\circ}$. Bisect $O C$ at $E$. Then $E$ is the centre for the $\operatorname{arc} C D$. The remaining flat arc $D B H$ is drawn with centre at $A$. To calculate $\pi^{\prime}$, the ratio of the perimeter to the diameter, take, for easy calculation, the radius $O C$ to be 4. Then $O E=2$ and since the angle $E O F$ is $60^{\circ}, E F=$ $\sqrt{ } 3$ and $O F=1$. Also $\tan \theta=E F / F A=(\sqrt{ } 3) / 5$ which makes $\theta$ in radians equal to $0 \cdot 33347$. $\beta=\pi / 3-\theta, A E=2 \sqrt{ } 7, A B=2+2 \sqrt{ } 7$. From these we can deduce that

$$
\begin{gathered}
\text { perimeter } / M N=\pi^{\prime}=\frac{5}{6} \pi+\frac{1}{2} \sqrt{ } 7 \times \theta=3.0591, \\
A B / M N=0.9114 .
\end{gathered}
$$

The construction of a Type B ring is easier. Divide the diameter $M N$ into three equal parts at $C$ and $E$. These are the centres for the small arcs. The flat closing arc is, as in

Type A, struck with centre at $A$. Making the calculations as before leads to:

$$
\text { perimeter } / M N=\pi^{\prime}=\frac{5}{6} \pi+\frac{1}{3} \sqrt{ } 10 \theta=2.9572
$$

where
and

$$
\tan \theta=\frac{1}{3}
$$

$$
A B / M N=0 \cdot 8604 .
$$

We find one or two sites where a slight modification to the above types has been used. At two sites a Type A construction was used but $O E$ was made equal to one-third of $O C$ instead of one-half. This can be called Type D . $\pi^{\prime}$ for Type D is 3.0840 and $A B / M N=$ 0.9343 . At one site Type B has been modified by making $O C=C M$. This modification makes $\pi^{\prime}=2.8746$ and reduces the diametral ratio to $0 \cdot 8091$.

## Egg-shaped rings

Ten sites are known with these peculiarly shaped rings. They can be classified into two types both of which are based on a Pythagorean or near Pythagorean triangle. In Type I (Fig. 4.3) two of these triangles are used placed base to base at $A B$. A semicircle is drawn


Fig. 4.3. Egg-shaped circle. Type I


Fig. 4.4. Egg-shaped circle. Type II.
with centre at $A$, an arc $E F$ is drawn with centre at $D$, and the pointed end of the egg is drawn with centre at $B$. The result of using triangles which have all sides integral is that, provided the semicircle has an integral radius, then all the other radii must also be integral. Any given Pythagorean triangle can be used in two ways depending on which side is chosen as the base and in fact we find the $3,4,5$ triangle turned both ways, but once the triangles are arranged the size and shape of the egg can still be varied infinitely by choosing different integers for the radius of the semicircle.

In Type II (Fig. 4.4) the triangles are placed together with a common hypotenuse. The arcs at each end are drawn with centres at the ends of this hypotenuse and joined by
straight lines parallel to the side of the triangle. As in Type I if one radius is integral so is the other.

The perimeter of a Type I egg can be found as follows. Referring to the figure, let

$$
\begin{aligned}
& r_{1}=\text { radius of the large end, } \\
& r_{2}=\#, ~ \#, ~ s m a l l ~ e n d, ~ \\
& r_{3}=\#, \# \text { arc } E F .
\end{aligned}
$$

Evidently $r_{3}=r_{1}+b$, but it is also equal to $r_{2}+a$, from which $r_{1}-r_{2}=a-b$.
The half-perimeter can obviously be written:

$$
\frac{1}{2} P=r_{1} \frac{1}{2} \pi+r_{3} \theta+r_{2} \beta,
$$

from which, with the above relations, we find

$$
\begin{gathered}
P=2 \pi r_{1}+\pi b-2 a \beta, \\
\tan \beta=b / c .
\end{gathered}
$$

$\beta$ being obtained from
Similarly we find that the perimeter of a Type II egg is given by

$$
P=2 \pi r_{1}+2 c-2 \theta \beta,
$$

$\theta$ being obtained from

$$
\tan \theta=c / b .
$$

## The ellipse

The earliest known study of the properties of the sections of a cone, of which the ellipse is one, seems to have been made by Menaechmus in the middle of the fourth century


Fig. 4.5. Ellipse drawn by rope. before Christ (Heath, 1921) but the ellipse may have been known to earlier Greeks. Our forefathers early in the second millennium B.C. were laying out ellipses but their approach was much simpler. Almost certainly their ellipses were set out either with a loop of rope round two stakes or with a rope tied to two stakes ( $F_{1}$ and $F_{2}$ in Fig. 4.5). In either method a third stake round which the rope could slide would be used to scribe the curve on the ground. The fixed stakes were at the points $F_{1}$ and $F_{2}$ and the stake at $P$ was moved round keeping the rope $F_{1} P F_{2}$ always tight. $F_{1}$ and $F_{2}$ are known as the foci, $D_{1}, D_{2}$ and $E_{1}$ and $E_{2}$ are the major and minor axes. The ratio $F_{1} F_{2} / D_{1} D_{2}$, that is $2 c / 2$ a, is known as the eccentricity (e). As the eccentricity gets smaller and smaller the ellipse gets nearer and nearer to being a circle. So we can regard an ellipse as being a circle with two centres.

A circle has a constant radius but an ellipse has the average of the two lengths $F_{1} P$ and $F_{2} P$ constant.

When Megalithic man set out a circle with a diameter of 8 units he found the circumference very nearly 25 units but in general he could not get nice whole numbers like these for both the diameter and the circumference simultaneously. Probably the attraction of the ellipse, and we know of over 30 set out by these people, was that it had an extra variable ( $F_{1} F_{2}$ ) and so it was easier to get the circumference near to some desired value. But the ellipse has, as it were, two diameters, the major and minor axes. How is it possible to get both of these and at the same time the focal distance $F_{1} F_{2}$ all integral? Looking at Fig. 4.5 we see that $F_{2} D_{2}$ is equal to $F_{1} D_{1}$. When $P$ is at $D_{2}$ the total length of the rope is $F_{1} D_{2}+F_{2} D_{2}$ and so is $F_{1} D_{2}+F_{1} D_{1}$ which is the major axis (2a). That is, half the length of the rope is the semi-axis major (a). So when $P$ is at $E_{1}$ we see that $F_{1} E_{1}$ is equal to $a$. Thus $a, b$, and $c$ are the sides of a right-angled triangle and if the triangle is Pythagorean we can have the major axis, the minor axis, and the focal distance all integral. Just as for the egg-shaped rings so for the ellipses it was desirable to start with a Pythagorean triangle. For both eggs and ellipses Megalithic man had a further very difficult task, namely, to get the perimeter integral. To be able to examine his success we must be able to calculate accurately the perimeters of his figures. We have seen how this can be done for the flattened circles and for the eggs.

Table 4.1. Perimeter of ellipse in terms of b/a. $2 \mathrm{a}=$ major axis, $\mathbf{2 b}=\boldsymbol{m i n o r}$ axis, P = perimeter

| $b / a$ | $P / 2 a$ | $b / a$ | P/2a | $b / a$ | P/2a |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.30 | $2 \cdot 1930$ | 0.54 | 2.4733 | 0.78 | 2.8067 |
| 0.32 | $2 \cdot 2135$ | 0.56 | 2.4994 | 0.80 | 2.8361 |
| 0.34 | $2 \cdot 2346$ | 0.58 | 2.5259 | 0.82 | 2.8658 |
| 0.36 | 2.2563 | 0.60 | 2.5527 | 0.84 | 2.8957 |
| 0.38 | 2.2786 | 0.62 | $2 \cdot 5798$ | 0.86 | 2.9258 |
| $0 \cdot 40$ | $2 \cdot 3013$ | 0.64 | 2.6072 | 0.88 | $2 \cdot 9561$ |
| 0.42 | 2.3246 | 0.66 | $2 \cdot 6349$ | 0.90 | 2.9865 |
| 0.44 | 2.3483 | 0.68 | 2.6629 | 0.92 | 3.0172 |
| 0.46 | $2 \cdot 3725$ | 0.70 | 2.6912 | 0.94 | 3.0481 |
| 0.48 | 2.3971 | 0.72 | 2.7197 | 0.96 | 3.0790 |
| 0.50 | 2.4221 | 0.74 | 2.7484 | 0.98 | 3-1101 |
| 0.52 | 2.4475 | 0.76 | 2.7774 | 1.00 | $3 \cdot 1416$ |

For the ellipse we require special tables. In the absence of these we can use Table 4.1 which gives the ratio $\pi^{\prime}$ of the perimeter to the major axis for values of b/a between 0.30 and 1.00 advancing by intervals of 0.02 . Simple linear interpolation will give $\pi^{\prime}$ for any intermediate value of $b / a$ without any appreciable error in the fifth figure.

A piece of thread and two drawing-pins can be used on a drawing-board to construct an ellipse but this is not a very satisfactory method. One of the best methods is that shown in Fig. 4.6. Two circles are drawn, the diameters being the major and minor axes. A radial line is drawn cutting the circles at $C$ and $B$. Lines parallel to the axes are drawn through $C$ and $B$ and where these meet is a


Fig. 4.6. Ellipse drawn on drawing board. point on the required ellipse. Thirty or forty radial lines can be drawn without any confusion and so an accurate ellipse can be completed.

The accuracy with which some of the large rings are set out shows it to be unlikely that a rope was used for the actual measurements although it must have been used in scribing the quadrants of the ellipses. The most accurate method available to these people was that still used today in a more sophisticated manner in measuring short base lines. This involves the use of two rods $A$ and $B$ each of a known length. $A$ and $B$ are laid down end to end and carefully levelled. Then $A$ is lifted over $B$ and again laid down touching $B$ at the other end. In this way moving $A$ and $B$ alternately the required length is set out.

One is entitled to ask what error would be introduced where this method was used with straight rods when measuring round a curve. Each rod in fact forms the chord of a short arc and we require to know the difference in length between the arc and the chord. Approximately the difference is given by

$$
\text { arc minus chord }=c^{3} / 24 R^{2} \text {, }
$$

where $c=$ length of the chord and $R=$ radius of the curve. As an example consider a circle of diameter 8 units measured with a rod lunit long. There are roughly 25 chords in the circle and the error in each is $1 / 24 \times 16$. So the total accumulated error is $25 / 384$ or 0.065 units. This error 1 in 384 ought with careful work to be just appreciable, but one would need level ground and carefully prepared supports for each rod. To detect the difference between the actual circumference of the circle, namely $8 \pi$ or $25 \cdot 133$, and that determined by the rods, $25 \cdot 065$, ought to be just possible.

## Number of lengths at a site which can be integral

To fix the position of $n$ points relative to one another by linear measurements we need $2 n-3$ lengths but the number of lines which can be drawn joining the points is $n(n-1) / 2$. Thus we can connect four points with five lines so that all are fixed relative to one another but we can still draw another line and the length of this line can be calculated from the length of the original five. We can say that only five are disposable.

So we get:

| $\boldsymbol{n}$ <br> points | Disposable <br> lengths | Possible <br> Lengths |
| :--- | :--- | :--- |
| 1 | 1 | 1 |
| 3 | 3 | 3 |
| 4 | 5 | 6 |
| 5 | 7 | 10 |
| etc. |  |  |

It follows that at a site with five circles only seven of the ten possible distances between the circles can be expected to be integral multiples of a unit length. An exception would be if the circles were in a straight line.

Chapter 4: Mathematical Background. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford 1971:27-33.

## 5

## MEGALITHICUNITOFLENGTH

(Thom 1971:34-53)
TODAY we use the yard as a standard unit of length. The word yard meant originally a rod of wood or a stick. The French verge has the same meaning and the Spanish word vara shows that this old length unit also meant originally a rod. In all three measures the idea was the same: the unit of length was carried about as a rod of wood just as today we carry a foot-rule or a metre stick. For our present discussion the most interesting is the vara, which has the following values in feet.

| 2.766 | Burgos | [Szymański, 1956] |
| :--- | :--- | :--- |
| 2.7425 | Madrid |  |
| 2.749 | Mexico | [W. Latto \& W. S. Olsen, private communication] |
| 2.778 | Texas \& California | [W. Latto \& W. S. Olsen, private communication] |
| 2.750 | Peru | [W. Latto $\&$ W. S. Olsen, private communication] |

It is one of the objects of this chapter to demonstrate unequivocally the existence of a common unit of length throughout Megalithic Britain and to show that its value was accurately 2.72 ft . We might speculate that this unit was left in the Iberian Peninsula by Megalithic people to become the vara of recent times and to be taken to America by Spain.

