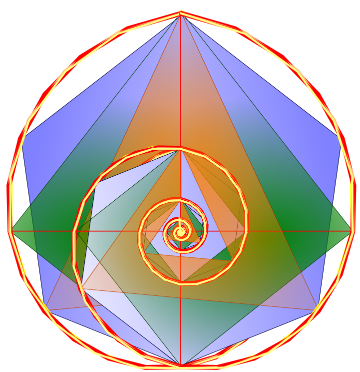
INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS



2023

PART FIVE

TIME AND TIDE:
THE SPIRAL FORM IN TIME AND PLACE

JNH

SPIRAL FORMS IN TIME AND PLACE

The initial pheidian equiangular spirals

After the preceding section it becomes clear that the preservation of knowledge, albeit convoluted, obscure and difficult was nevertheless continued forward over the intervening centuries albeit with the degree of success yet to be fully determined. As for the emphasis on the spiral form, perhaps this is best examined in terms of inquiries into Nature in keeping with the main aspects of the matter provided earlier by Ovid concerning the three-fold number with, perhaps, understandable interest in the many spirals which abound in nature fueling an ongoing inquiry into the details, and (where possible) the mechanics involved.

Either way these results lead next to a theoretical base (hereafter the *Pheidian planorbidae*), namely the ϕ-*Series* parameters ***T, R*** and ***Vi*** with initial emphasis on exponents 4, 5, 6 and 7 which generate distance relation (9), the mean sidereal period (*T*) for **Jupiter**, Synodic cycle ***SD1*** (*S*), and the mean sidereal period (*T*) of **Saturn**. Thereafter cube roots of these four periods yield the *inverse* velocities (***Vi***) from exponents **4/3**, **5/3**, **6/3** and **7/3** (hence the title "Thirds") and the *relative* velocities ***Vr*** from their respective reciprocals.

x	Planets/Syn	ϕ-*Series* $T = \phi^x$	ϕ-*Series R*	Thirds	Sixths	ϕ-*Series* Vi	ϕ-*Series* Vr
4	*Synodic* (*SD*)	6.8541019662	3.608281187	4/3	8/6	1.899547627	0.526441130
5	JUPITER #4	11.090169944	4.973080251	5/3	10/6	2.230040415	0.448422366
6	*Synodic* (*SD*1)	17.944271910	6.854101966	6/3	12/6	2.618033989	0.381966011
7	SATURN #3	29.034441854	9.446602789	7/3	14/6	3.073532624	0.325358511

Table 1. ϕ-*Series* mean periods (years), mean distances (a.u.) & mean velocities (*Vi*), (*Vr*) ref. unity.

The central value: *Vi* = ϕ^2 = 2.61803398875 already plays a major role in phyllotaxis and from ancient and modern sources provides a close approximation for the ratio of the mean synodic arcs for Saturn and Jupiter. Furthermore, it is also the reciprocal of the Pierce limiting constant ϕ^{-2} (0.38196601125), the *relative velocity* (*Vr*) of *SD1* between Jupiter and Saturn, also the mean *distance* for Mercury (*R*), and lastly, the period (*S*) of Mercury-Venus *Synodic* 7-8.

Plus, ***earlier and independently***, it must be acknowledged that the above constants and the distance variant: $\phi^{4/3}$ (1.899547627) were derived by **K. P. Butusov** (1997)[34] as ***Planet Period*** and ***Planet Distance Laws*** which are essentially relations (8) and (9) as developed and applied here. The main difference being, it would seem, that the latter also integrated relation (1) but not as the general synodic formula initially applied here to complete Benjamin Pierce's stalled planetary framework.

Spira Solaris

Thus the spiral *Spira Solaris, V*i = 2.61803398875, so-named because of its central importance above commencing with the equiangular "square" followed by the corresponding equiangular spiral which is inverted as shown below to provide a series of scalable spirals conforming to the free-swimming orientation of nautiloids and ammonites.

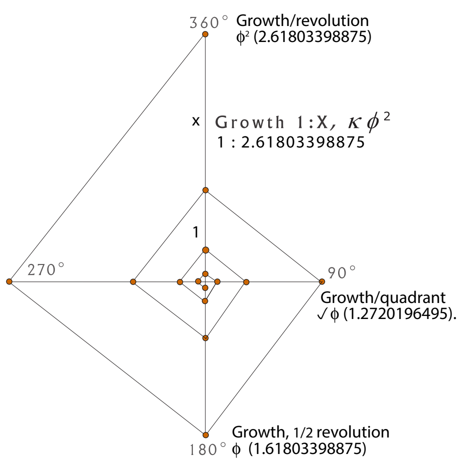


Fig.2a. The equiangular "square" spiral.

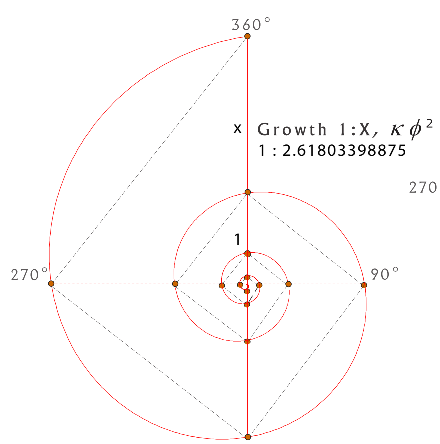


Fig.2b. Equiangular spiral "Spira Solaris."

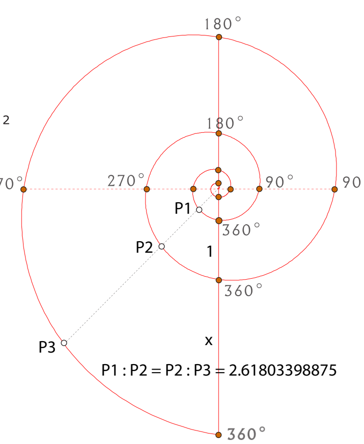


Fig. 2c. "Spira Solaris" Inverted

The Pheidias Spiral

Applied to an expansion factor provided by *Phi* itself (*w* = 1.61803398875 per revolution) the name *Pheidias Spiral* owes its origins to William Schooling's investigation of the mathematical relationships inherent in the *Phi*-series published in Cook's *The Curves of Life* (1914).[36] Shown in Figure 3 in its original form the *Pheidias* spiral is inverted, augmented by inner whorls, then color-coded to standardize with Spira Solaris and subsequent test spirals.

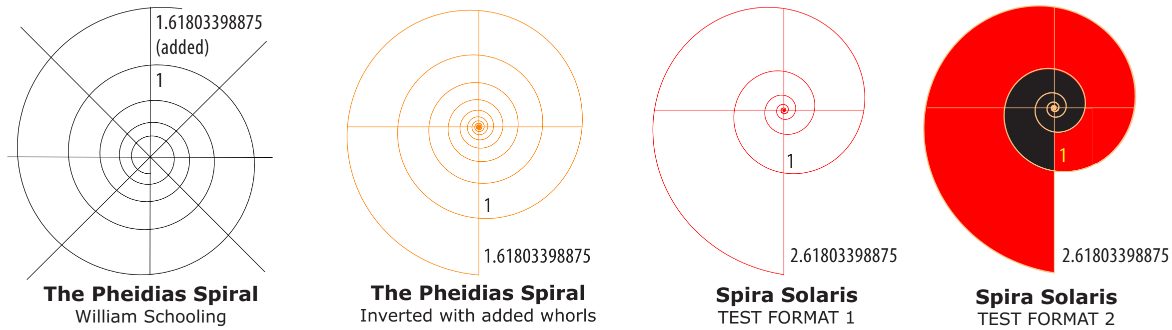


Fig.3. The Pheidias Spiral and Spira Solaris equiangular Test spirals. Line: 1, Solid: 2. Test spirals are equal in *size*, not *scale*.

Generation of equiangular spirals

At this juncture it should be noted that decades before the general availability of electronic computers a surprising number of investigations concerning growth, form and phyllotaxis were carried out during the first quarter of the previous century. It is, however, the wide-ranging *Curves of Life* (1914) by Sir Theodore Andreas Cook and even more expansive *On Growth and Form* (1917) by Sir d'Arcy Wentworth Thompson which are of immediate interest. It is the latter which provides the mathematical details concerning equiangular spirals, in particular those associated with *The shape of a nautiloid spiral* [37] with attendant whorl-to-whorl growth (w), the equiangles, and the following formula for the radius vectors:

$$r = e^{2\pi \operatorname{Cot} \alpha} \qquad (1)$$

This was expanded by wider descriptions of the general form of the coiled shell in 1966 [38] and 1967 [39] by David Raup (1933-2015) whereas the initial approach employed by Thompson included a table of corresponding growth factors ($w = 1/r$) and another in the form $r : 1$ with corresponding equiangles, stating " Here we have $r = e^{2\pi \operatorname{Cot}\alpha}$, or Log r = log e x 2π x cotα, from which we obtain the following figures," [40] explaining in a footnote: "It is obvious that the ratios of opposite whorls, or of radii 180° apart, are represented by the square roots of these values; and the ratios of whorls or radii 90° apart, by the square roots of these again." Which returns the inquiry to the *Rotation of the Elements,* the equiangular "square" and additional insights from Aristotle's cryptic statement in *On the Heavens,* namely: [41]

> . . . . . It is agreed that there are only three plane figures which can fill a space, the *triangle,* the *square* and the *hexagon*, and only two solids, the *pyramid* and the *cube*.