To demonstrate the actual size of the Megalithic yard it might be logical to confine the argument to data based on what were intended to be true circles, since everyone will admit that such circles exist, and actually we obtain an identical value if we do so restrict ourselves, but in view of the fact that the existence of other definite shapes has been demonstrated in a number of papers already published it seems better in the present analysis to strengthen the case by including all the available data. That is we shall include true circles, both types of flattened circles, both types of eggs, and compound rings: but exclude ellipses and the Avebury ring. The latter will be seen to play its own part and provide a check on the whole result.

Using all the data but excluding circles where the uncertainty in the measured diameter exceeds 1 ft , we desire to find definite answers to the following questions.

1. Can it be definitely established that a universal unit of length was in use in all parts of the country?
2. If so, what was its value ?
3. Was it ever subdivided, and if so, how ?
4. Was a different unit, perhaps a multiple, used for the longer distances ?
5. In setting out circles was the measurement made to the inner side of the stone, to the centre of the stone, or to the outside ?
6. Was the same unit used for circles for alignments, and for the distances between circles?

The difficulty we encounter at the outset is that nearly all sites are in a ruinous condition. Stones have fallen out of place or have been bodily displaced by growing trees, by earth movement, or worst of all by well-meaning persons who have re-erected fallen stones without proper excavation to determine the original position and without leaving a record of their activities. Thus a statistical approach is necessary making use of the formulae and methods given in Chapter 2. But it is first necessary to obtain estimates of the diameters, distances between circles, etc., from the sites. It is useless to attempt to measure these quantities directly on the ground. One must first have an accurate survey and for our purpose most published surveys are quite unsuitable. So practically all the data used here are from the author's surveys, except the measurements at Callanish, where Somerville has made a reliable survey perhaps only inaccurate in azimuth and then only by a few minutes of arc, and measurements from a recent survey of Stanton Drew by Prain and Prain.

To obtain the diameter of a circle from a large-scale survey one can use a statistical 'least squares' method (Thom, 1955). This was used for the earlier surveys but latterly, using fairly complete circles, a simpler and very much more rapid method was found to be sufficient. For this method a carefully drawn circle is passed through the stones. The exact size chosen is unimportant as is also the position of the centre. Divide the ring into four quadrants. Mark what appears to be the centre of the base of each stone and measure the distance of this centre from the circle; positive if the stone centre is outside the circle, negative when it is inside. Find the mean for each quadrant separately. Call these means $\delta_{\mathrm{ne}}, \delta_{\mathrm{se}}, \delta_{\mathrm{sw}}$, and $\delta_{\mathrm{nw}}$. Then the required diameter is the diameter of the superimposed circle increased by

$$
\frac{1}{2}\left(\delta_{\mathrm{ne}}+\delta_{\mathrm{se}}+\delta_{\mathrm{sw}}+\delta_{\mathrm{nw}}\right)
$$

The chosen centre of the superimposed circle should now be moved to the north-east by $\frac{1}{2}\left(\delta_{\mathrm{ne}}-\delta_{\mathrm{sw}}\right)$ and to the north-west by $\frac{1}{2}\left(\delta_{\mathrm{nw}}-\delta_{\mathrm{se}}\right)$. The diameter should then be corrected for tape stretch if this has been determined.

In the case of a flattened circle the same kind of procedure can be applied with slight modification provided that the geometrical construction is definitely known.

If it is necessary to use fallen stones then measure to the centre of the end which lies nearest to the superimposed circle. It is seldom possible to say on the site which way the stone has fallen. Often the original top of a fallen stone is found to be lying lower than the original base. This is because the builders sometimes packed small stones round the base of upright stones and these have prevented the lower end of the fallen stone from sinking so much as the top. Diameters obtained from fallen stones only cannot be accurate especially if all have fallen out or all in. Sometimes the stump of a broken stone can be found by prodding with a bayonet. Where only part of a circle remains it may still
be possible to obtain an accurate diameter provided the remaining stones are small and upright.

Surveys of some of the circles used will be found in the figures and some have been published elsewhere. These are all on a very much reduced scale but the diameters given in Tables 5.1 and 5.2 were determined from the original surveys, which were plotted to scales which varied from $1 / 32$ to $1 / 264$ according to the size of the circle. The diameters tabulated for Types A, B, and D are the longest diameter of the figure and for the egg-shapes I and II the shortest.

We first demonstrate that there is a presumption amounting to a certainty that a definite unit was used in setting out these rings. It is proposed to call this the Megalithic yard (MY). Two of these might be called the Megalithic fathom. Obviously if the radius is an integral number of yards the diameter will be the same integral number of fathoms. It will appear that the Megalithic yard is 2.72 ft and so the Megalithic fathom is 5.44 ft .

Table 5.1. Circles and rings of which the diameter is known to $\pm 1$ foot or better

$$
\begin{aligned}
y & =\text { diameter (feet) } \\
e_{1} & =y-2 \cdot 72 m_{1} \\
e_{2} & =y-5 \cdot 44 m_{2} \quad m_{1} \text { and } m_{2} \text { integers }
\end{aligned}
$$

For flattened circles the diameter given is the longest and for eggs the shortest.

| Site | Diameter, $y$ <br> (ft) | $m_{1}$ | $\epsilon_{1}$ | $m_{2}$ | $\epsilon_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Scotland, circles |  |  |  |  |  |
| B 7/4 | $10 \cdot 8$ | 4 | $-0.08$ | 2 | $-0.08$ |
| A $2 / 8$ | $11 \cdot 2$ | 4 | +0.32 |  | +0.32 |
| P 2/14 | $12 \cdot 7$ | 5 | $-0.90$ | 2 | +1.82 |
| P 1/13 | $16 \cdot 4$ | 6 | $+0.08$ | 3 | $+0.08$ |
| B 1/10 | 16.9 | 6 | $+0.58$ | 3 | $+0.58$ |
| N 2/3 | $20 \cdot 5$ | 8 | $-1.26$ | 4 | -1.26 |
| B $2 / 4$ | $20 \cdot 6$ | 8 | $-1.16$ | 4 | $-1.16$ |
| G 4/9 | $20 \cdot 9$ | 8 | -0.86 | 4 | -0.86 |
| A $2 / 12$ | 21.0 | 8 | -0.76 | 4 | $-0.76$ |
| P 2/6 | 21.0 | 8 | -0.76 | 4 | -0.76 |
| B4/2 | $21 \cdot 3$ | 8 | -0.46 | 4 | -0.46 |
| A $2 / 5$ | 21.4 | 8 | $-0.36$ | 4 | -0.36 |
| M 2/14 | 21.8 | 8 | $+0.04$ | 4 | +0.04 |
| B 7/2 | 22.0 | 8 | +0.24 | 4 | $+0.24$ |

Table 5.1. (cont.)

| Site | $\text { Diameter, } y$ (ft) | $m_{1}$ | $\epsilon_{1}$ | $m_{2}$ | $\epsilon_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| G 8/2 | $23 \cdot 2$ | 9 | $-1.28$ | 4 | +1.44 |
| H 1/1 | 24.0 | 9 | $-0.48$ | 4 | +2.24 |
| N 2/2 | 24.0 | 9 | $-0.48$ | 4 | +2.24 |
| $\mathrm{P} 2 / 8{ }_{1}$ | 27.5 | 10 | +0.30 | 5 | $+0.30$ |
| P $2 / 8{ }_{2}$ | 27.5 | 10 | $+0.30$ | 5 | $+0.30$ |
| P 2/3 | 28.0 | 10 | $+0.80$ | 5 | $+0.80$ |
| B 7/19 | $30 \cdot 1$ | 11 | +0.18 | 6 | $-2.54$ |
| B 7/17 | 32.0 | 12 | $-0.64$ | 6 | -0.64 |
| B 3/4 | $32 \cdot 5$ | 12 | -0.14 | 6 | -0.14 |
| B 2/7 | $33 \cdot 4$ | 12 | $+0.76$ | 6 | $+0.76$ |
| B 6/1 | $35 \cdot 6$ | 13 | +0.24 | 7 | $-2.48$ |
| B 1/18 | 37.6 | 14 | $-0.48$ | 7 | $-0.48$ |
| B 7/6 | 39.2 | 14 | +1.12 | 7 | +1.12 |
| B 2/5 | $43 \cdot 6$ | 16 | $+0.08$ | 8 | $+0.08$ |
| M 2/14 | $44 \cdot 1$ | 16 | $+0.58$ | 8 | $+0.58$ |
| A $2 / 8$ | 44.2 | 16 | $+0.68$ | 8 | $+0.68$ |
| B 1/5 | $45 \cdot 0$ | 17 | -1.24 | 8 | +1.48 |
| B 2/16 | $46 \cdot 8$ | 17 | $+0.56$ | 9 | -2.16 |
| P 2/1 | 48.5 | 18 | -0.46 | 9 | -0.46 |
| B 1/16 | 49.0 | 18 | $+0.04$ | 9 | $+0.04$ |
| B 3/1 | 49.7 | 18 | $+0.74$ | 9 | $+0.74$ |
| A 8/6 | 54.9 | 20 | $+0.50$ | 10 | $+0.50$ |
| B 3/7 | 56.4 | 21 | $-0.72$ | 10 | $+2.00$ |
| B 2/17 | 56.9 | 21 | -0.22 | 10 | $+2.50$ |
| B 1/23 | 57.0 | 21 | $-0.12$ | 10 | $+2.60$ |
| B 7/2 | 59.1 | 22 | -0.74 | 11 | $-0.74$ |
| B 2/4 | 59.2 | 22 | -0.64 | 11 | -0.64 |
| B 2/1 | 59.3 | 22 | -0.54 | 11 | -0.54 |
| B 6/2 | 63.0 | 23 | +0.44 | 12 | -2.28 |
| B 1/6 | 64.0 | 24 | $-1.28$ | 12 | $-1.28$ |
| A $1 / 2$ | $65 \cdot 1$ | 24 | -0.18 | 12 | $-0.18$ |
| B 2/3 | $66 \cdot 9$ | 25 | $-1 \cdot 10$ | 12 | $+1.62$ |
| B 1/26 | $67 \cdot 2$ | 25 | $-0.80$ | 12 | +1.92 |
| B 6/1 | 68.4 | 25 | $+0.40$ | 13 | -2.32 |
| B 7/19 | $69 \cdot 1$ | 25 | +1.10 | 13 | $-1.62$ |
| B 2/16 | $73 \cdot 3$ | 27 | -0.14 | 13 | +2.58 |
| B 2/8 | $74 \cdot 1$ | 27 | $+0.66$ | 14 | -2.06 |
| B 7/18 | $74 \cdot 3$ | 27 | $+0.86$ | 14 | $-1.86$ |
| B 3/1 | $75 \cdot 1$ | 28 | -1.06 | 14 | -1.06 |
| B 7/12 | 76.0 | 28 | $-0.16$ | 14 | $-0.16$ |
| G 4/14 | $82 \cdot 1$ | 30 | +0.50 | 15 | $+0.50$ |
| B 7/15 | $82 \cdot 9$ | 30 | +1.30 | 15 | +1.30 |
| B $2 / 2$ | $83 \cdot 2$ | 31 | -1.12 | 15 | +1.60 |
| G 4/3 | 89.1 | 33 | $-0.46$ | 16 | $+2.26$ |
| B 4/4 | 92.0 | 34 | -0.48 | 17 | -0.48 |
| B 7/1 | 103.9 | 38 | $+0.34$ | 19 | $+0.34$ |
| B 7/1 ${ }_{2}$ | $104 \cdot 2$ | 38 | +0.84 | 19 | $+0.84$ |
| B 1/8 | $108 \cdot 4$ | 40 | $-0.40$ | 20 | -0.40 |
| B 5/1 | $110 \cdot 0$ | 40 | $+1.20$ | 20 | $+1.20$ |
| B 7/16 | $113 \cdot 2$ | 42 | -1.04 | 21 | $-1.04$ |
| B 7/15 | 119.9 | 44 | $+0.22$ | 22 | $+0.22$ |
| N 1/13 | 188.3 | 69 | $+0.62$ | 35 | $-2 \cdot 10$ |