One can certainly generate all that is needed from relation (1) and above information provided by Thompson for any equiangular spiral required, but this relation, involving e, logarithms, radians and trigonometric functions is cumbersome, and also, as it turns out, unnecessary. Identical results for r per degree can in fact be obtained by the expansion of Aristotle's three figures, *i.e.*, addition of an equiangular triangle (120°) to the equiangular square (90°) followed by an equiangular hexagon (60°), etc. At which point, extending the process downwards to include 45°, 30°, 15°, 36°, 24°, 18°, 12°, 6°, 3°, 2° until, leading to an equiangular 360-gon (triacosihexacontigon) all the "spaces" are filled, *i.e.,* comprised of 1° segments. Or, better stated, the straight-line bases of narrow triangles per degree sufficient to generate *all* the equiangular spirals shown here according to the desired expansion rates per revolution. Furthermore, because the growth factors are already known, equiangles play no role in the generation of the spirals, nor do e, π, logarithms or trigonometric functions.

What *are* required for test purposes are standard formats and ranges which originally pertained to Spira Solaris, Pheidias and immediate vicinity in an astronomical context. Thus mean periods of revolution naturally expanded downwards to unity (*i.e.*, mean period of Earth) and outwards to that of Saturn, followed by the inverse velocity *Vi*. It is this constant that underlies each particular pheidian spiral subject to exponentiation per degree and continuity per revolution in the present treatment.

For example, a six-whorl spiral for *Spira Solaris* commences at 0° and extends to 6 • 360° = 2160° with the radius vectors per degree generated from the *pheidian* constant *SD*1 $Vi = w = \phi^2$ = 2.61803389875:

$$r = k \cdot w^{n/360} \quad (w = \text{pheidian growth rate, } k = \text{the starting point, } n = 0°, 1, 2, 3, ..., 2160°) \qquad (2)$$

$$r = k \cdot 2.61803389875^{(n/360)} \quad (k = \text{selected choice, } w = \phi^2, \quad n = 0°, 1, 2, 3, ..., 2160°) \qquad (2k)$$

with 1/4 for the 90° exponent (90°/360°), 1/2 for 180° and 360° per cycle. Thus back to the equiangular square plus all radius vectors for each intervening degree per revolution until 2160° is reached. The precise number of whorls selected is a matter of practical convenience; in general the lower the expansion rate (w) the greater the number of whorls and 360° cycles required, and *vice versa* for the larger rates of growth.

The Pheidian Planorbidae

Although the name "Pheidias" in the present context is provided by William Schooling's Fig. 389 of Sir Theodore Andrea Cook's *The Curves of Life* (1914:421), the name *Pheidian Planorbidae* owes its origins to something entirely different. In fact the name originated from early attempts by the writer to fit equiangular spirals to a variety of shells, ammonites, and in particular, the configuration of the earliest ammonite, *Psiloceras Planorbis.* Still subject to further refinement, the best fit for the latter is a pheidian growth rate of 1.8995476 per revolution, and therefore (perhaps coincidentally) *Phi*-series relation (9), the planet-to-planet increase in heliocentric distance ($\phi^{4/3}$).

The Pheidian Planorbidae *Thirds*

In so much as the present inquiry began with the Peirce planetary framework followed by the sequential inclusion of the Fibonacci, Lucas and *Phi*-series, the Golden Section (or Three-fold number) not only remains the underlying constant throughout, it also incorporates a dynamic quality provided by the *Spiral of Pheidias* with a growth rate *w* of 1.61803398875 per revolution. As does the square of this constant, *i.e.*, *Spira Solaris* = ϕ^2 = 2.61803398875, with the latter pair also the *Phi*-series constants for the planet-synodic-planet and planet-to-planet increases of *S* and *T*, *i.e.*, relations 7 and 8 respectively. All of which are expressed in terms of the inverse velocity (*Vi*) that increases by a multiplication factor of $\phi^{1/3}$ = 1.1739884997, thus from the lowest value in the set commencing at Mars the inverse velocities are all *exponential thirds.* Accordingly: $\phi^{1/3}$, $\phi^{2/3}$, $\phi^{3/3}$, $\phi^{4/3}$ $\phi^{5/3}$, $\phi^{6/3}$ and $\phi^{7/3}$ yield growth factors of *w* = *Vi* per revolution for seven sequential equiangular spirals with those of *Pheidias* and *Spira Solaris* naturally included.

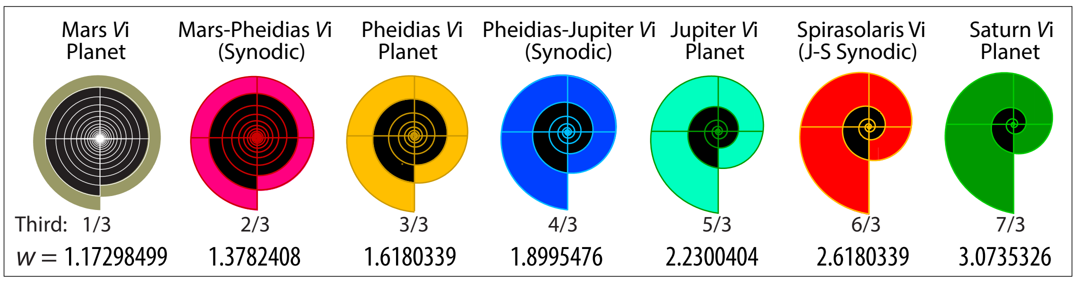


Fig. 3b. *The Pheidian Planorbidae.* Exponential *Thirds*, $w = \phi^{1/3}$ through $\phi^{7/3}$ (not to scale; $w = \phi^{N/3}$)

Does this assign the *origins* of the equiangular spirals to the complex, interactive motions of the planets *per se*, and and nothing else? Not necessarily, for there still remains an alternate possibility, which is that the very structure of the Solar System is itself a pheidian reflection of larger, *a priori* set of conditions and that the "three-fold number" is (or may perhaps be), as Aristotle states, "present in all things whatsoever."

As for the *Pheidian Planorbidae,* they at least provide a relatively narrow focus applied to planispiral ammonites (as Figure 3b shows) plus two additional benefits. Because the three-fold number in this planetary context embraces periods of revolution (*T*), heliocentric distances (*R*) *and* orbital velocity (*Vr*, *Vi*), the growth rate is truly present, *i.e.*, it is not limited to the expansion rate per revolution alone, it also provides *periodicity*. And, even though the periods may intuitively seem to be too high, the range for *Vi* from Earth to Saturn can be still checked against growth factors and morphospace contours for 405 ammonites assigned by David Raup (1967: 46-48).[42]

PLANETS N Synodics #	ϕ^x = x	*Phi*-series *T* (Years)	*Phi*-series *R* Distance (a.u.)	*Third* x/3	*Phi*-series *Vi* = Velocity *Vi*
Earth Synodic	0	1 (Earth)	1 (Earth)	1	*Thirds* $w = \phi^{N/3}$
MARS #6	1	1.618033989	1.378240772	1	1.173984997
Synodic 6-5	2	2.618033989	1.899547627	2	1.378240772
PHEIDIAS #5	3	4.236067977	2.618033989	3	1.618033989
Synodic 5-4	4	6.854101966	3.608281187	4	1.899547627
JUPITER #4	5	11.09016994	4.973080251	5	2.230040415
Synodic (SD1)	6	17.94427191	6.854101966	6	2.618033989
SATURN #3	7	29.03444185	9.446602789	7	3.073532624

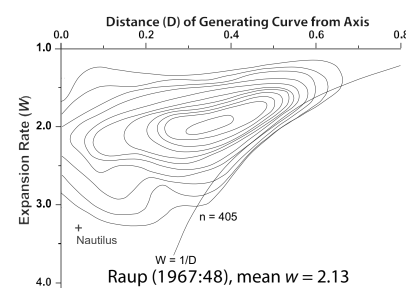


Table 2. *Phi*-Series data (Earth-Saturn), exponents (x), *T*, *R*, *Vi* (Inverse velocity); Raup (1967:48) *w* distribution.

Particularly relevant here (as Raup explains in his Summary) is *the problem of explaining why ammonoids generally have a W value less than 3.0 but greater than 1.25, and a D value less than 0.65.*[43] While *Vi* = *w* from 3.073533 through 1.173985 lies within the range range assigned by Raup as seen by their inclusion in Figure 4, with the THIRDS also added to the right vertical axis. But although the range for ammonite growth *w* given by Raup is in keeping with *Vi* = $\phi^{1/3}$ through $\phi^{7/3}$, further intermediate data is necessary to fill the logarithmic gaps between the seven additions.

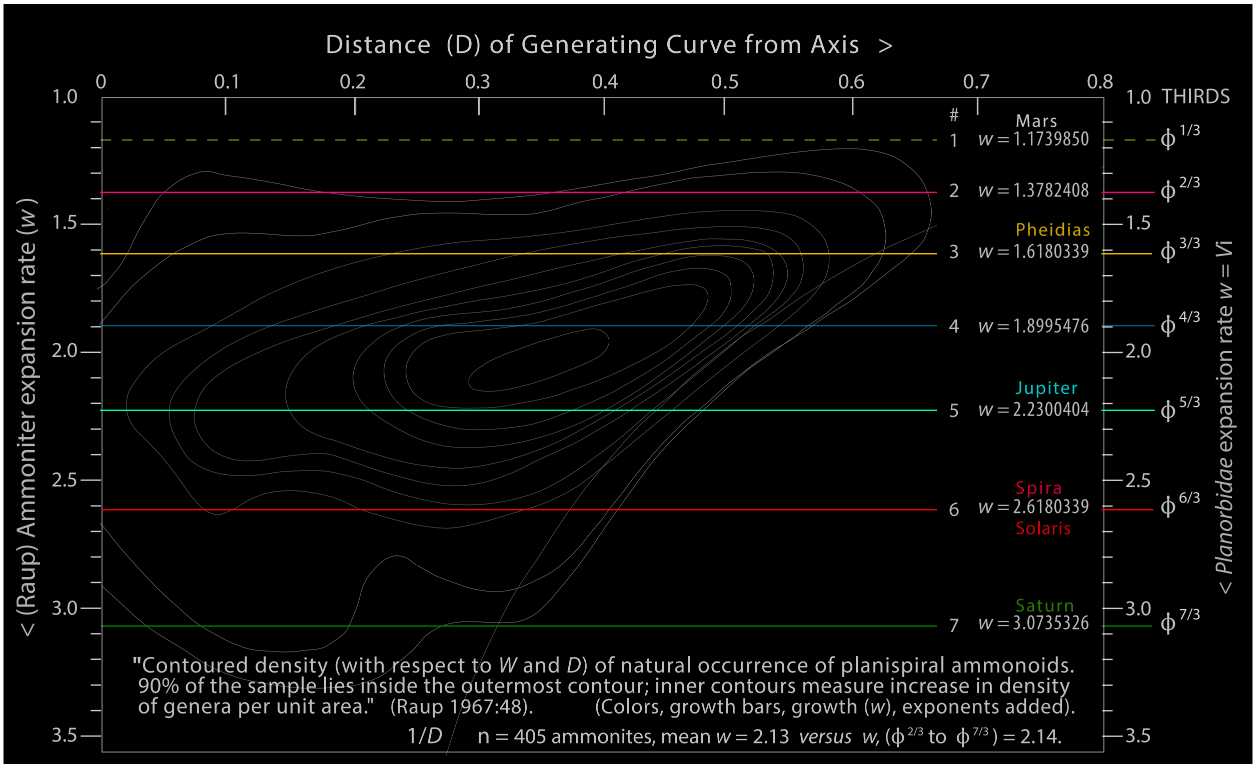


Fig. 4 *Pheidian Planorbidae* Thirds 1-7 added to Raup morphospace contours (*TEXT-Fig-4*.1967:48)

The Pheidian Planorbidae *Sixths*

This deficiency is initially met by considering the Lucas-based right triangles of Table 3 and the relation: √Lucas = *T* as a possible replacement for the ϕ-*Series* to increase the number of intervals between each planet. However, this is best achieved by the ½ ϕ-*Series* with geometric means Gm1s and Gm2s filling the logarithmic gaps.

POSITIONS	Lucas	√LUCAS = *T*	ϕ-*Series* = *T*	ϕ^x x	POSITIONS	ϕ^x x	½ ϕ-*Series* = *T*	½ ϕ-*Series* *V*i, 6th	
Earth *Synodic*			1 EARTH	0			(also, $T = Vi^3$)	(*w/Revolution*)	#
	1	1			Gm2	0.5	1.272019649	1.083505882	1
MARS	3	1.732050808	1.618033989	1	MARS	1	1.618098875	1.172984997	2
	4	2			Gm1	1.5	2.058171027	1.272019649	3
Synodic	7	2.645751311	2.618033989	2	*Synodic*	2	2.618033989	1.378240772	4
	11	3.316624790			Gm2	2.5	3.330190677	1.493331984	5
Pheidias	18	4.242640687	4.236067977	3	Pheidias	3	4.236067977	1.618033989	6
	29	5.385164807			Gm1	3.5	5.388361704	1.753149344	7
Synodic	47	6.855654600	6.854101966	4	*Synodic*	4	6.854101966	1.899547627	8
	76	8.717797887			Gm2	4.5	8.718552381	2.058171703	9
JUPITER	123	11.09053651	11.09016994	5	JUPITER	5	11.09016994	2.230040415	10
	199	14.10673598			Gm1	5.5	14.10691408	2.416261907	11
Synodic SD1	322	17.94435844	17.94427191	6	*Synodic*SD1	5	17.94427191	2.618033989	12
	521	22.82542442			Gm2	6.5	22.82546647	2.836655227	13
SATURN	843	29.03446228	29.03444185	7	SATURN	7	29.03444185	3.073532624	14

Table 3b. *The Pheidian Planorbidae SIXTHS* 1-14 , ½ ϕ-*Series V*i from Earth through Saturn.

The new data increase sequentially by $\phi^{1/6}$ = 1.08350588217 (hence the title *Sixths*). Represented by color-coded line spiral indicators #2 to #15 applied across Raup's morphospace contours, the exponential *Sixths* have also been added to to the right vertical axis in composite Figure 5a with test format 2 solid spirals shown in Figure 5b:

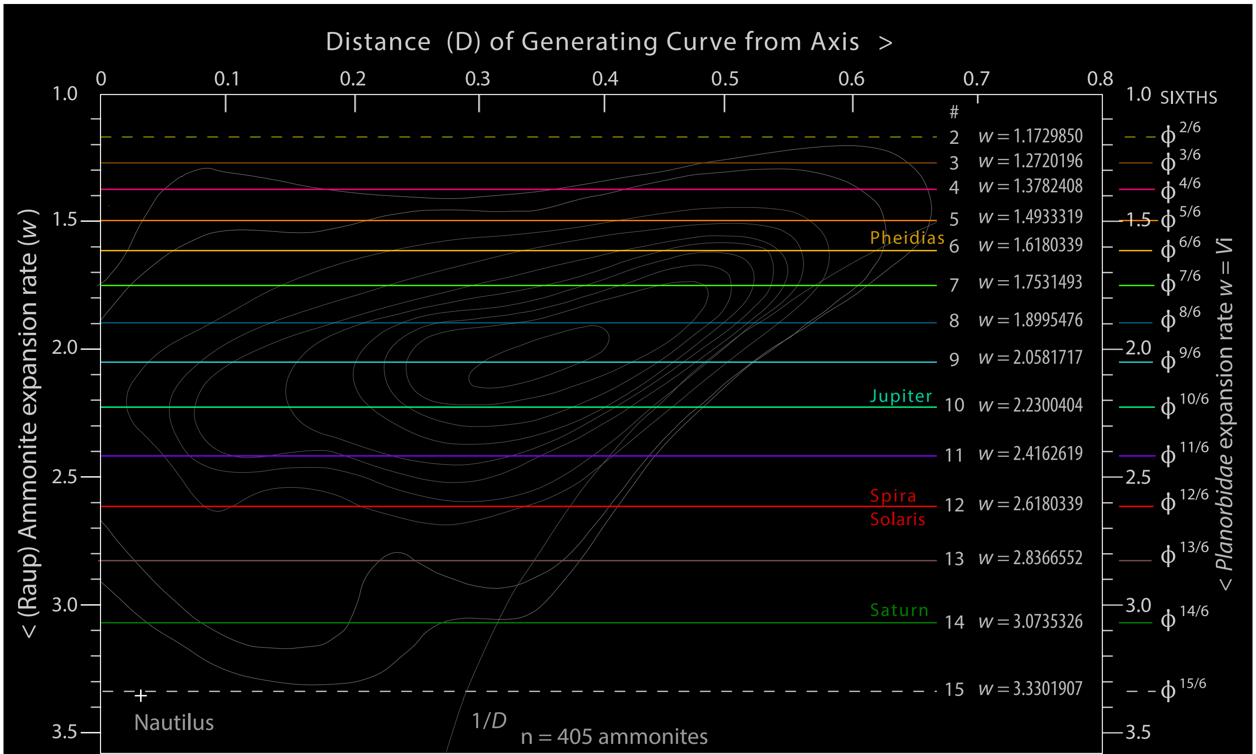


Fig. 5a *Pheidian Planorbidae Sixths* 1-15 added to Raup morphospace contours (*TEXT-Fig-4*.1967:48)

PHEIDIAN SIXTHS : PHI-SERIES TEST SPIRALS, MARS - SATURN (Format 2)

Fig. 5b *Pheidian Planorbidae Spirals:* Exponential *Sixths*, $w = \phi^{3/6}$ through $\phi^{14/6}$ (1.27201965 - 3.07353262, *not to scale*; $w = \phi^{N/6}$)

Despite the overall distribution of the *Pheidian planorbidae* in Figure 5a the latter might still be dismissed as mere coincidence, while the mean inverse velocity (Vi) is both unexpected and little used in modern astronomy. Yet the grouping is nevertheless fundamentally correct. This becomes more complex, however, when the parameters for the Inferior planets are included.

Pheidian *Planorbidae* and Inferior planet *relative velocities* (Vr)

The ½ ϕ-*Series Planorbidae* from #3 to #16 are shown in Figure 6 with further details -*T*, *R* and *Vi* - provided in the upper section of Table 4. The lower section from IMO2 to Venus employs the reciprocal velocity *Vr* for the Inferior planets. In so much as the reciprocal spirals are identical in *form* to those of the Superior planets based on *Vi*, the corresponding *Vr* spirals for the Inferior planets need not be immediately displayed.

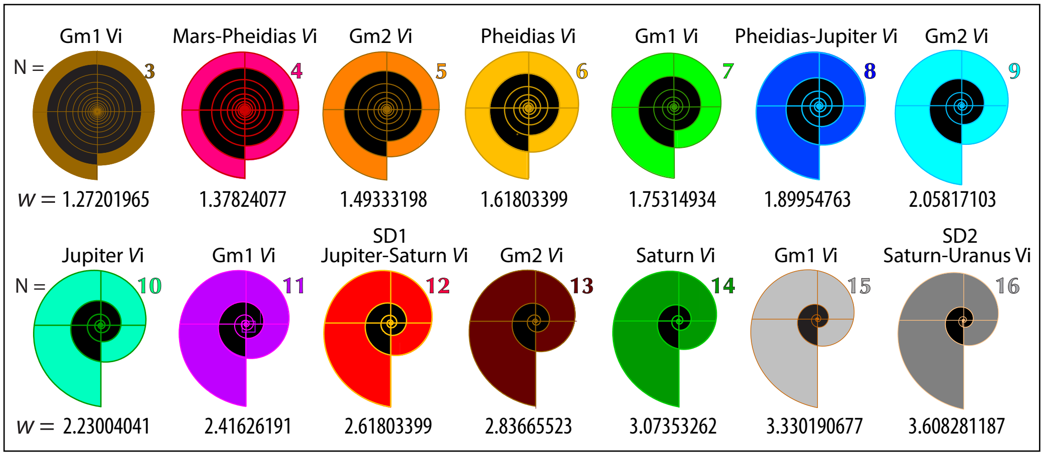


Fig. 6. *Phedian Planorbidae*(*Sixths*). Equiangular ammonite/nautiloid test spirals 3 -16.