Table 5.1. (cont.)

| Site | Diameter, $y$ <br> $(\mathrm{ft})$ | $m_{1}$ | $\epsilon_{1}$ | $m_{2}$ | $\epsilon_{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Scotland, Type A

| G 7/4 | $37 \cdot 7$ | 14 | $-0 \cdot 38$ | 7 | -0.38 |
| :--- | :--- | :--- | :--- | ---: | ---: |
| B 7/12 | $43 \cdot 0$ | 16 | -0.52 | 8 | -0.52 |
| H $1 / 1$ | $43 \cdot 3$ | 16 | -0.22 | 8 | -0.22 |
| G $4 / 12$ | $54 \cdot 5$ | 20 | $+0 \cdot 10$ | 10 | +0.10 |
| G 7/2 | $65 \cdot 5$ | 24 | +0.22 | 12 | +0.22 |
| B 7/16 | $66 \cdot 8$ | 25 | $-1 \cdot 20$ | 12 | +1.52 |
| G 3/7 | $69 \cdot 3$ | 25 | +1.30 | 13 | -1.42 |

Scotland, Type B

| A 1/2 | 16.0 | 6 | -0.32 | 3 | -0.32 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| B 1/9 | 28.0 | 10 | +0.80 | 5 | +0.80 |
| B 2/6 | 58.9 | 22 | -0.94 | 11 | -0.94 |

Scotland, egg-shaped rings

| B 7/18 | $38 \cdot 3$ | 14 | +0.22 | 7 | +0.22 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| G 9/15 | 43.9 | 16 | +0.38 | 8 | +0.38 |
| B 2/4 | 76.1 | 28 | -0.06 | 14 | -0.06 |
| B 7/1 | 103.6 | 38 | +0.24 | 19 | -0.24 |
| G 9/10 | 136.0 | 50 | +0.00 | 25 | +0.00 |

Scotland, compound rings
B 7/10
$60 \cdot 0$
$22+0.16$
$11+0 \cdot 16$
England and Wales, circles

| S $2 / 4$ | 11.9 | 4 | +1.02 | 2 | +1.02 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S 5/2 | $12 \cdot 0$ | 4 | $+1 \cdot 12$ | 2 | +1.12 |
| W $2 / 1$ | $13 \cdot 2$ | 5 | -0.40 | 2 | +2.32 |
| S 5/2 | 13.6 | 5 | 0.00 | 2 | +2.72 |
| W $8 / 3$ | $17 \cdot 2$ | 6 | $+0.88$ | 3 | $+0.88$ |
| S 5/2 | $19 \cdot 3$ | 7 | $+0.26$ | 4 | $-2.46$ |
| S 2/5 | $22 \cdot 3$ | 8 | +0.54 | 4 | +0.54 |
| L 2/13 | 24.0 | 9 | -0.48 | 4 | +2.24 |
| W 11/2 | $24 \cdot 3$ | 9 | -0.18 | 4 | +2.54 |
| L 5/1 | 27.7 | 10 | $+0.50$ | 5 | +0.50 |
| S 5/2 | $30 \cdot 9$ | 11 | +0.98 | 6 | $-1.74$ |
| L 3/1 | 31.5 | 12 | $-1 \cdot 14$ | 6 | $-1 \cdot 14$ |
| W 13/1 | $32 \cdot 7$ | 12 | $+0.06$ | 6 | +0.06 |
| S 5/2 | $34 \cdot 3$ | 13 | $-1.06$ | 6 | +1.66 |
| D 1/3 | $35 \cdot 5$ | 13 | $+0.14$ | 7 | $-2.58$ |
| S 2/4 | 41.2 | 15 | +0.40 | 8 | -2.32 |
| W 6/2 | 42.0 | 15 | +1.20 | 8 | $-1.52$ |
| W 11/2 | $43 \cdot 7$ | 16 | $+0.18$ | 8 | $+0.18$ |
| W 9/4 | 43.7 | 16 | $+0.18$ | 8 | $+0.18$ |
| S 5/2 | $46 \cdot 8$ | 17 | $+0.56$ | 9 | -2.16 |
| S 1/2 | $49 \cdot 6$ | 18 | $+0.64$ | 9 | $+0.64$ |
| L 1/6B | 49.7 | 18 | $+0.74$ | 9 | $+0.74$ |
| L 1/13 | 49.7 | 18 | $+0.74$ | 9 | $+0.74$ |
| L 1/6D | 52.0 | 19 | $+0.32$ | 10 | $-2.40$ |
| S 1/10 | $53 \cdot 6$ | 20 | -0.80 | 10 | -0.80 |

Table 5.1. (cont.)

| Site | $\text { Diameter, } y$ (ft) | $m_{1}$ | $\epsilon_{1}$ | $m_{2}$ | $\epsilon_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L 1/6C | $54 \cdot 6$ | 20 | $+0.20$ | 10 | $+0 \cdot 20$ |
| W 5/2 | $55 \cdot 5$ | 20 | $+1 \cdot 10$ | 10 | $+1 \cdot 10$ |
| W 11/5 | 58.6 | 22 | $-1.24$ | 11 | $-1.24$ |
| S 5/2 | $64 \cdot 8$ | 24 | -0.48 | 12 | -0.48 |
| W 11/4 | 68.2 | 25 | $+0.20$ | 13 | -2.52 |
| S 1/11 | $71 \cdot 6$ | 26 | $+0.88$ | 13 | $+0.88$ |
| W 9/2 | $73 \cdot 2$ | 27 | -0.24 | 13 | +2.48 |
| W 11/2 | $76 \cdot 3$ | 28 | $+0.14$ | 14 | $+0.14$ |
| S 1/14 | $77 \cdot 8$ | 29 | $-1.08$ | 14 | +1.64 |
| S 2/3 | 81.4 | 30 | -0.20 | 15 | -0.20 |
| S 1/6 | 81.5 | 30 | $-0.10$ | 15 | $-0.10$ |
| L 2/13 | $86 \cdot 0$ | 32 | $-1.04$ | 16 | $-1.04$ |
| L 1/3 | $93 \cdot 7$ | 34 | $+1.22$ | 17 | +1.22 |
| S 6/1 | 103.6 | 38 | $+0.24$ | 19 | $+0.24$ |
| S 2/1 | $104 \cdot 5$ | 38 | +1.14 | 19 | +1.14 |
| S 1/1 | $107 \cdot 6$ | 40 | $-1.20$ | 20 | $-1.20$ |
| S 1/5 | 108.3 | 40 | $-0.50$ | 20 | $-0.50$ |
| S $2 / 1$ | 108.5 | 40 | $-0.30$ | 20 | $-0.30$ |
| S 1/1 | 113.7 | 42 | -0.54 | 21 | $-0.54$ |
| S 5/2 | 129.7 | 48 | -0.86 | 24 | $-0.86$ |
| S 1/4 | $147 \cdot 0$ | 54 | +0.12 | 27 | +0.12 |

England and Wales, Type A

| S $1 / 3$ | $38 \cdot 6$ | 14 | +0.52 | 7 | +0.52 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| D $1 / 9$ | $54 \cdot 2$ | 20 | -0.20 | 10 | -0.20 |
| S $1 / 16$ | 71.6 | 26 | +0.88 | 13 | +0.88 |
| D $2 / 2$ | 76.0 | 28 | -0.16 | 14 | -0.16 |
| D $2 / 1$ | 93.3 | 34 | +0.82 | 17 | +0.82 |
| L $1 / 1$ | 107.8 | 40 | -1.00 | 20 | -1.00 |
| S $1 / 8$ | 139.7 | 51 | +0.98 | 26 | -1.74 |

England and Wales, Type B

| D $1 / 7$ | 47.7 | 18 | -1.26 | 9 | -1.26 |
| :--- | :--- | :--- | :--- | ---: | ---: |
| S 2/2 | 67.4 | 25 | -0.60 | -12 | +2.12 |
| S 1/13 | 82.6 | 30 | +1.00 | 15 | +1.00 |
| D $1 / 8$ | 86.6 | 32 | -0.44 | 16 | -0.44 |

England and Wales, Type D

| L $1 / 10$ | 88.9 | 33 | -0.86 | 16 | +1.86 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| S 1/7 | 150.7 | 55 | +1.10 | 28 | -1.62 |

England and Wales, egg-shaped rings

| $\mathrm{W} 11 / 3$ | 59.8 | 22 | -0.04 | 11 | -0.04 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mathrm{~S} 1 / 1$ | 136.7 | 50 | +0.70 | 25 | +0.70 |

England and Wales, compound rings

| W 5/1 | 38.2 | 14 | +0.12 | 7 | +0.12 |
| :--- | :--- | :--- | :--- | ---: | :--- |
| W 6/1 | 86.9 | 32 | -0.14 | 16 | -0.14 |

Table 5.2. Diameters known with less accuracy

| Site |  | Diameter <br> (ft) | Site |  | Diameter <br> (ft) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Scotland |  |  |  |  |  |
| N $2 / 3$ | Shin River | $13 \cdot 6$ | B 3/3 | Raedykes | $60 \pm$ |
| M 1/9 | Ardnacross | 12 | P 1/19 | Croftmoraig | 60 |
|  |  | 15 | M 4/2 | Balemartin | $65 \pm$ |
| P 1/4 | Weem | $15 \cdot 4$ | B 1/11 | Balquhain | $67 \cdot 4$ |
| P 1/10 | Fowlis Wester | 15 | B 1/7 | Kirtkown of Bourtie | 71 |
| G 8/7 | Dere Street | 19 | B 2/18 | Tillyfourie Hill | $72 \pm$ |
| P 1/7 | Aberfeldy | 19 | B 2/11 | Cairnfauld | $75 \pm$ |
| H7/9 | Strathaird | $21 \pm$ | P 1/5 | Weem | 76 |
| G 9/11 | Ninestone Rig | 21 | B 1/23 | Yonder Bognie | $80 \pm$ |
| P 2/4 | Courthill | $22 \cdot 8$ | B 4/1 | Carnoussie House | 84 |
| P 1/14 | Tullybeagles Lodge | 23 | B 1/13 | Old Rayne | $86 \pm$ |
| B 7/10 | Easter Delfour | 23.6 | H 4/2 | Gramisdale (S) | 88 |
| B 4/1 | Carnoussie House | 27 | H 4/1 | " (N) | $87 \pm$ |
| P 1/14 | Tullybeagles Lodge | $31.4 \pm$ | B 2/14 | Leylodge | 97 |
| H 6/5 | Bernera (Barra) | $32 \pm$ | B 6/2 | Moyness | 98.5 |
| G 7/3 | Wamphrey | 38 | G 6/2 | Auldgirth | $100 \cdot 0$ |
| P 1/19 | Croftmoraig | 41 | H 3/17 | Pobull Fhinn | $124 \pm$ |
| G 7/4 | Loupin Stanes (W) | $44 \pm$ | G 7/5 | Girdle Stanes | 131土 |
| B 1/1 | Strichen | 44 | H $3 / 18$ | Sornach Coir Fhinn | 139 |
| B 3/4 | Raedykes | 47 | N 1/5 | Forse, Latheron | $157 \cdot 5$ |
| P 3/2 | Blackgate | 55 | G 6/1 | Twelve Apostles |  |
| B 1/26 | Loanhead | 54 |  | Type B | 288.4 |
| B 2/14 | Leylodge | 54 (?) | G 7/6 | Whitcastles Special Type | $184 \cdot 8$ |
| England |  |  |  |  |  |
| L 2/11 | Castlehowe Scar | $21 \pm$ | Type A |  |  |
| L 6/2 | Staintondale | 32 | L 1/6 | Burnmoor E | 104 |
| S 4/3 | Hampton Down | $35 \cdot 6$ | L 2/14 | Orton | 146 |
| D 1/4 | Ninestone Close | $42 \cdot 5$ |  |  |  |
| W 9/5 | St. Nicholas | $43 \pm$ | Type B |  |  |
| L 1/9 | Glassonby | $46 \cdot 5$ | L 2/12 | Harberwain | 49 |
| L 2/10 | Gunnerkeld | $48 \pm$ | S 4/2 | Kingston Russell | 91 |
| L 1/12 | Lacre | 53 | L 3/4 | Lilburn | $100 \pm$ |
| L 1/6 | Burnmoor A | 71 | S 1/12 | Porthmeor | 113 |
| L 2/10 | Gunnerkeld | $100 \pm$ | L 1/7 | Long Meg, etc. | 358.8 |
| L 1/14 | Dean Moor | 110 |  |  |  |
| L 1/2 | Elva Plain | 113.4 |  |  |  |
| W 4/1 | Penbedw Hall | 116土 |  |  |  |
| S 3/1 | Stanton Drew | $372 \cdot 4$ |  |  |  |

Let us take, in the notation of Chapter 2, $2 \delta=5 \cdot 44$, and examine the residuals $\left(\epsilon_{2}\right)$ of the diameters from integral fathoms. That is, we put

$$
\epsilon_{2}=\left|y-5 \cdot 44 m_{2}\right|
$$

where y is the diameter in feet and $\mathrm{m}_{2}$ is the integer which gives the smallest $e_{2}$.
For all the 145 diameters in Table 5.1 we find $\sum e_{2}^{2}=238.68$, so

$$
s^{2}=\sum e_{2}^{2} \quad / n=1.646 \text { and s }^{2} / \delta^{2}=0.222 .
$$

If there were an a priori reason for expecting that the diameters were set out in units of 5.44 ft then we might enter Fig. 2.1 with $n=145$ and $\mathrm{s}^{2} / \delta^{2}=0 \cdot 222$. We should find that the probability that the hypothesis is correct is so high that the point is off the figure to the right showing a probability level of about $10^{-5}$ or 0.001 per cent. If we deny that there is an a priori reason for the hypothesis then we calculate

$$
C=\sqrt{ } n\left(\frac{1}{3}-\mathrm{s}^{2} / \delta^{2}\right)
$$

and find $C=1 \cdot 33$, thus showing that we can accept the existence of the fathom for the diameter and so of the yard for the radius with complete confidence.
From a common-sense point of view we do not need to depend on this last step. We can say that the Megalithic fathom was first demonstrated in 1955. So from that date we expect all future work to show the same unit. Thus the many surveys made subsequent to 1955 can be analysed by the method used on p . 40 and will show such a probability level as to remove all doubt.

In setting out a circle one uses the radius and so it is probable that it was the half-fathom or yard of 2.72 ft which was the unit. It will, however, be shown later that the yard was sometimes halved when used for alignments and ellipses. Accordingly we shall analyse the diameters in terms of the yard to determine the exact value of the latter and of the additive constant $\beta$; thus we allow for the possibility of half-yards happening sometimes in the radius.

We write

$$
\epsilon_{1}=y-2 \cdot 72 m_{1}
$$

The values of $m_{1}$ and $\epsilon_{1}$ are given in columns 3 and 4 (Table 5.1). We first show that the residuals do not increase seriously with increasing size of circle. To do this the data are divided into four groups according to the size of the diameter and the variance is found for each group. The results are shown in Table 5.3.

Table 5.3

| Group | Diameters between | $n$ | $\Sigma \epsilon_{1}^{2}$ | $s^{2}=\left(\Sigma \epsilon_{1}^{2}\right) / n$ | $\sigma=\sqrt{ } s^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 and 31 ft | 35 | 15.27 | 0.436 | 0.66 |
| 2 | $31 " 54 ~ "$ | 35 | 14.53 | 0.416 | 0.65 |
| 3 | $54 " 76$ | 37 | 19.31 | 0.522 | 0.72 |
| 4 | $76 " 189 "$ | 38 | 21.91 | 0.577 | 0.76 |

It will be seen how far wrong it would be to take a proportional to the diameter. In fact it appears that $\sigma$ changes so little that the formulae for constant $\sigma$ are appropriate. Making the rather long calculations indicated we find from the figures in Table 5.1 the values in Table 5.4.

Table 5.4


Using the formulae of p. 9 the values in Table 5.5 immediately follow.

## Table 5.5

|  | $\beta$ | $2 \delta$ | $\Sigma m_{1} y i \Sigma m_{1}^{2}$ |
| :--- | :--- | :--- | :--- |
| England, circles only | +0.23 ft | 2.704 ft | 2.719 ft |
| Scotland ", ", | -0.28 | 2.729 | 2.719 |
| Britain ", | 0.00 | 2.719 | 2.719 |
| England, alltypes | +0.22 | 2.714 | 2.21 |
| Scotland ", ", | -0.25 | 2.728 | 2.719 |
| Britain ", | -0.07 | 2.722 | 2.720 |

The range of the values obtained for $\beta$ is merely a reflection of the essential difficulty of determining this quantity. One or two poorly determined diameters, especially at the lower end of the scale, can have a large effect. A reasonably accurate value can only be expected if we have a large number of measurements well spread through the scale. For this reason we must give a high weight to the result for Britain as a whole, namely -0.07 ft . But the standard error of this quantity as determined by the formula of p. 9 is $\pm 0.06 \mathrm{ft}$. So we have no reason for believing $\beta$ to be significantly different from zero. Suppose that the stones in the circles have an average thickness of $b \mathrm{ft}$, measured radially, and suppose that the erectors measured the diameters to the inner side of the stones. If we now come along and measure to the centres of the stones then all our deduced diameters will be too large by roughly $b$ and will be represented by

$$
2 \cdot 72 m_{1}+b .
$$

The thickness of the stones actually ranges from under a foot to several feet. So the supposition that the erectors measured to the inside of the stones must be wrong. The explanation of $\beta$ being zero is of course that the erectors measured to the stone centres. Perhaps where the stones formed a retaining wall the measurement may have been taken to the outside of the wall, i.e. the outside relative to the rubble filling behind the wall.

Taking then $\beta=0$ we are entitled to use the simpler formulae of Case 1 (a) and to deduce the value of the yard from $\sum m_{1} y / \sum m^{2}{ }_{1}$. These values are given in the last column (Table 5.5). An estimate of the standard error can be made by the formulae given and so
the data of the main table (Table 5.1) give finally

$$
1 \mathrm{MY}=2 \cdot 720 \pm 0 \cdot 003 \mathrm{ft} .
$$

A further conclusion is that this unit was in use from one end of Britain to the other. It is evident from Table 5.5 that it is not possible to detect by statistical examination any difference between the values determined from the English and Scottish circles. There must have been a headquarters from which standard rods were sent out but whether this was in these islands or on the Continent the present investigation cannot determine. The length of the rods in Scotland cannot have differed from that in England by more than 0.03 in or the difference would have shown up in Table 5.5. If each small community had obtained the length by copying the rod of its neighbour to the south the accumulated error would have been much greater than this.

## Circles of which the diameters are known with less accuracy

A list of these circles with their estimated diameters is given in Table 5.2. The uncertainty may be because of an indifferent survey but in most cases it lies in the ruinous condition of the site. An example is the circle in Strathaird, Skye. Here only three upright stones remain, unfortunately adjacent. The others were never seen, being buried in peat, but prodding revealed their position roughly. The circle almost certainly belongs to the 8-MY diameter group but of course no accuracy is possible.

## Circles from other sources

There are many circles in Britain which the author has not yet surveyed. The number is certainly fifty but it may well be 100 . Published plans of a number of these will be found scattered in books and journals and many more circles are mentioned as being in existence or as having vanished. Even when the published plans are based on accurate surveys the scales are usually too small to permit the diameters to be estimated nearer than $\pm 1$ per cent. The surveys by R. H. Worth (1953) seem to be reliable, but when he states a diameter there is a suspicion that he refers to the inside measurement.

A list of circles from various sources is given in Table 5.6. Circles surveyed by the author are excluded. An attempt was made to survey the Fedw Circle but it was found to
be in such a ruinous condition that without clearing and excavation nothing could be done. Apparently in 1861 it was still complete. Many other recorded sites were visited only to find the ground cleared. Of the great circle, the Gray Yauds, only one stone remained. In several places the local people admitted that there had been stones there and in others the near-by walls built with large stones showed where the circle had gone.

The diameters given in the table for the various rings at Stonehenge were scaled from the published Ministry of Works survey. This survey carries two scales, one in metres and
one in feet, but unfortunately these scales differ by about $1 \frac{1}{4}$ per cent. All that one can do is to take a mean.

Table 5.6. Circles from other sources

| Site | Diameter (ft) | Source | Remarks |
| :---: | :---: | :---: | :---: |
| Langston Moor | 58.0 | Worth | Ref. 33 |
| Cordon Whitemoor | 67.0 | " |  |
| Down Ridge | 82.0 | " |  |
| Buttern Hill | 82.0 | " |  |
| Scorhill | 89.0 | " |  |
| Sherberton | 97.0 | ", |  |
| Image Wood | $11 \cdot 3$ | Keiller | Ref. 13 |
| Cairnwell | 28.0 | " |  |
| Binghill | $33 \cdot 6$ | " |  |
| Raes of Cluny | $54 \cdot 2$ | " |  |
| Auld Kirk o' Tough | $102 \cdot 7$ |  |  |
| Egryn Abbey | 111.0 | Hawkes |  |
|  | $159 \cdot 0$ |  |  |
| Crick Barrow | 92.0 | North |  |
| Rempston | 76 | Piggott |  |
| The Fedw Circle | $77 \cdot 6$ | Arch. Camb. 1861 |  |
| Zadlee | $27 \cdot 2$ | St. Act. East Lothian |  |
| East Lothian 240 | $40 \cdot 5$ | " " " | Type A |
| Cerrig Pryfaid | $71 \cdot 5$ | R. Com. on An. Mon., Vol. 1 |  |
| The Druids' Circle | 83 | ". " " " " |  |
| Isle of Purbeck | 76 | Antiquity |  |
| Barpa Langass | $82 \cdot 6$ | St. Act. 137 N. Uist | Type A |
| Brogar | $340 \cdot 5$ |  |  |
| Stonehenge |  |  |  |
| Aubrey Holes | 285 |  |  |
| Y Holes | 177 |  |  |
| Z Holes | 128 |  |  |

In spite of these difficulties and uncertainties the mean value of the yard as deduced by the usual statistical method from the values in Table 5.6, with or without Stonehenge, is again $2 \cdot 72 \mathrm{ft}$.

## The sizes of the circles

The erectors of Megalithic monuments were evidently interested in getting the dimensions of their structures to be multiples of certain units of length. Since they were capable of measuring to a high degree of accuracy how does it come about that many circles which seem to have been undisturbed have mean diameters which differ by appreciable amounts
from what were presumably their nominal diameters? It will be shown that in a significant number of cases the discrepancy is produced by a small adjustment made by the erectors to the diameter, to bring the circumference nearer to an integer. This desire to have both dimensions integral has a further consequence in that at many sites it affects the integer chosen for the diameter.

The distribution of known circle diameters is shown in Fig. 5.1. Here each circle is represented by a small gaussian area placed at the appropriate diameter. The ordinates of the gaussians are added so that we get a kind of histogram showing the favoured diameters. The true circles are shown above the base line and the flattened circles below. The circles listed in the main table (Table 5.1) have diameters known with an uncertainty between $\pm 0 \cdot 3$ and $\pm 1.0 \mathrm{ft}$. These are shown by hatched areas. The circle diameters listed in Table 5.6 have been collected from various published sources and are considered to have an uncertainty of about 1 ft . Accordingly the same gaussian area has been used but unshaded. The diameters in Table 5.2 are uncertain but those which are considered to be known to about $\pm 1 \cdot 5 \mathrm{ft}$ are shown with a wider and flatter area unshaded. A key to the areas used is given in the figure.

Three scales are shown. Below the base the diameter is given in feet and immediately above the histogram in Megalithic yards. Above this again the corresponding circumference is given in Megalithic yards. In examining the figure bear in mind that some of the diameters may be in error by 1 ft or more, although the half-width of the shaded gaussians is only about half this.

The figure is in itself a pictorial proof of the existence of the Megalithic yard, but it contains much more information. Most of the diameters are seen to lie near to an even number of yards. In other words the radii are integers. But there are also concentrations at odd numbers, so the designers frequently used half-yards for the radius. We shall see that in the alignments also the yard was sometimes divided in two and even in four.

In the histogram there are concentrations at diameters $10,20,30$, and 40 MY but only a few at 15,25 , and 35 . The concentrations at $4,8,12,16,20,24$, and 28 are obvious. The reason for this last sequence becomes evident when we consider the circumferences.

## The circumferences

Perhaps the most striking feature of the circumferences shown in Fig. 5.1 is that large concentrations occur at $12 \frac{1}{2}, 25,37 \frac{1}{2}, 50,62 \frac{1}{2}, 75$, and $87 \frac{1}{2}$, all multiples of $12 \frac{9}{2}$.