SUPERIOR PLANETS (Half *Phi -Series*)

	SATURN	Gm2	SD1 (Syn)	Gm1	JUPITER	Gm2	(Synodic)	Gm1	PHEIDIAS	Gm2	(Synodic)	Gm1	MARS
T	29.0344419	22.8254665	17.9442719	14.10691409	11.0901699	8.71855238	6.85410197	5.38836170	4.23606798	3.33019068	2.61803399	2.05817103	1.61803399
R	9.44660279	8.04661287	6.85410197	5.838321602	4.97308025	4.23606798	3.60828119	3.07353262	2.61803399	2.23004042	1.89954763	1.61803399	1.37824077
Vi [1]	3.07353262	2.83665523	2.61803399	2.416261907	2.23004046	2.05817103	1.89954763	1.75314934	1.61803399	1.49333198	1.37824077	1.27201965	1.17398499
Vr	0.32535851	0.35252786	0.38196601	0.413862419	0.44842237	0.48586827	0.52644113	0.57040206	0.61803399	0.66964346	0.72556263	0.78615138	0.85179964

[1] Inverse velocity *Vi* = Planorbidae 14 through 2

INFERIOR PLANETS (Half *Phi -Series*)

	IMO 2	Gm2	(Synodic)	Gm1	IMO 1	Gm2	(Synodic)	Gm1	MERCURY	Gm2	(Synodic)	Gm1	VENUS
T	0.03444185	0.04381071	0.05572809	0.07088723	0.09016994	0.11469794	0.14589803	0.18558517	0.23606798	0.30028311	0.38196601	0.48586827	0.61803399
R	0.10585816	0.12427589	0.14589803	0.17128210	0.20108262	0.23606798	0.27714026	0.32535851	0.38196601	0.44842237	0.52644113	0.61803399	0.72556263
Vi	0.32535851	0.35252786	0.38196601	0.41386242	0.44842237	0.48586827	0.52644113	0.57040206	0.61803399	0.66964346	0.72556263	0.78615138	0.85179964
Vi [2]	3.07353262	2.83665523	2.61803399	2.41626191	2.23004042	2.05817103	1.89954763	1.75314934	1.61803399	1.49333198	1.37824077	1.27201965	1.17398499

[2] Relative velocity *Vr* for Planorbidae 14 through 2.

Table 4. ½ ϕ-*Series* planetary framework (x = 7 to -7) and reciprocal velocities *Vi* : *Vr*.

The situation with respect to the Solar System is more complex and also more variable. The inclusion of Pheidias is provided by the Mars-Jupiter geometric mean in Table 5.

SUPERIOR PLANETS (Solar System)

	SATURN	Gm2	SD1 (Syn)	Gm1	JUPITER	Gm2	(Synodic)	Gm1	Pheidias [3]	Gm2	(Synodic)	Gm1	MARS
T	29.5351971	24.2050488	19.8368199	15.3426915	11.8667298	9.65106718	7.84909567	6.08944437	4.72428089	3.84220025	3.12481478	2.42427747	1.88079027
R	9.55490959	8.36765902	7.32793094	6.17448919	5.20260319	4.53297746	3.94953908	3.33466168	2.81551043	2.45312680	2.13738547	1.80463018	1.52367934
Vi	3.09110168	2.89269062	2.70701513	2.48485195	2.28092157	2.12907902	1.98734473	1.82610560	1.67794828	1.56624609	1.46197998	1.34336524	1.23437407
Vr	0.32350925	0.34569891	0.36941057	0.40243846	0.43841928	0.46968665	0.50318396	0.54761346	0.59596593	0.63846927	0.68400389	0.74439919	0.81012719

INFERIOR PLANETS (Solar System)

	IMO 2 [4]	Gm2	(Synodic)	Gm1	IMO 1	Gm2	(Synodic)	Gm1	MERCURY	Gm2	(Synodic)	Gm1	VENUS
T	0.03389857	0.04288440	0.05425221	0.07001434	0.090355921	0.11430748	0.14460813	0.18662177	0.24084183	0.26026754	0.32001897	0.41249404	0.61518332
R	0.10474200	0.12251791	0.14331059	0.16987312	0.201359014	0.23553192	0.27550435	0.32656894	0.38709831	0.40764301	0.46786133	0.55413015	0.72332982
Vi	0.32363869	0.35002558	0.37856385	0.41215667	0.448730447	0.48531631	0.52488509	0.57146210	0.62217225	0.63846927	0.68400389	0.74439919	0.85048799
Vr	3.08986539	2.85693404	2.64156235	2.42626182	2.228509355	2.06051184	1.90517892	1.74989731	1.60727194	1.56624609	1.46197998	1.34336525	1.17579556

[3] The period (T) for Pheidias is the MARS-JUPITER geometric mean. [4] Period for IMO 2 from the mean of the Mercury-Venus, IMO 1-Mercury reduction factors.

Table 5. The modern Solar System, (*T,S, Gms*), (*R*) and reciprocal velocities *Vi* : *Vr*.

Lastly,inter-relationships between the Inferior and Superior planets for the *unmodified Phi*-series are as follows:

PLANETS Synodics	N #	MODERN *T* (Julian Years)	ϕ^x = x	*Phi*-series *T,S* (Years)	*Phi*-series (*R*) Distance (a.u.)	*Phi*-series (*Vi*) Inverse Velocity	*Phi*-series (*Vr*) Velocity (Ref.1)
Uranus	2	83.7474068	9	76.01315562	17.94427191	4.236067977	0.236067977
Synodic	3-2	45.3602193	8	46.97871376	13.01969312	3.608281187	0.277140264
Saturn	3	29.4235194	7	29.03444185	9.446602789	3.073532624	0.325358512
Synodic	4-3	19.8588721	6	17.94427191	6.854101966	2.618033989	0.381966011
Jupiter	4	11.8565250	5	11.09016994	4.973080251	2.230040414	0.448422366
Synodic	5-4	7.84767877	4	6.854101966	3.608281187	1.899547627	0.526441130
Pheidias	5	4.72214968	3	4.236067977	2.618033989	1.618033989	0.618033989
Synodic	6-5	3.12552908	2	2.618033989	1.899547627	1.378240772	0.725562630
Mars	6	1.88071105	1	1.618033989	1.378240772	1.173984997	0.851799642
Earth/*Syn*	7-6	0.91422728	0	1.000000000	1.000000000	1.000000000	1.000000000
Venus	7	0.61518257	-1	0.618033989	0.725562630	0.851799642	1.173984997
Synodic	8-7	0.39580075	-2	0.381966011	0.526441130	0.725562630	1.378240772
Mercury	8	0.24084445	-3	0.236067978	0.381966011	0.618033989	1.618033989
Synodic	9-8	0.14474748	-4	0.145898034	0.277140264	0.526441130	1.899547626
IMO 1	9	0.09041068	-5	0.076806725	0.201082619	0.448422366	2.230040414
Synodic	10-9	(0.0556507)	-6	0.055728090	0.145898034	0.381966011	2.618033989
IMO2	10	(0.0344447)	-7	0.040434219	0.105858161	0.325358512	3.073532624

Table 6. Modern Periods (*T,S*) and *Phi*-series, x, *T*, *R*, Velocity *Vi* (Inverse) and *Vr* (relative), IMO 2 to Uranus.

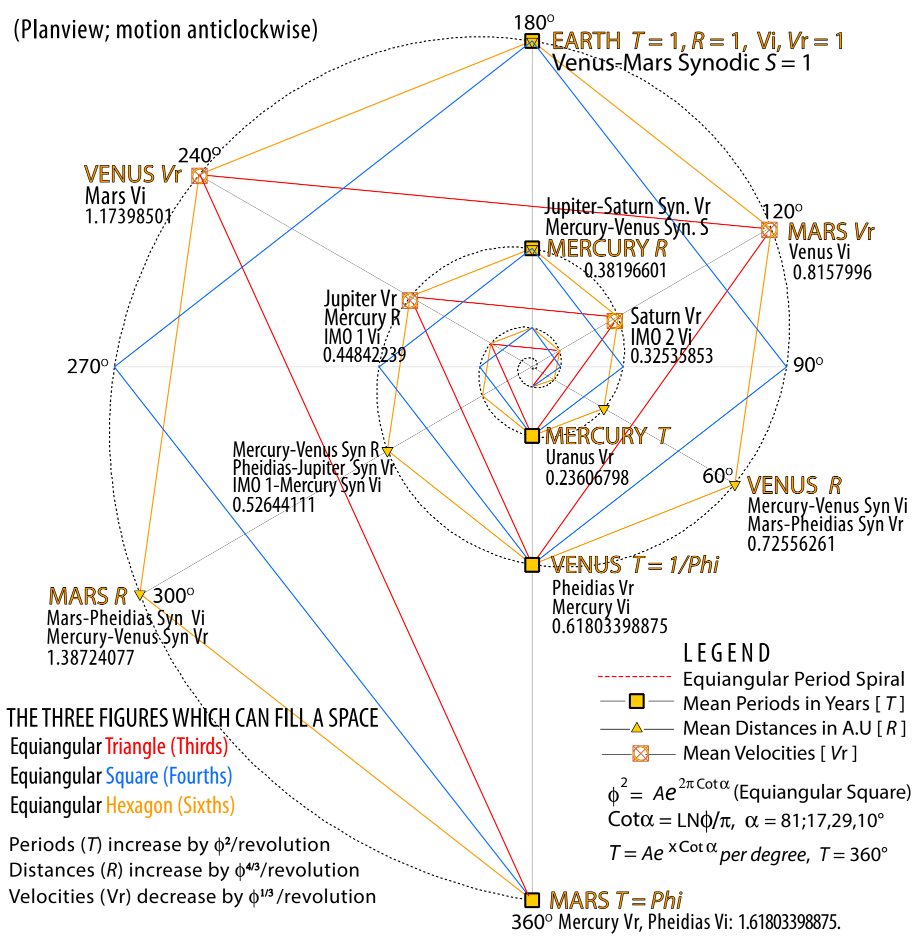


Fig. 7. Spirasolaris from Mercury to Mars, the three-fold number and unmodified *Phi-series*.

whereas *the actual deficiencies* in the Solar System between Mars and Jupiter also need to be addressed.

Planorbidae 3 through14 and the Mars-Jupiter Gap

The complete set of *Pheidian planorbidae* from 3-14 provides a reasonable fit with the Raup morphospace contours but in the Solar System the absence of a body between Mars and Jupiter also excludes the synodic positions and the associated Gms generated by the theoretical ½ ϕ-*Series*. Nevertheless, despite such deficiencies adjustments can be calculated for the Solar System, resulting in solid spirals 3b, 4b and 7b with relatively small positive changes for the final five positions from Jupiter outwards:

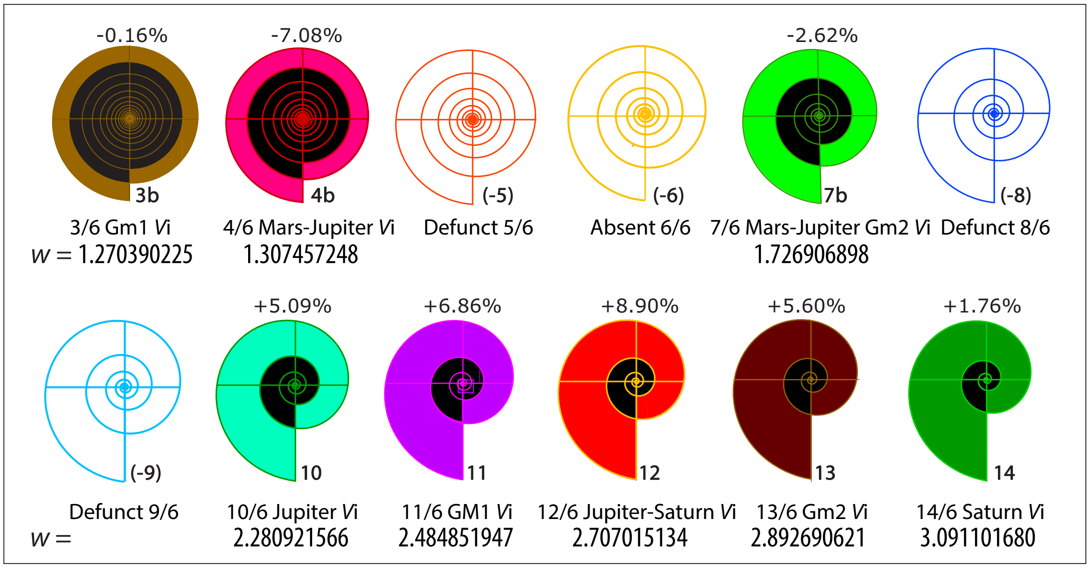


Fig. 8. *Solar System Planorbidae* 3-14. Missing: Nos. 5, 6, 8 & 9; retained/restored: #3b, 4b and 7b.

In other words, remaining with the ½ ϕ-*Series* format, instead of *seven* successive periods between Mars and Jupiter only *three* Solar System periods - Gm1, the Mars-Jupiter Synodic and Gm2 between Mars and Jupiter) now exist.

Planorbidae spirals and Inferior planet *relative velocities* (*Vr*)

However, despite the differences between the modern periods of revolution (*T*) for Mars and Jupiter and the ½ ϕ-*Series,* the modern estimate for Gm2 *Vi* is 1.72694432, whereas the comparable value for the extended ½ ϕ-*Series* is $w = \phi^{7/6} = 1.75314934$. In addition, although *Pheidias* and associated periods on either side are absent, among extinct ammonites examples exist that approximate the mean *relative velocity Vr* of Mercury = 1.607281127 versus the *missing* inverse velocity *Vi* = ϕ (1.61803398875) of the absent *Pheidias*. Also noteworthy are the similarities for #7 (1.75314931) versus 1.7498973, and 1.7269069 in the modern Solar Solar. Relative velocities (*Vr*) for the inferior planets and the corresponding equiangular spirals (shown as involutes in Fig. 9) are as follows:

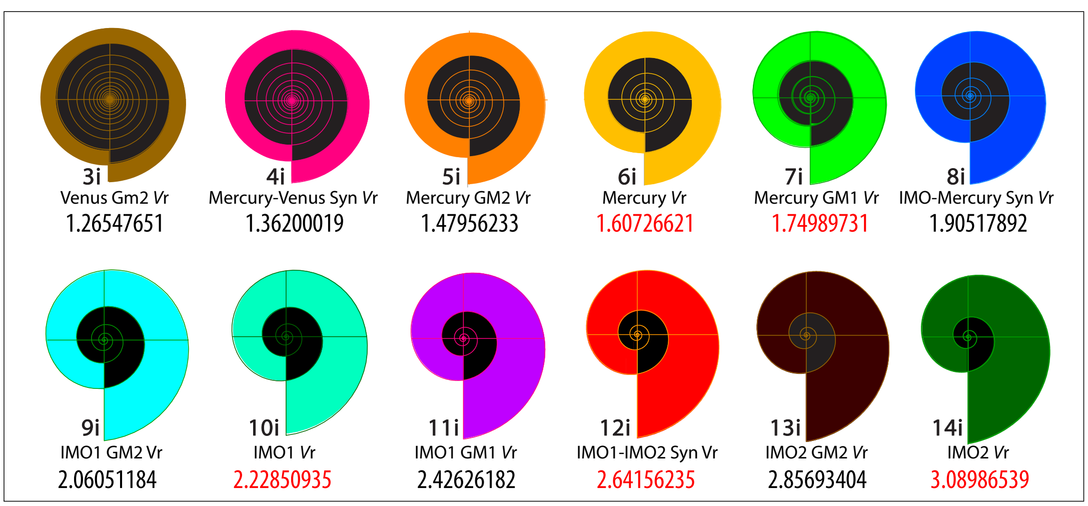


Fig. 9. Comparable *Solar System Velocities (Vr)* for the Inferior Planets [Venus] to IMO2 (#3i to 14i) shown with relative velocities (*Vr*) of interest emphasized in red.

The Mars-Jupiter Gap revisited

The absence of a suspected body between Mars and Jupiter, associated periods and the theoretical configuration for the ½ ϕ-*Series* compared to the present Solar System nonetheless includes the retention and/or "restoration" of *planorbidae* #7 (Gm2), albeit with a different origin as shown in the inset table in Figure 10.