If we accept the approximation $\pi=3 \frac{1}{8}$ then a circle with diameter 4 has a circumference of $12 \frac{1}{2}$. So the above sequence of circumferences follows from a diameter sequence of 4 , 8,12 , etc. This immediately explains why there are so many circles with a diameter of 8 or 16 , since these have circumferences very close to 25 and 50 MY .

For the larger circles the error in taking $\pi=3 \frac{1}{8}$ would begin to show up seriously. In fact with a diameter of 28 the approximation $\pi=3 \frac{1}{7}$ gives $P=88$ and we may suppose that this rather than the poorer value of $3 \frac{1}{8} \times 28$ or $87 \cdot 5$ was the reason for the four circles having this diameter. The reader may also have noticed the small groups at diameters 7, 14 , and 21.


Turning to the large circles beyond the range of Fig. 5.1 we find

| Aubrey Holes, Stonehenge | $D \fallingdotseq 105 \mathrm{MY}$ | giving | $P=329 \cdot 87$ |
| :--- | :---: | ---: | ---: |
| Avebury (inner ring) | 125 |  | $392 \cdot 70$ |
| Brodgar | 125 |  | $392 \cdot 70$ |
| Stanton Drew | 137 |  | $430 \cdot 39$ |

If these circumferences are all intended to be multiples of $2 \frac{1}{2}$ we can write them 330, $392 \cdot 5$ and 430 giving the interesting approximations for $\pi: 3 \frac{1}{7}, 3 \cdot 1400$, and $3 \cdot 139$.

Looking again at the histogram it is seen that many of the peaks do not lie at what might be called the nominal diameters. Consider, for example, the concentrations at or near to $10,18,30$, and 38 MY . Multiplying by $\pi$ we find $31 \cdot 4,56 \cdot 5,94 \cdot 2$, and $119 \cdot 4$. Assuming that a multiple of $2 \frac{1}{2}$ was required these circles were perhaps enlarged slightly to bring the circumferences nearer to $32 \cdot 5,57 \cdot 5,95$, and 120 . It will be seen that the peak in all four concentrations lies a little to the right of the nominal diameter but falls just short of the circumference which is a multiple of $2 \frac{1}{2}$. There is ample evidence that, when they wanted, these people could measure with an accuracy better than 1 in 500 , so it is certain that they knew what they were doing when they made adjustments of this kind. Since we do not know the reason for their preoccupation with integers we cannot tell how worried the designer would be when other considerations forced him to use a diameter which had to be adjusted to make the circumference fit. Had he to demonstrate his solution to a visiting inspector ?

In the next section a statistical examination will be made of the above idea that when the diameter and the circumference were irreconcilable an adjustment was made to the diameter so that the circumference fitted a little better.

## The adjustment of the diameter

The examination of the diameter distribution (Fig. 5.1) has given the definite impression that it was more important to have the perimeter a multiple of $2 \frac{1}{2}$ than to have it an integer. Further evidence comes from the eggs and ellipses. At Woodhenge, for example, the perimeters are all multiples of 10 and to attain this the integral condition for practically all the radii was sacrificed although the basic $12,35,37$ triangle was retained. At Moel ty Uche (W 5/1) the enclosing circle has a diameter of 14 so its circumference is $3 \frac{1}{7} \times 14$ or 44 . But this was not enough and an elaborate geometrical construction was used to obtain a perimeter of $42 \cdot 8$ as an approximation to $42 \frac{1}{2}$.

Accordingly, in the examination to be made we assume that the target for the circumferences of circles $(\mathrm{P})$ was a multiple of $2 \frac{1}{2}$. Further we shall assume arbitrarily that the condition was satisfied if P was within one quarter of $2 \frac{1}{2}$ (i.e. $0 \cdot 625$ ) of being a multiple. So we find that the following diameters satisfy each of these when multiplied by $\pi$ being within 0.625 of a multiple of $2 \frac{1}{2}$.

| 4 | 16 | 28 | 39 | 51 |
| ---: | ---: | :--- | :--- | :--- |
| 7 | 19 | 31 | 42 | 54 |
| 8 | 20 | 32 | 43 | etc. |
| 11 | 23 | 35 | 46 |  |
| 12 | 24 | 36 | 47 |  |
| 15 | 27 | 38 | 50 |  |

We now proceed to examine what the designer did when he used a diameter which did not satisfy. All such circles are listed in Table 5.7, with the actual measured diameters in feet designated by $y$ and in Megalithic yards by $d$. The actual circumferences are designated $P_{\mathrm{a}}(=\pi d)$ and the amount by which this exceeds a multiple of $2 \frac{1}{2}$ is given under $\epsilon_{p a}$. The nominal diameters appear under $D(\mathrm{MY})$ and the corresponding circumferences under $P_{\mathrm{n}}(=\pi d)$. The excess of $P_{\mathrm{n}}$ over a multiple of $2 \frac{1}{2}$ is called $\epsilon_{p n}$. These values of $\epsilon_{p n}$, are, of course, all greater than $0 \cdot 625$.

We see that of the five circles having a nominal diameter $D$ of 9 all are set out with an actual diameter $d$ slightly smaller than 9 . Had the diameter been 9 the circumference would have been $P=28 \cdot 27$. By reducing the diameter slightly the circumference was brought nearer to $27 \frac{1}{2}$, a multiple of $2 \frac{1}{2}$.

Table 5.7. Circles for which $\pi_{x}$ (nominal diameter) is not near a multiple of $2 \frac{1}{2}$ MY, i.e., where
$\left|\pi \boldsymbol{D}-2 \frac{1}{2} m\right|>0.625$
Material from Table 5.1 only

$$
\begin{aligned}
& y=\text { actual diam. (ft) } \quad D=\text { nominal diameter (MY) } \\
& d=">"(\mathrm{MY}) \quad P_{\mathrm{n}}=\pi d \\
& P_{\mathrm{a}}=\pi d(=1 \cdot 155 \mathrm{y}) \\
& \epsilon_{a}=(d-D) \quad \epsilon_{p a}=\left(P_{a}-2 \frac{1}{2} m\right) \quad \epsilon_{p n}=\left(P_{\mathrm{n}}-2 \frac{1}{2} m\right) \\
& m=\text { appropriate whole number }
\end{aligned}
$$

| Site | ${ }_{(\mathrm{ft})}^{y}$ | $\stackrel{d}{(\mathrm{MY})}$ | $\epsilon_{d}$ | $\begin{aligned} & \left.P_{a} . \dot{\text { M }}\right) \end{aligned}$ | $\epsilon_{p a}$ | $\begin{aligned} & D \\ & \text { (MY) } \end{aligned}$ | $\begin{aligned} & P_{n} \\ & (\mathrm{MY}) \end{aligned}$ | $\epsilon_{\nu n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P 2/14 | 12.7 | $4 \cdot 67$ | -0.33 | 14.67 | -0.33 | 5 | 15.71 | $+0.71$ |
| W $2 / 1$ | $13 \cdot 2$ | $4 \cdot 85$ | -0.15 | 15.24 | +0.24 |  |  | " |
| S 5/2 | $13 \cdot 6$ | 5.00 | 0 | 15.71 | +0.71 |  |  |  |
| P 1/13 | $16 \cdot 4$ | 6.03 | +0.03 | 18.94 | -1.06 | 6 | 18.85 | $-1.15$ |
| B 1/10 | $16 \cdot 9$ | 6.21 | +0.21 | 19.52 | -0.48 | " |  |  |
| W 8/3 | 17.2 | 6.32 | +0.32 | 19.87 | -0.13 |  |  |  |
| G 8/2 | $23 \cdot 2$ | $8 \cdot 53$ | -0.47 | 26.80 | -0.70 | 9 | 28.27 | +0.77 |
| H1/1 | 24.0 | 8.82 | -0.18 | 27.72 | +0.22 |  |  |  |
| N $2 / 2$ | 24.0 | 8.82 | -0.18 | 27.72 | +0.22 | " | " | " |
| L 2/13 | 24.0 | 8.82 | -0.18 | 27.72 | +0.22 | " | " |  |
| W 11/2 | 24.3 | 8.93 | -0.07 | 28.07 | +0.57 |  |  |  |
| P 2/8 | 27.5 | $10 \cdot 11$ | $+0.11$ | 31.76 | -0.74 | 10 | $31 \cdot 42$ | -1.08 |
| P 2/8 | " | " | " | " | " | " | , | " |

\{Table 5.7 continued on $p .49$ below\}

Table 5.7 (cont.)

| Site | (ft) | $\begin{aligned} & d \\ & (\mathrm{MY}) \end{aligned}$ | $\epsilon_{d}$ | $\begin{aligned} & P_{a} \\ & (\mathrm{MY}) \end{aligned}$ | $\epsilon_{p a}$ | $\begin{aligned} & D \\ & (\text { MY }) \end{aligned}$ | $\begin{aligned} & P_{n} \\ & (\mathbf{M Y}) \end{aligned}$ | $\epsilon_{p n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| L 5/1 | 27.7 | 10.18 | +0.18 | 31.99 | -0.51 | 10 | $31 \cdot 42$ | -1.08 |
| P 2/3 | 28.0 | 10-29 | +0.29 | $32 \cdot 34$ | -0.16 |  |  |  |
| S 512 | $34 \cdot 3$ | 12.61 | -0.39 | 39.62 | -0.38 | 13 | $40 \cdot 84$ | $+0.84$ |
| D 1/3 | 35.5 | 13.05 | +0.05 | 41.00 | +1.00 |  |  |  |
| B 6/1 | 35.6 | 13.09 | +0.09 | 41.11 | +1.11 |  |  |  |
| B 1/18 | 37.6 | 13.82 | -0.18 | 43.43 | +0.93 | 14 | 43.98 | -1.02 |
| B 7/6 | 39.2 | 14.41 | +0.41 | 45.28 | +0.28 |  |  |  |
| B 1/5 | $45 \cdot 0$ | 16.54 | -0.46 | 51.97 | -0.53 | 17 | 53.41 | $+0.91$ |
| B 2/16 | $46 \cdot 8$ | 17.21 | +0.21 | 54.05 | -0.95 | " | " | " |
| S 5/2 | $46 \cdot 8$ | 17.21 | +0.21 | 54.05 | -0.95 |  |  |  |
| P 2/1 | 48.5 | 17.83 | -0.17 | 56.02 | +1.02 | 18 | 56.55 | -0.95 |
| B 1/16 | 49.0 | 18.01 | +0.01 | 56.60 | $-0.90$ | " | " | " |
| S 1/2 | 49.6 | 18.24 | +0.24 | 57.29 | -0.21 | " | " | " |
| B 3/1 | 49.7 | 18.27 | +0.27 | 57.40 | -0.10 | " | " |  |
| L $1 / 4$ | " | " | " | " | " | " | " | " |
| L 1/13 |  |  |  |  |  |  |  |  |
| B 3/7 | 56.4 | 20.74 | -0.26 | $65 \cdot 14$ | +0.14 | 21 | 65.97 | +0.97 |
| B 2/17 | 56.9 | 20.92 | -0.08 | 65.72 | +0.72 | " | " | " |
| B 1/23 | 57.0 | 20.96 | -0.04 | $65 \cdot 83$ | +0.83 |  |  |  |
| W 11/5 | 58.6 | 21.54 | $-0.46$ | 67.67 | +0.17 | 22 | 69.12 | -0.88 |
| B7/2 | 59.1 | 21.73 | -0.27 | 68.26 | +0.76 | " | " | " |
| B 2/4 | 59.2 | 21.76 | -0.24 | 68.38 | +0.88 | " | " | " |
| B 2/1 | 59.3 | 21.80 | -0.20 | 68.49 | +0.99 |  |  |  |
| B 2/3 | $66 \cdot 9$ | 24.60 | -0.40 | 77.27 | -0.23 | 25 | 78.54 | +1.04 |
| B 1/26 | 67.2 | 24.71 | -0.29 | 77.62 | +0.12 | " | " | " |
| W 11/4 | 68.2 | 25.07 | +0.07 | 78.77 | -1.23 | " | " | " |
| B6/1 | 68.4 | 25.15 | +0.15 | 79.00 | $-1.00$ |  | " | " |
| B 7/19 | 69.1 | 25.40 | +0.40 | 79.81 | -0.19 |  |  |  |
| S 1/11 | 71.6 | 26.32 | +0.32 | 82.70 | +0.20 | 26 | 81.68 | -0.82 |
| S 1/14 | 77.8 | 28.60 | $-0.40$ | 89.86 | -0.14 | 29 | $91 \cdot 11$ | +1.11 |
| S $2 / 3$ | 81.4 | 29.93 | -0.07 | 94.02 | -0.98 | 30 | 94.25 | -0.75 |
| S 1/6 | 81.5 | 29.96 | -0.04 | 94.13 | -0.87 | " | " | " |
| G 4/14 | $82 \cdot 1$ | $30 \cdot 18$ | +0.18 | 94.83 | -0.17 | " | " | " |
| B 7/15 | 82.9 | $30 \cdot 48$ | +0.48 | 95.75 | +0.75 |  |  |  |
| G 4/3 | 89.3 | 32.83 | $-0.17$ | 103.14 | +0.64 | 33 | 103.67 | $+1.17$ |
| B 4/4 | 92.0 | 33.82 | -0.18 | $106 \cdot 26$ | -1.24 | 34 | 106.81 | +0.69 |
| L 1/3 | 93.7 | 34.45 | +0.45 | 108.22 | +0.72 |  |  |  |
| S 1/1 | 107.6 | 39.56 | -0.44 | 124.28 | -0.72 | 40 | 125.66 | -0.66 |
| S 1/5 | 108.3 | 39.82 | -0.18 | 125.09 | $+0.09$ | " | " | " |
| B 1/8 | 108.4 | 39.85 | -0.15 | $125 \cdot 20$ | +0.20 | " | " | " |
| S.2/1 | 108.5 | 39.89 | -0.11 | 125.32 | +0.32 | " | " | " |
| B 5/1 | 110.0 | $40 \cdot 44$ | +0.44 | 127.05 | -0.45 |  |  |  |
| B 7/15 | 119.9 | 44.08 | +0.08 | 138.48 | +0.98 | 44 | 138.23 | +0.73 |
| S 5/2 | 129.7 | 47.68 | -0.32 | 149.80 | -0.20 | 48 | 150.80 | +0.80 |
| N 1/13 | 188.3 | 69.23 | +0.23 | 217.49 | -0.01 | 69 | 216.77 | -0.73 |
| $\Sigma \epsilon^{2}$ |  | 4.0114 |  | 24.6227 |  | 46.8803 |  |  |
|  |  | $\begin{gathered} 58 \\ 0.0691 \end{gathered}$ |  |  | $58$ | 58 |  |  |
| $s^{\text {a }}=$ |  |  |  | 0.425 |  |  | 08 |  |
| $\underset{s^{2} / \delta^{2}}{\text { Quantum }}=2 \delta$ |  | 1.000.276 |  | 2.5 |  | 2.5 |  |  |
|  |  | 0.272 |  | 0.517 |  |  |
| Probability level |  |  |  | 8\% |  | - |  |  |

We see that in the same way and for the same reason the four circles with a nominal diameter of 10 were increased and of the six with $D=18$ five were increased, thus in both sets improving the circumference. An examination of the whole table shows, however, that not all have suitable adjustments. This may be due to errors or uncertainties in the determination of the diameters. So we must apply a statistical method to see if the improvements are significant.

The ' lumped variance ' of the actual circumference $P_{\mathrm{a}}$ is

$$
s^{2}=\left(\sum \epsilon_{\mathrm{pa}}^{2}\right) \div n=0 \cdot 425 \text { (see Table 5.7) }
$$

The quantum $2 \delta$ is $2 \frac{1}{2}$. This makes $s^{2} / \delta^{2}=0.272$ and so from Fig. 2.1 we see that the probability level is about 6 per cent. In the ordinary way this is not low enough for acceptance of the hypothesis, but in the context it is definitely significant. We are dealing with a set of data chosen so that with no adjustments to the diameters we ought to get something very far from significant. To show this, suppose that these circles had been set out with their exact nominal diameters. The residuals would then have been those in the last column, which is seen to give $s^{2}=0 \cdot 808$ or $s^{2} / \delta^{2}=0 \cdot 517$. As explained on p .11 , a random distribution would give $s^{2} / \delta^{2}$ very near to $\frac{1}{3}$. This has become 0.517 because we are dealing with nominal diameters chosen because they do not fit. The improvement shown by the actual diameters over the nominal (from 0.517 to 0.277 ) is so great that adjustments in the right direction must have been made by the builders in a significant number of cases. Uncertainties in the surveys would act in a random manner and could produce no improvement of this magnitude.

Full adjustment of the diameters would, in most cases, make the diameter too far from the integral value and so has not in general been made. To look into this the actual diameters in the table have been analysed to see if they have remained near enough to integers to continue to show significance. It will be seen from the figures below the first four columns that the probability level is also about 8 per cent. Thus the adjustments made have left the diameters near enough to integers to show some significance ( 8 per cent probability level) while making the circumferences significantly multiples of $2 \frac{1}{2}$.

The statistical examination just made thus bears out the impression formed from a visual examination of the histogram in showing that when the diametral and circumferential conditions were irreconcilable a compromise was effected.

## Possible effect of the above adjustments to diameters on the derived value of the yard

In the investigation into the value of the Megalithic yard made on p. 42 all diameters of reasonable accuracy were used. We have just seen that when the circumference did not measure up to a multiple of $2 \frac{1}{2}$ the erectors usually changed the diameter slightly from its integral value. Since this adjustment may have had an effect on the derived value of the
yard, the calculation has been repeated retaining only those circles where little or no adjustment was necessary, i.e. where $\pi D$ was within 0.625 of a multiple of $2 \frac{1}{2}$, thus excluding the circles in Table 5.7.

There is some evidence that the non-circular rings were also adjusted, so it would be safer to exclude those with nominal diameters giving perimeters which do not satisfy. The ratios of the perimeter to the main diameter for flattened circles are

$$
\begin{array}{ll}
\text { Type A - } 3.0591 & \text { Type B }-2.9572 \\
& \text { Type D }-3.0840
\end{array}
$$

With this information we can calculate the perimeters from the nominal diameters and discard those outside the range. From the remainder we find the values in Table 5.8 below.

## Table 5.8

|  | $n$ | $\sum y$ | $\sum m$ | $\sum m^{2}$ | $\sum m y$ |
| :--- | :--- | :--- | ---: | :--- | ---: |
|  |  |  |  |  |  |
| Circles | 55 | 2866.0 | 1054 | 27536 | 74863.2 |
| Non-circular rings | 16 | 1119.4 | 411 | 13665 | 37232.5 |
| All | 71 | 3985.4 | 1465 | 41201 | 112095.7 |

From these we can deduce that both for the circles alone and for all, $\beta$ is small $(+0.03$ and $-0 \cdot 02$ ). So we put $\beta=0$ and find $2 \delta=2.719$ for the circles and 2.721 for circles and rings together. Thus we obtain a complete check on the previously deduced value.

A possible criticism of these methods of deducing the value of the yard is that we must start off the calculation by using an initial value. Since we used 2.72 as initial value and end up with 2.720 one might wonder if the calculation means anything. In fact, the process is one of successive approximation and what we are seeing here is the result of many years of preliminary work. Some confirmation of the yard can also be obtained from the consideration of the distances between circles given in the next section.

## Distances between circles

The distance between circle centres is a satisfactory length to examine for investigating the use of the yard in longer distances. If an accurate survey of a circle exists the centre can be found with a precision equal to or greater than that of the diameter. The distance between two such centres is then an unambiguous length unaffected by any additive constant. There are many places where there are two, three, or more circles in a group. Details from those so far surveyed will be found in Tables 5.9 and 5.10. It will be seen that four of the
sites used have three circles in each. If the directions (azimuths) of the sides of the triangle formed by the circle centres are controlled by other considerations (perhaps astronomical) then the length of only one side is disposable; once one side is fixed in length the lengths

Table 5.9. Distances between circles
$l=$ distance between circle centres (feet)

| Site |  | $l$ | Site |  | $l$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| L 1/6 | Burnmoor | $122 \cdot 3$ | W 2/1 | Penmaen-Mawr | 829.0 |
| " | " | $150 \cdot 4$ | W 11/2 | Trecastle | 144.2 |
| " | " | 340 | W 11/4 | Usk River | 365-8 |
| " | " | 420 | B 7/1 | Clava | 189.2 |
| " | " | 1568 | " | " | $232 \cdot 7$ |
| " | " | 1307 |  |  | 413.7 |
| " | " | 1297 | B 1/26 | Loanhead | $65 \cdot 3$ |
| " | " | 1238 | B 1/27 | Sands of Forvie | 132.8土 |
| " | " | 1375 | " | " " | $144 \cdot 0 \pm$ |
| S $11 / 1$ | Hurlers | 1515 419.1 | B3/3 | Raedykes | ${ }_{315.5} 24.8$ |
| " | " | 204.0 | B 4/1 | Carnoussie | 163.1 |
|  |  | $215 \cdot 9$ | G 7/4 | Loupin Stanes | $65 \cdot 5$ |
| S 2/1 | Grey Wethers | 128.1 | N $2 / 3$ | Shin River | $119 \cdot 8$ |
| S 3/1 | Stanton Drew | 381.0 | P 1/14 | Tullybeagles | 54.0 |
| " | " | 711.5 | P 2/8 | Shianbank | $70 \cdot 5$ |
| " | " | $1054 \cdot 0$ |  |  |  |

Table 5.10. Distances between circles Megalithic yards

| Site | $L$ | Site | $L$ | Site | $L$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| P 1/14 | $19 \cdot 9$ | B 4/1 | 60.0 | L 1/6 | 154.4 |
| B 1/26 | 24.0 | B 7/1 | $69 \cdot 6$ | S 1/1 | $154 \cdot 1$ |
| G 7/4 | $24 \cdot 1$ | S 1/1 | 75.0 | S 3/1 | $261 \cdot 6$ |
| P 2/8 | $25 \cdot 9$ | S 1/1 | 79.4 | W $2 / 1$ | $304 \cdot 8$ |
| N 2/3 | 44.0 | B 7/1 | $85 \cdot 6$ | S 3/1 | $387 \cdot 5$ |
| L 1/6 | $44 \cdot 9$ | B 1/27 | $90 \cdot 7$ | L 1/6 | $455 \cdot 1$ |
| S 2/1 | 47-1 | B 3/3 | 116.0 | L 1/6 | $476 \cdot 8$ |
| B 1/27 | $48 \cdot 8$ | L 1/6 | 125.0 | L 1/6 | $480 \cdot 5$ |
| B 1/27 | $52 \cdot 9$ | W 11/4 | $134 \cdot 5$ | L 1/6 | $505 \cdot 5$ |
| W 11/2 | 53.0 | S 3/1 | $140 \cdot 1$ | L 1/6 | 557.0 |
| L 1/6 | $55 \cdot 3$ | B 7/1 | 152.1 | L 1/6 | $576 \cdot 5$ |

of the other two follow. At one site there are five circles and so even if there is no restriction on the azimuths out of the ten sides only seven are disposable (p. 33). So to construct such a figure with all ten sides integral multiples of a unit is in general impossible. A great deal of trial and error might result in an approximate solution. Perhaps such an approximation was attempted at Burnmoor (L 1/6), where out of ten lengths eight seem to lie within $\frac{1}{2}$ MY of being multiples of $2 \frac{1}{2}$ MY, but this site needs to be re-surveyed before we can be certain of the longer lengths. The points just mentioned should be remembered in examining the material presented in Tables 5.9 and 5.10.

In the first table all the distances between circles are collected and given in feet. In the second (Table 5.10) the distances have been converted to Megalithic yards and arranged
in order of size. It may be noticed that the first ten items, i.e. up to a distance of 53 yds , all lie very close to a whole number. These are swamped by the larger values further down the table, as is evident in the analysis given in Table 5.11, but they do contribute to the favourable $s^{2} / \delta^{2}$ shown by the half-yard. The use of $2 \frac{1}{2}$ MY and 5 MY as units for measuring the longer distances is brought out. The importance of the former measure will become apparent in the study to be made later of the perimeters of ellipses and egg-shaped rings. We have already seen how it affected the diameters of circular rings.