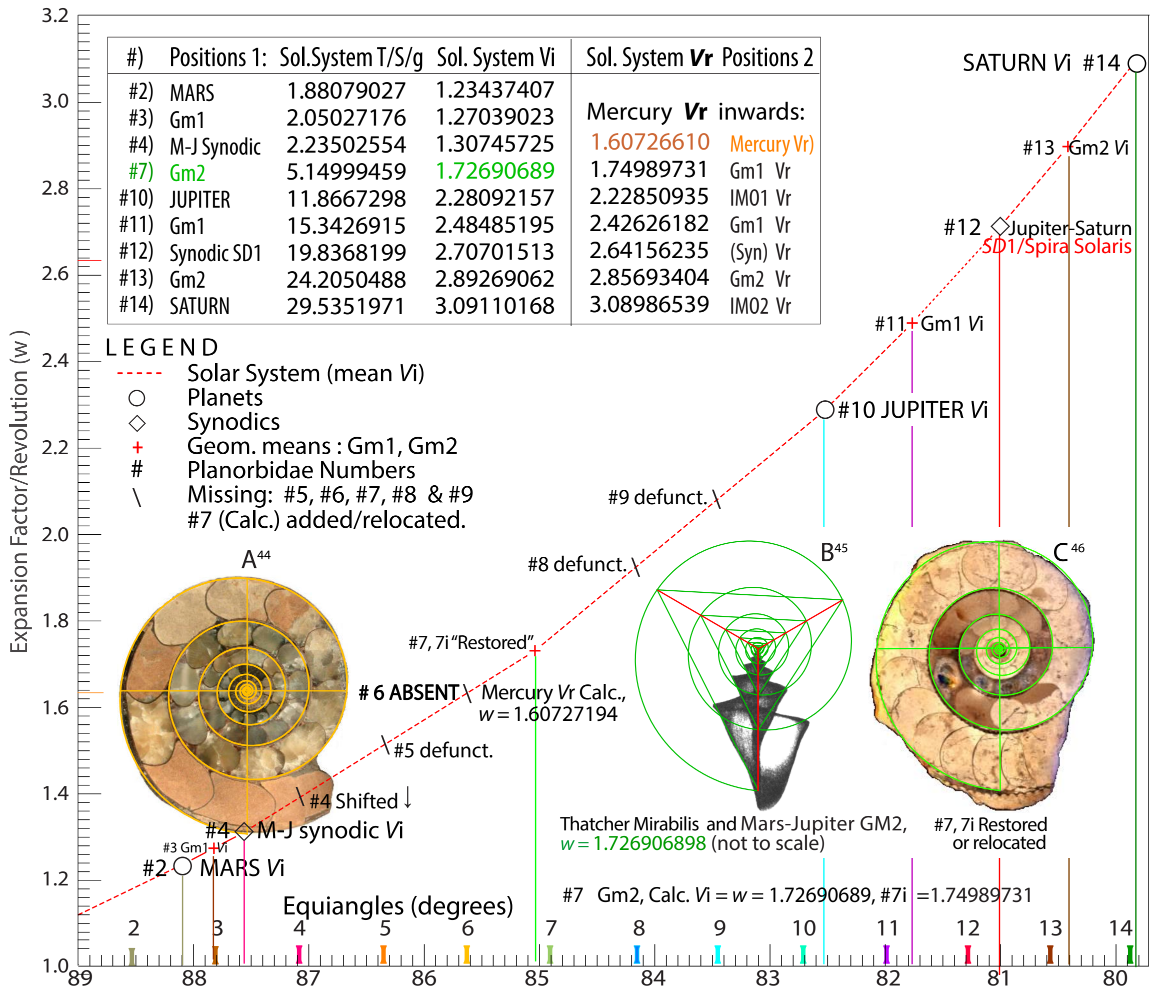


Fig. 10. The Mars-Jupiter Gap, *planorbidae* #6 and the Solar System; possibly relocated #7and/or #7i ?

In short, *planorbidae* #5, #6, #7, #8 and #9 are no longer applicable, with only the Mars-Jupiter Synodic remaining, along with associated real-time intervals between the two planets. Plus, perhaps not entirely justified, inclusion of data from the inferior planets, that of Mercury in particular.

Numerous questions therefore arise concerning not only the full set of *planorbidae*, but also what can be made of the occurrence of spirals belonging to #6 (*Pheidias*) and #7 in particular among the defunct ammonites and their apparent existence in the Solar System. Furthermore, *pentagonal, hexagonal*, *hexakaidecagonal* and *icosagonal* figures occur (*i.e.*, 16 and 20 se*pta* per revolution for the latter pair).

Initial Tests

As far as the initial tests of *pheidian* spirals applied to ammonites and shells are concerned there remain a number of qualifiers before commencing, *e.g.*, the following supplied by Peter Ward (1992: 85):[47]

> Nautiluses (and apparently the ammonites as well) were not creatures that grew throughout their lives. Like humans, they reached a certain adult size and then quit growing. *The slowing of growth immediately preceding the final adult size is marked by changes in the spacing of the last two or three septa formed within the shell and by changes in the shape of the outer shell wall.* (italics supplied)

It is at this point, however, that David Raup's treatment of ammonoid spirals comes to mind, particularly his initial range for *w* from 1.25 to 3, especially since the inclusion of both Gm1s and Gm2s to the Pheidian planorbidae Sixths permits the following comparison with the growth factors and the equiangular spirals determined by the latter:

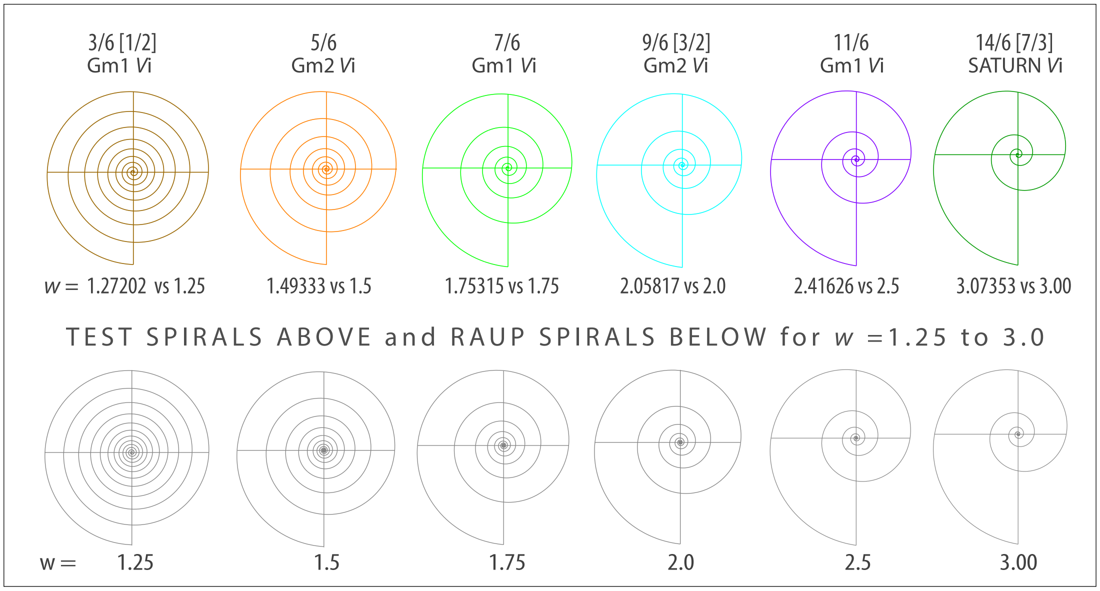


Fig. 11. Test spirals within the Raup range *w* = 1.25 to 3.00 and comparable planorbidae. Saturn excluded, the first five planorbidae are all successive odd-numbered geometric means (GM1, GM2: #3, #5, #7, #9 and #11).

Questions which arise now are why Raup was unable to proceed further with this promising line of inquiry with its numerical progressions. And also, why the golden ratio/golden section did not surface during the investigation at least as an approximation via the least error geometric mean between 1.5 and 1.75, *i.e.*, 1.6201852 followed in turn by the generally low error GMs for the rest of his estimates given in Table 6 below.

ASSIGNMENTS	Exp. x	½ ϕ-*Series T*	½ ϕ-*Series R*	6Ths	½ ϕ-*Series Vi*	Raup *w/Gm*	Diff (%)
EARTH (Unity)	0	1	1		1		
GM2	0.5	1.2720196495	1.1739849967	1	1.083505882	*?*	
MARS (Vi)	1	1.6180339887	1.3782407725	2	1.172984997	*?*	
GM1*	1.5	2.0581710273	1.6180339887	3	1.272019649	**1.25**	-2.20%
Synodic	2	2.6180339887	1.8995476270	4	1.378240772	*1.3693*	-0.89%
GM2*	2.5	3.3301906768	2.2300404146	5	1.493331984	**1.5**	0.67%
{ PHEIDIAS Vi }	3	4.2360679775	2.6180339887	6	1.618033989	*1.62018*	0.22%
GM1*	3.5	5.3883617041	3.0735326237	7	1.753149344	**1.75**	-0.31%
Synodic	4	6.8541019662	3.6082811871	8	1.899547627	*1.9034*	0.38%
GM2*	4.5	8.7185523808	4.2360679775	9	2.058171703	**2**	-5.82%
JUPITER (Vi)	5	11.090169944	4.9730802506	10	2.230040415	*2.2361*	0.60%
GM1*	5.5	14.106914085	5.8383216016	11	2.416261907	**2.5**	8.37%
Synodic	6	17.944271910	6.8541019662	12	2.618033989	*?*	
GM2	6.5	22.825466466	8.0466128743	13	2.836655227	*?*	
SATURN (Vi)	7	29.034441854	9.4466027887	14	3.073532624	**3**	-7.35%

*Comparable ½ ϕ-*Series T* GMs for the Raup growth factors w = 1.25 - 3.

Table 6. ***Pheidian Planorbidae*** *SIXTHS* 1-14 , ½ ϕ-*Series Vi* and the Raup growth factor estimates, *w* =1.25 to 3.

On the other hand, it is a major step to embrace planetary motion in the first place, not to mention such counter-intuitive concepts as inverse velocity in the second, especially without historical guidance. This said, it was only a year after Raup's 1967 paper that Nicole Oresme's *Le Livre du ciel et du monde* was published in Madison in 1968. It was this work concerned with both the commentary by Averroes on Aristotle's "On the Heavens" and additional insights by Nicole Oresme which supplied the weighty and erudite statement given earlier. It is worth repeating:

> *Said Aristotle, never-failing friend of Truth: The three-fold number is present in all things whatsoever, nor did we ourselves discover this number, but rather, nature discovers it for us.*

Test Procedures adopted for Ammonites and Nautiloids

There is one further point concerning Raup's pioneering analyses, which is that his methodology for establishing the growth factor *w* must have been relatively limited with some degree of mechanical measurements required to arrive at his given estimates for growth per revolution. The present treatment which follows - which is likely already superceded by more sophisticated methods - nonetheless produces accurate, double-precision equiangular radius vectors per degree for successive whorls, thus 2160 successive data points for 6-whorl spirals, and 2880 data points for 8-whorl spirals, etc. Furthermore, based on the same data, the various figures included also maintain the same accuracy, being the specific points which correspond to 120° for the triangle, 90° for the square, 72° the pentagon, 60° for the hexagon and 45° for the octagon, etc., and so on down to the 1° points of the spirals themselves. Thus the following colour-coded equiangular test spirals for figures that occur among ammonites:

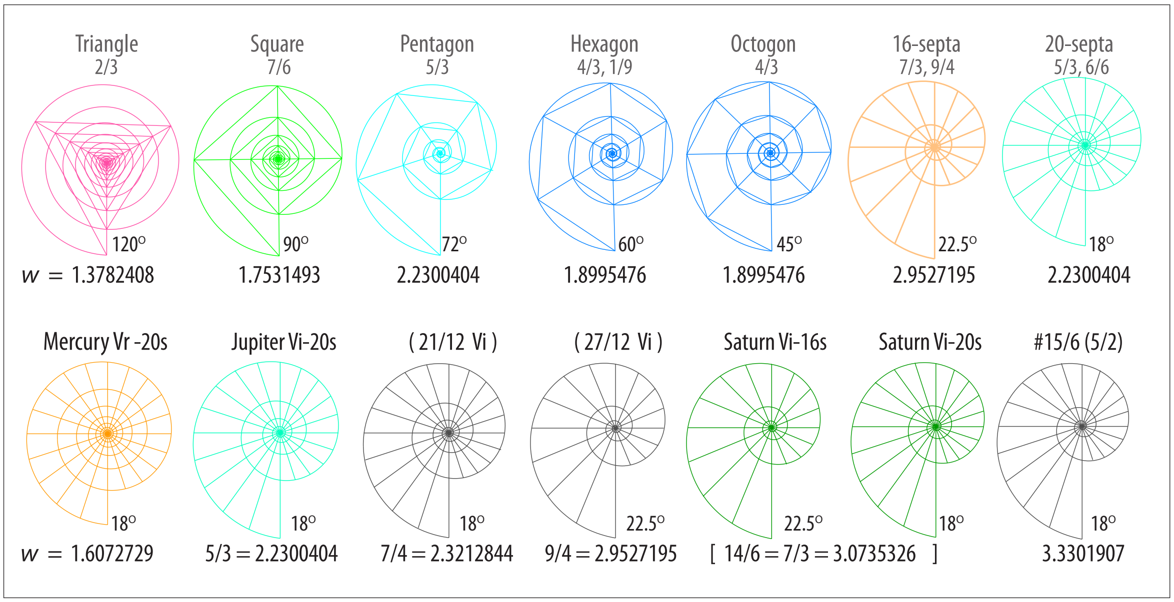


Fig. 12. Growth factors *(w)* with 3, 4, 5, 6, 8,12,16 and 20X configurations for the *Pheidian Sixths* and Solar System.

Additional information is also provided by various integral figures known to occur among ammonites that extend to the outermost regions shown in Figures 12 and 13, albeit to a far lesser extent for the Solar System equiangular spirals of potential interest in the latter group.

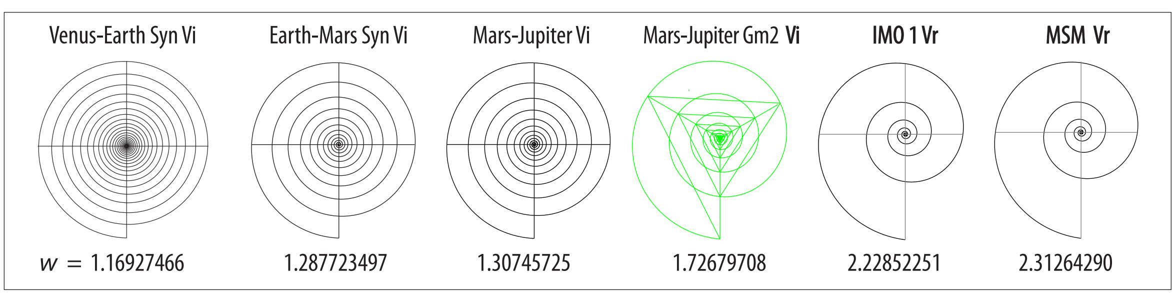


Fig. 13. Solar System growth factors (w) and equiangular spirals of additional interest.

In Figure 13, MSM is the mean *synodic* month in days with the relative velocity *Vr* finally expressed with respect to unity; see Tables AP1 and AP 2 in Part One for further details. Included earlier in Figure10, the Mars-Jupiter Gm2 of 1.72697078 associated with the Mars-Jupiter Gap is at this stage perhaps best examined with respect to the spiral configurations of certain sea-shells. One shell in a particular in this context is *Thatcher Mirabilis* with a provisional growth factor of $w = \phi^{7/6} = 1.75314934$, thus an equiangular spiral in close proximity to $w = 1.72697078$ and therefore a potential replacement for the region of special interest.

Initial test procedures for ammonites and nautiloids

The superb line drawings of ammonites in the *Treatise on invertebrate paleontology* [48] provide a major resource for the refinement of test techniques once the last *septa* is repositioned to the lower, free-swimming position. At this point, however, it is necessary to acknowledge that this is a highly specialized field, even before such topics as time scales, variants and extinctions add further complexity. This said, the limited range of the growth factors (w) along with the similarities in form and standard orientation suggests that planispiral ammonites nevertheless provide a workable test set for the spirals introduced in Figures 5b and 6 for *Pheidian planorbidae* numbers 3 through 14.

Lastly, in fitting all double-precision generated *planorbidae* spirals to test subjects, apart from colors, density and line widths, modifications were limited to scaling and rotation alone. As for the fit, the proportion and the form of test subjects were equally inviolable, each confined to reorientation and scaling. For spatial considerations the test subjects were also displayed in the same size. The best fit was taken to be the middle spiral of three consecutive *planorbidae* as shown in Figure 14 and the larger, more complex drawing of *Manticoceras* assigned to Figure 15. For the quadruple test set *planorbidae* #9 is too small (-), #10 the best fit and #11 too large (+).

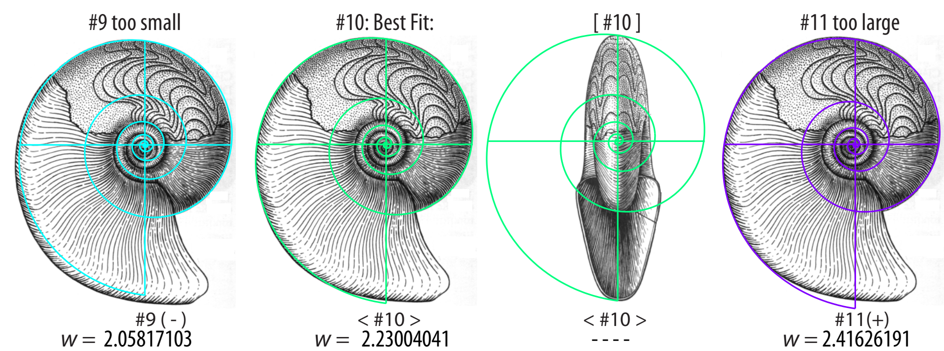


Fig. 14. *Manticoceras* [50] and *planorbidae* #9 (-), #10 (Jupiter *V*i, $w = 2.230004041$) and #11 (+).

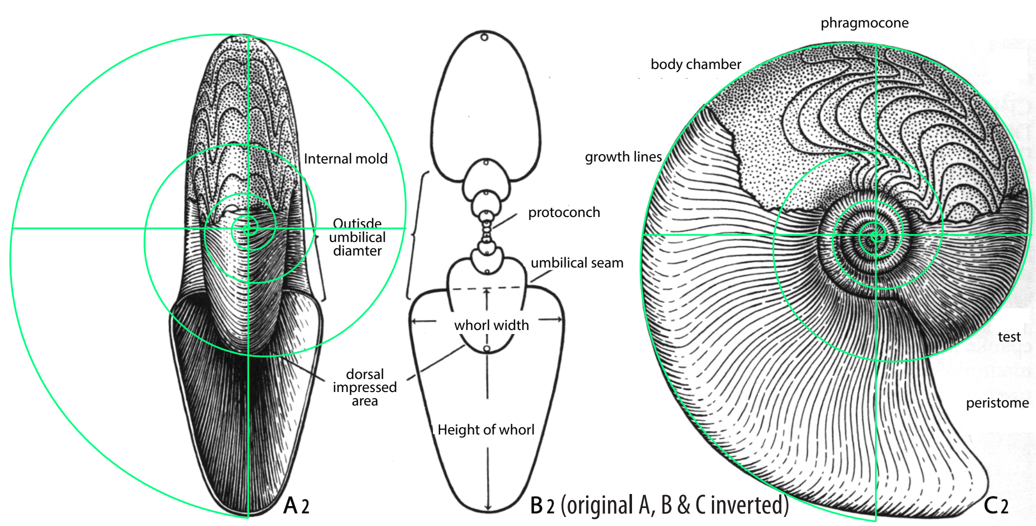
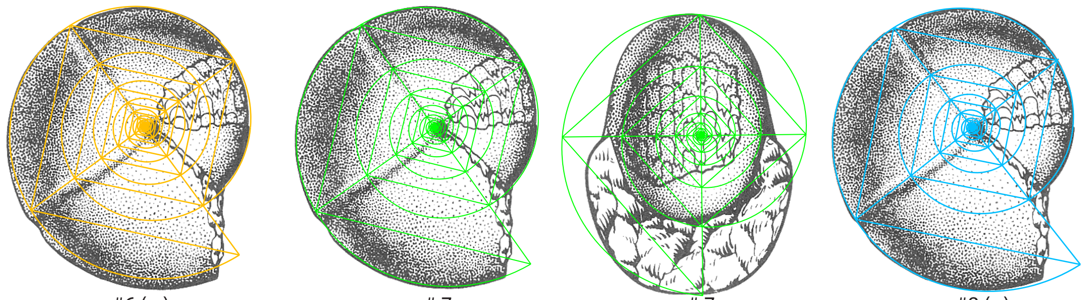


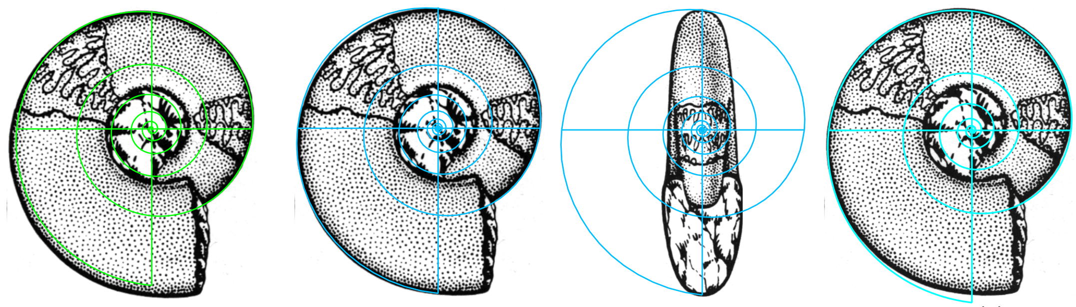
Fig. 15. *Manticoceras* [49] and *planorbidae* #10 (Jupiter *V*i, $w = 2.230004041$).