## Table 5.11. Probability Levels

| Assumed quantum <br> $2 \delta(\mathrm{MY})$ | $\Sigma \boldsymbol{\epsilon}^{\mathbf{2}}$ | $\boldsymbol{s}^{\mathbf{2}}=\frac{\Sigma \boldsymbol{\epsilon}^{2}}{n}$ | $s^{2} / \delta^{\mathbf{2}}$ | Probability level <br> (Fig. 2.1) |
| :--- | ---: | :--- | :--- | :--- |
| 0.5 | 0.34 | 0.0103 | 0.165 | $0.06 \%$ |
| 1.0 | 2.44 | 0.074 | 0.296 | 30 |
| 2.5 | 12.79 | 0.39 | 0.25 | 5 |
| 5.0 | 45.04 | 1.36 | 0.22 | 2 |
| 10.0 | 395.04 | 12.0 | 0.48 | - |

To appreciate fully the arrangement of the Burnmoor circles let us anticipate the astronomical results for this site. In Table 8.1 it will be seen that at least six of the lines joining the circles give important declinations, one of the lines in both directions. So the azimuths were controlled astronomically, and yet looking at Table 5.10 it appears that of the ten lengths seven are within one unit of being a multiple of 5 and several are much closer. The surrounding mountains helped with the astronomical part of the problem, but to solve this and at the same time satisfy the length requirement seems almost impossible. We shall see at Castle Rigg in the same district another perhaps more striking but not more remarkable example of the combination of geometrical design with astronomical requirements. We begin to see why these people went to such trouble to mark permanently the five points they had established after presumably years of experiment.

## The residuals

It is of interest to examine the data in Tables 5.1 and 5.9 by making a histogram of their residuals from the Megalithic yard. For the circles and rings we take

$$
\epsilon=|R-2.72 \mathrm{~m}| \text {, where } R \text { is the radius, }
$$

and for the distances between circles we take

$$
\epsilon=|I-2.72 m| .
$$

As indicated we do not distinguish between positive and negative values.

A histogram for both lots combined is shown in Fig. 5.2 (a). The evidence for the Megalithic yard is of course the high pile at the left, which simply shows the tendency for the various measurements to cluster together round multiples of 2.72 ft . There is also a cluster at the right indicating that some 30 per cent of the measures contain a half-yard.


Fig. 5.2. Histograms of deviations, $\epsilon=|I-2.72 \mathrm{~m}|$, compared with some suggested gaussians. In each case the full line shows the sum of the gaussians. (a) Radii and distance between circles. (b) Distance between stones. (c) and (d) Radii and distance between circles and distance between stones.

There is a suspicion of a concentration in the middle. If real, this indicates the occasional use of the quarter-yard. Three gaussian distributions have been drawn in, each having the same standard deviation ( $0 \cdot 28$ ). Summing these gives the full line, which is seen to approximate to the actual histogram, but we are on insecure ground here because there seems to be no rigid mathematical method of investigating this arbitrary subdivision and in fact other gaussians can be drawn to represent the data.

## The distances between stones

In view of the difficulty of assigning a probability level to the idea that the yard was subdivided into halves and quarters it was decided to seek other evidence. This is to be found in the distances between stone centres where the stones obviously belong to the same line. We exclude the distances between stones in circles. A histogram of these separately showed nothing. The diverging stone rows in Caithness were also excluded. An analysis of these has already been published (Thom, 1964) and this shows that the spacing in these rows was controlled by other considerations.

A list was made of the distances between the centres of the stones in all the other alignments and stone rows in the author's surveys. Where there is an obvious gap in the row the distance between the stones on either side of the gap was used. Even if a stone is really missing this procedure cannot affect the issue, because if each of the original intervals was an integer then the sum (i.e. the distance measured) is also an integer. All distances over 30 ft were arbitrarily neglected.

It should be said that no two individuals will make the same selection. The reason becomes obvious if Figs. 12.9 and 12.13 are examined. Where one man would include a given stone as being in the line another would pass it over. A first survey of the material was published in 1961. A re-measurement of all the surveys in 1964 (with of course much new material) showed many differences in selection. But the over-all picture remains the same. The up-to-date histogram will be found in Fig. 5.2 (b). This shows the deviations from the integral yard. Here again we have concentrations at the ends (yard and half-yard) and the possibility of the quarter-yard. The combination of all the data in Fig. 5.2 (a) and (b) is shown in Fig. 5.2 (c). An attempt has again been made to explain the observed distribution by three gaussians of the same standard deviation. This would seem to be the best explanation, but the kind of thing suggested in Fig. 5.2 (d) cannot be entirely ruled out. Here we assume that the majority of the circles set out to the full yard were carelessly done ( $s . d=0.56$ ) but that a few and also those using the half-yard were more carefully laid out (s.d. $=0 \cdot 17$ ). The fit is seen to be reasonable, but the idea of two distinct groups is less attractive.

One fact emerges very dearly and that is that there is no evidence whatsoever of the yard being subdivided into three. This would have shown up by the appearance of a lump in the histograms two-thirds along from the left, i.e. at 0.91 ft . This is just where all three histograms are low; in fact where the combined histogram is at its lowest.

Chapter 5: Megalithic Unit of Length. A. Thom, Megalithic Sites in Britain, Clarendon Press, Oxford 1971:34-53.

## 13

## THE EXTINCTION ANGLE

THE lowest apparent altitude at which we can see a particular star on a perfectly clear night is called its extinction angle. This angle depends primarily on the magnitude of the star and to a lesser extent on the observer. The atmospheric conditions are of secondary importance because the definition excludes anything but the clearest conditions. We must not think of the man-made conditions which exist in Britain today in any inland position where no matter what the direction of the wind the atmosphere carries sufficient smoke to reduce visibility on the horizon to a few miles. Think rather of clear conditions on the north-west coast, where we can see mountain perhaps a hundred miles away.

Table 8.1 shows the collected azimuths of observed lines and the consequent declinations. A star, with a date attached, is named in column 8 if between 2000 and 1600 B.C. its declination was near the deduced value. In a number of cases the horizon altitude was below the probable extinction angle and so Neugebauer's value for the latter was used instead of the observed altitude. These lines are collected in Table 13.1 and the reverse process applied That is, we assume a date arid calculate the altitude which, with the latitude and the observed azimuth, will give the declination of the star at that date This altitude is assumed to be the extinction angle for the star, but it will only be the true extinction angle if we have assumed the correct date or if the star's declination did not change seriously with time. On plotting all the values so obtained on the magnitudes of the stars, we ought to obtain the relation between extinction angle and magnitude. What we in fact find is a rather scattered picture which nevertheless helps us to form an opinion on the legitimacy of associating these lines with stars. We could only expect to find a nice tidy line if we were in a position to assign the correct date to each site. All we can do is to use a mean date for the whole country and by trying two or three such dates we may be able to see which suits best. It will be realized that this procedure makes use only of the small number of lines which have a low enough horizon altitude.

The method of calculation can be arranged to make use of Table 3.1, relating declination with azimuth, altitude, and latitude.

Let $h_{\mathrm{E}}$ be the extinction angle. Correcting this for refraction gives the corresponding true altitude $h_{\mathrm{r}}$.

Put $\delta_{0}=$ declination as calculated from the observed azimuth and the known latitude with zero true altitude.
Table 13.1. Extinction angles deduced by assuming a date

| Site | Az. | $\delta_{0}$ | $a$ | Star | Mag. | 1800 B.C. |  |  | $\frac{1900 \text { в.с. }}{h_{F}}$ | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\delta$ | $\left(\delta-\delta_{0}\right) / a$ | $h_{E}$ |  |  |
| B 3/3 | $259 \cdot 2$ | $-5.84$ | 0.84 | Bellatrix | 1.7 | $-5.07$ | $+0.92$ | +1.16 | $+0.84$ | Uncertain outlier |
| G 4/14 | $78 \cdot 2$ | $+6.76$ | 0.82 | Altair | 0.9 | 6.98 | 0.27 | 0.72 | 0.90 |  |
| H $1 / 1$ | $77 \cdot 8$ | 6.40 | 0.86 | " |  |  | 0.67 | 1.07 | 1.27 |  |
| H 1/3 | 280 | $5 \cdot 25$ | 0.83 | Procyon | 0.5 | $6 \cdot 28$ | 1.24 | 1.58 | 1.34 | Azimuth uncertain |
| H 3/12 | $281 \cdot 9$ | $6 \cdot 36$ | 0.85 | Altair | 0.9 | 6.98 | $+0.73$ | $1 \cdot 12$ | $1 \cdot 24$ | Azimuth definite |
|  | " | , | " | Procyon | $0 \cdot 5$ | $6 \cdot 28$ | $-0.09$ | 0.41 | 0.80 |  |
| L 1/10 | 354.0 | $35 \cdot 36$ | $1 \cdot 00$ | Deneb | 1.3 | 36.64 | $+1.28$ | 1.62 | $1 \cdot 54$ | Good outlier |
| M 1/4 | 342 | $31 \cdot 58$ | 0.98 | Capella | 0.2 | 32.47 | 0.91 | 1.28 | 0.75 | Azimuth uncertain |
| P 1/2 | $13 \cdot 5$ | 32.77 | 0.99 | Capella | $0 \cdot 2$ | 32.47 | $-0.30$ | 0.24 | $-0.22$ | Azimuth uncertain |
| S 5/3 | $340 \cdot 2$ | 35.91 | 0.96 | Deneb | $1 \cdot 3$ | 36.64 | $+0.76$ | $1 \cdot 26$ | $+1.25$ |  |
| S 5/4 | $31 \cdot 0$ | 32.48 | 0.92 | Capella | $0 \cdot 2$ | 32.47 | $-0.01$ | $0 \cdot 48$ | $-0.01$ |  |
| S 6/1 | $28 \cdot 7$ | 32.71 | 0.93 |  | " |  | $-0.26$ | 0.26 | $-0.20$ |  |
| W 2/1 | $18 \cdot 6$ | $34 \cdot 56$ | 0.98 | Deneb | $1 \cdot 3$ | $36 \cdot 64$ | $+2 \cdot 12$ | 2.40 | $+2.30$ | Peculiar site |
| W $5 / 1$ | $17 \cdot 3$ | $35 \cdot 15$ | 0.98 | " | " | " | $1 \cdot 52$ | 1.83 | 1.76 |  |
| " | $349 \cdot 8$ | 36.41 | 0.99 | " | " | " | 0.23 | $0 \cdot 68$ | 0.62 | Azimuth uncertain |

Then we can with sufficient accuracy write the actual declination as

$$
\text { declination }=\delta_{0}+a h_{T},
$$

where $\delta_{o}$ and a are both found from Table 3.1, a being $d \delta / d h$ or the change in declination produced by unit change in $h_{T}$.

Let $\delta$ be the star's known declination at the assumed date. Then if we are correct in associating the observed azimuth with the star,

$$
\begin{aligned}
\delta & =\delta_{0}+a h_{T}, \\
h_{T} & =\left(\delta-\delta_{0}\right) / a .
\end{aligned}
$$

from which
Applying the appropriate refraction difference we obtain the extinction angle $h_{E}$.
The above method of reduction is applied in Table 13.1 to all germane lines in Table 8.1. Although the existence of the intermediate calendar dates may not yet be fully accepted it seemed better to omit lines which would belong to these instead of trying to associate them with stars. These lines were not omitted in the calculation made in Thom, 1966, because when that paper was written the existence of intermediate dates dividing the year into thirty-two parts was only beginning to be suspected. It will be noticed that the accurate line at H $3 / 12$ has been associated with two different stars, Altair and Procyon. Since both assumptions yield reasonable extinction angles we may assume that this line was an indicator for both these stars. The different magnitudes, by producing different extinction angles, allowed the line to be used with two different declinations. While the foresight (Craig Hasten, see p. 130) makes such a definite mark it might not be visible in starlight, but this is no real objection to this line. The backsight Clach Mor à Chè is so accurately orientated that the identification of the stars would have been quite possible even if no fire was lit at the foresight. There would of course be no danger of confusing these two stars one with another.

The calculation is shown in some detail assuming a mean date of 1800 B.C., and the results only for 1900 B.C. The extinction angles so found are plotted in Fig. 13.1 (a) and (b). On the whole it will be seen that 1800 B.C. gives a more reasonable set of points than 1900, and certainly shows better agreement with Neugebauer's values, which are indicated by a dotted line.

Amongst the points are two or three ascribed to Deneb. For finding extinction angle this is the most useful star because its declination is practically independent of the date.

Apparently no improvement would be obtained by trying 1700 B.C. and so it appears that if we are right in associating these lines with the stars shown the resulting date is not far from 1800 B.C. A totally different approach is possible and that is to assume the true


Fig. 13.1. Extinction angles deduced from observed azimuths assuming (a) 1900 B.C. and (b) 1800 B.C. Unreliable values are shown by open rings. Neugebauer's values are shown by a dotted line.
extinction angle $h_{T}$ to be linear with magnitude and the stars' declinations to be linear with time. Every line then gives an equation with three unknowns. Solving these by the usual least-squares method yields the date and extinction angle. This method was tried at an earlier stage of the investigation and gave reasonable results, but it is felt that there are objections, even if an extension was made to include lines with horizon altitudes above the extinction angle. There is not enough material for a fully fledged statistical calculation of this kind and an over-all mean date is all that could be obtained.

Chapter 13: The Extinction Angle. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford, 1971: 160-163.

## 14

## CONCLUSIONS

We have in foregoing chapters tried to assess Megalithic man's knowledge of metrology, geometry, and astronomy. An attempt has been made to present the evidence in such a way that the reader can form his own opinions. Perhaps in this summary the author may be allowed to give his own conclusions. The reader of necessity sees the subject from a different standpoint and the earlier chapters on statistics, mathematics, and astronomy were inserted to make it easier for him to understand the author's viewpoint and methods of working.

Once we had discovered how the flattened circles were designed it became obvious that we were dealing with a people who had mastered elementary geometrical construction. When the egg-shaped rings were studied they revealed the remarkable interest shown by the builders in units of measurement and the concomitant attempt to discover Pythagorean triangles. This led to the discovery that the obsession with integral lengths extended also to perimeters; witness the manner in which integral diameters were so often slightly adjusted to make the circumference more nearly a multiple of the larger unit. The ellipse may have been extensively used because of the greater freedom it presented in choosing sizes which would satisfy the desire to use integers in the perimeter as well as in the straight dimensions. Symmetrical figures were the rule and yet the greatest circle of all, at Avebury, shows no symmetry. The other great site at Callanish shows symmetry only in the Type A ring at the centre. There is only one obvious explanation of the skew construction used at Callanish and that is that the alignments were for astronomical purposes. The fact that these alignments and the axis of the small ellipse lead from one of the auxiliary centres of the main ring shows that peculiar attempt to combine geometrical construction with astronomical azimuths which achieves its most spectacular success at Castle Rigg. In this connexion the circles at Burnmoor are not far behind, although there we need to dig a little deeper to appreciate fully what was achieved. The greatest and most remarkable circle in Britain, if not in the world, is at Avebury. Its greatness does not lie in its size alone but in the remarkable manner in which its arcs are built up from a basic Pythagorean triangle so that each retains an integral character, and in the exceedingly high precision of the setting out, a precision only surpassed today in high-class surveying. Avebury provides the final proof of the exact size of the Megalithic yard and demonstrates the use of the larger linear units, $2 \frac{1}{2}$ and 10 yds .

It is strange that the beauty of design achieved at Moel ty Ucha or Easter Delfour is not and cannot ever have been apparent on the site. Nor can it ever have been obvious
that these designs incorporated those peculiar integral ratios which form the main theme of all the constructions. These features cannot have meant much to the majority of the people any more than they would to the man in the street today. Yet to invent designs with these properties probably took years of many men's time. Perhaps the proportions were worked out on the sands of the seashore, only to be expounded to the chosen few.

So much for metrology and geometry. What about astronomy? The evidence mutely presented by Ballochroy shows unequivocally the intense interest in the solstices. The division of the year into eight parts will hardly be denied by anyone. The evidence for the division into sixteen parts has been growing for many years and is of such a nature that it only falls into place when the idea is worked out in detail. Whether we are prepared to accept it or not a similar, albeit smaller, body of evidence is accumulating for the division into thirty-two parts. The idea that these parts were always either eleven or twelve days fits better than any other arrangement. The great body of the information on the calendar has come from the north as far down as Wales. The paucity of the south country in this respect may be due to destruction of sites or to the difficulties associated with tree-covered horizons. But until more evidence comes along we cannot exclude the possibility of a different form of calendar.

To a people so interested in the sun much thought must have been given to the possibility of predicting eclipses. Soon it would be apparent that this involved a study of the moon. As far back as 1912 Somerville suggested that there was a lunar line in the Callanish layout. The author, through a fear of building evidence subjectively, resisted accepting lunar lines until the final evidence came objectively. When the first histogram of the possible lunar lines was plotted it showed a double peak corresponding to the two limbs of the moon. This result was unexpected and it was so unlikely to have happened by accident that it seemed desirable to look more closely into a number of sites where the indication of the necessary azimuth at the site itself was weak. This study showed up that Megalithic man was well acquainted with the small amplitude ripple on the moon's declination and has left such definite indicators that we can, with their help alone, determine its magnitude. We do not know of any technique which could have been used to examine this oscillation with the moon at the nodes, but they could have made a measurement of its period and may have connected it with the eclipse year.

Attempts to date the sites by stellar declinations depend on being able to associate an observed azimuth with a particular first-magnitude star. If the evidence put forward in the chapter on extinction angle is accepted then one is entitled to go one step further and construct a histogram, on a date basis, of all the associations in Table 8.1. Three such histograms are shown in Fig. 14.1. The interval is fifty years and when a date is at the boundary between two intervals half has been allotted to each of the intervals. Fig. 14.1 (a) shows all lines in England, Wales, and Scotland south of the Clyde. Fig. 14.1 (b) shows all lines north of the Clyde, and Fig. 14.1 (c) is for all Britain.


Fig. 14.1. Histograms of dates from star declinations (see Table S.1), (a) south of Clyde, (b) north of Clyde, (c) all Britain. Dates like 1800 B.C. have been put half each way.

It is difficult to think of a reason for the clumping together of the dates in both (a) and (b) other than that many of the observed azimuths really were set out for first-magnitude stars, It may be noted that the centre of the concentration is about 1860 B.C. for the southern lines and about 1810 for the northern, and that most of the stellar lines were erected between say 2000 and 1700 B.C.

Because of the slow rate of change in the obliquity of the ecliptic it is difficult to get an accurate date from a solstitial site, but in Thom, 1954, a value close to 1800 B.C. was obtained by an elaboration of this method. There is a possibility that the lunar lines which showed up the small oscillation mentioned above will give a more accurate measure of the obliquity than the solar lines. Already in Chapter 12 it appears that they show a mean date for the north of 1800 B.C. $\pm 100$ and there seems to be hope of improving the accuracy.

But the whole position will be much more satisfactory when archaeologists date the sites by entirely different means and the kind of data used in this book can be used to make a detailed study of the astronomical work of Megalithic man.

When we think of the conditions under which these people worked and the limited material aids which they could employ we begin to appreciate what they did achieve. There are hundreds of sites throughout Britain which can surely teach us a great deal more if they are examined in an unbiased manner. Whatever we do we must avoid approaching the study with the idea that Megalithic man was our inferior in ability to think.

Archaeology is today advancing so rapidly that its findings may link up with the major findings of the present study and may give a meaning to much that is obscure.

Chapter 14: Conclusions. Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford 1971:164-166.

## LISTOFDISTRICTS

| H 1 | Lewis |  | Caithness | B 1 | Aberdeen N |
| :---: | :---: | :---: | :---: | :---: | :---: |
| H2 | Harris |  | Sutherland | B 2 | " " S |
| H 3 | N. Uist | N 3 | Ross \& Cromarty | B 3 | Kincardine |
| H4 | Benbecula |  |  | B 4 | Banff |
| H 5 | S. Uist | P 1 | Perth W of Tay | B 5 | Elgin |
| H 6 | Barra | P 2 | " E | B 6 | Nairn |
| H7 | Skye |  | Forfar | B 7 | Inverness (mainland) |
| H 8 | The Small Isles | P 4 | Fife | W 1 Anglesey |  |
|  |  | P 5 | Kinross |  |  |
| M 1 | Mull N | P 6 | Clackmannan | W 2 | Caernarvon |
| M 2 | " "S | P 7 | Stirling | W 3 | Denbigh |
| M 3 | Coll |  |  | W 4 | Flint |
| M 4 | Tiree |  | Ayreshire | W 5 | Merioneth |
| M 5 | Ardnamurchan | G 2 | Wigtownshire W | W 6 | Montgomery |
| M 6 | Morven | G 3 | "" E | W 7 | Cardigan |
| M 7 | Appen | G 4 | Kirkcudbright W | W 8 | Radnor |
| M 8 | Benderloch | G 5 | "" E | W 9 | Pembroke |
| M 9 | Lismore | G 6 | Dumfriesshire W | W 10 Carmarthen <br> W 11 Brecknock |  |
|  |  | G 7 | Dumfriesshire |  |  |
| A 1 | Lorne | G 8 | Roxburgh | W 1 | Glamorgan |
| A 2 | Argyll | G 9 | Midlothian | W 1 | Monmouth |
| A 3 | Knapdale |  | East Lothian |  |  |
| A 4 | Kintyre | " | Berwick | D 1 | Derbyshire |
| A 5 | Colonsay |  | Peebles | D 2 | Shropshire |
| A 6 | Jura |  |  |  |  |
| A 7 | Islay |  | L 1 Cumberland |  |  |
| A 8 | Arran |  | L 2 Westmoreland |  |  |
| A 9 | Bute |  | L 3 Northumberland |  |  |
| A 10 | Kerry \& L. Fyne E |  | L 4 Durham |  |  |
| A 11 | Loch Lomond |  | L 5 Lancashire |  |  |
|  |  |  | L 6 Yorkshire |  |  |
| S 1 | Cornwall |  |  |  |  |
| S 2 | Devon |  |  |  |  |
| S 3 | Somerset |  |  |  |  |
| S 4 | Dorset |  |  |  |  |
| S 5 | Wiltshire |  |  |  |  |
| S 6 | Oxfordshire |  |  |  |  |

## APPENDIX

## On calculating the azimuth line from the coordinates of the two ends

TODAY, as more and more of the country is covered by large-scale Ordnance Survey maps plotted on the National Grid, it is seldom necessary to use geographical coordinates (latitude and longitude) for the calculation of an azimuth.

The National Grid is a transverse Mercator projection. In the sense that the axis of the usual Mercator projection is the equator, the axis of the National Grid is a north-south line at longitude $2^{\circ} \mathrm{W}$. This does not mean that the origin of coordinates is at $2^{\circ} \mathrm{W}$. The origin is displaced to the south-west so that both coordinates are always positive for the land areas of Britain.

No error greater than $\frac{1}{4}$ minute of arc will be introduced by the following simple procedure. If great accuracy is required the special tables published for the Ordnance Survey must be used.

Read the coordinates of the two points from the largest-scale Ordnance Survey map available and to the greatest accuracy possible. Treat these coordinates as simple Cartesian coordinates and so calculate the azimuth from its tangent. Then for the end of the line at which the azimuth is wanted find the difference between grid north and true north. This is stated on the maps but interpolation may be awkward, especially with the 1 -inch maps. So it is often easier to calculate the correction from the relation

$$
\text { correction }=\Delta \lambda \sin \varphi,
$$

where $\Delta \lambda$ is the amount by which the longitude differs from $2^{\circ} \mathrm{W}$. and $\varphi$ is the latitude. The sign of the correction will be apparent from the values given in the map margin. The latitude and longitude can easily be obtained with sufficient accuracy from the 1 -inch Ordnance map.

If one end of the line lies in an area not yet covered by the maps then it is perhaps best to convert the geographical coordinates of the point to grid coordinates by the tables referred to above. It should be mentioned that the explanation of the use of the tables is contained in another small publication also issued by H.M. Stationery Office.

Appendix: (Azimuth Calculations). Thom, A. Megalithic Sites in Britain, Clarendon Press, Oxford, 1971:163.

Further selections from Megalithic Sites in Britain:

1. Introduction, Statistical Methodology, Requisite Tools, The Megalithic Yard, Conclusions
2. Circles, Rings, Megalithic Astronomy
3. The Calendar, Indications of Lunar Declinations
4. The Outer Hebrides, Variety of Sites

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Many plans will be found in the journals of archaeological societies in various parts of Britain, in the County Inventories of the Royal Commission on Ancient Monuments, and in the Old and New Statistical Account (Scotland).

# MEGALITHIC SITES IN BRITAIN (1971) 

## A. Thom

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[^0]:    For positive amp. interpolate the decl. from the table for the nearest of the three latitudes given and then apply the correction for latitude shown, is greater

    For negative amp. the decl. is negative and the correction for latitude is
    The correction for altitude is always positive if the true altitude is positive.