Initial Tests

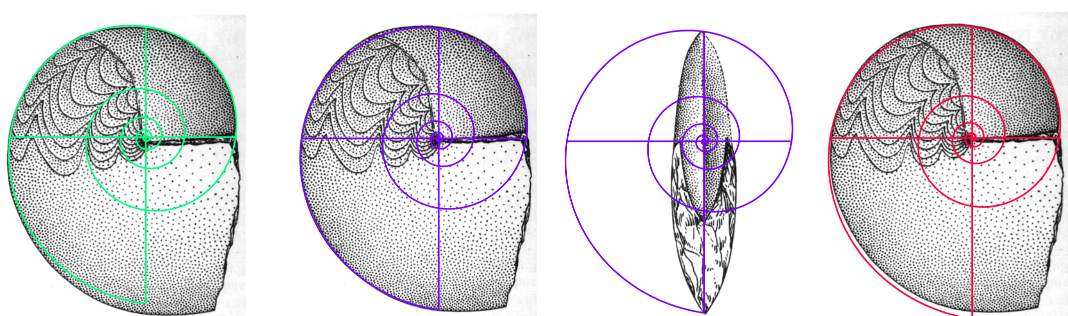
Sequential Pheidian Planorbidae applied to ammonite drawings in the *Treatise on Invertebrate Paleontology* (1957) are shown in Figure 16A_D. The inverted, scaled drawings for this initial test feature growth factors (w) from #6 (the spiral of *Pheidias*) through *Spiral Solaris* (#12) plus #13. The first (15A) is a lone example which also shows a vestigial "square" figure in its structure, as does Figure 16 which follows (albeit for planorbidae #7), while integral pentagonal, hexagonal and octagonal figures also follow in due order.



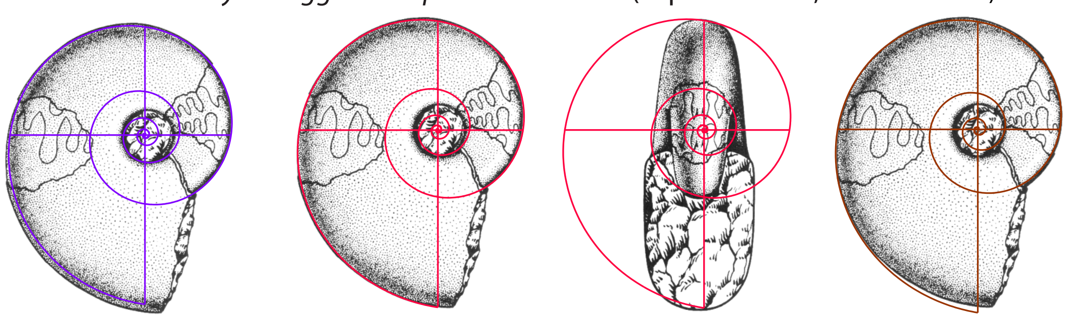
A. *Peritrochia ganti* [51] and *planorbidae* #7 (*Pheidias* Gm1 *Vi*, *w* = 1.75314934).



B. *Wellerites mohri* [52] and *planorbidae* #8 (Jupiter Synodic *Vi*, *w* = 1.89954763).



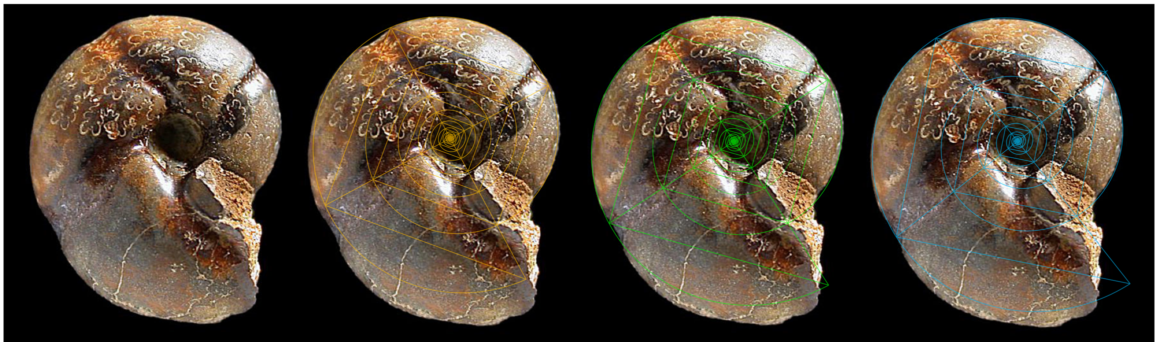
C. *Timanites keyserlinggi* [53] and *planorbidae* #11 (Jupiter Gm1 *Vi*, *w* = 2.41626191).



D. *Pronorite arkansasensis* [54] and *planorbidae* #12 (Spira Solaris, *w* = 2.61803399).

Fig. 16A-D. *Sequential Pheidian Planorbidae* applied to technical ammonite drawings in the *Treatise on Invertebrate Paleontology* (1957) with best fit growth factors (*w*) for *planorbidae* #6 through #13. (all reoriented and rescaled).

Next, the process is applied to high quality photographs of ammonites which feature integral squares, pentagons hexagons and octagons (Figures 16 through 19). Once again, the preferred spirals and figures are *middle values* between three *consecutive Planorbidae* with the first spiral marginally *too small* and third marginally *too large*.

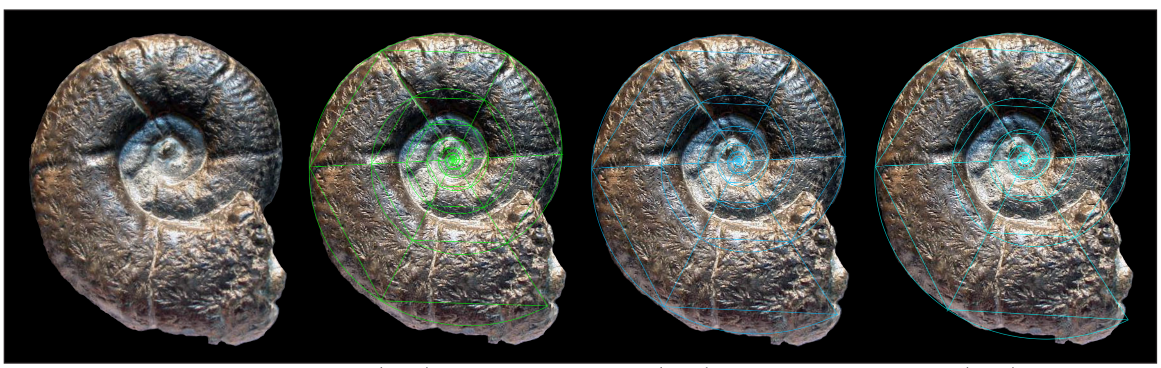


Hervé Chatelier, Ammonites.fra (#0781) $\phi_k = 6/6$, $w = 1.618033$ $\phi_k = 7/6$, $w = 1.75315$ $\phi_k = 8/6$, $w = 1.89954$

Fig. 17. *Calliphylloceras biciolae* (Meneghini 1874) juv.[55] *Pheidian* spirals/*squares*: #6 (-) <best fit #7>; #8 (+).

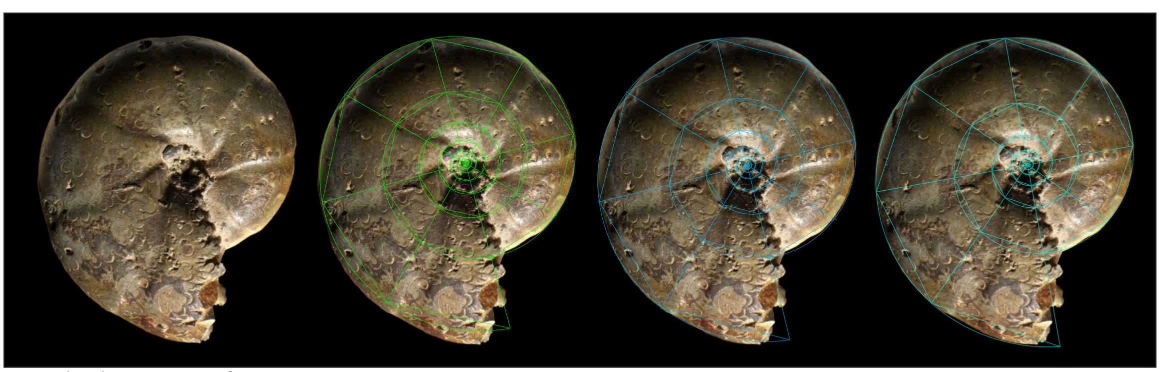
Rene Bau.club.fr. $\phi_k = 9/6$, $w = 2.0582$ $\phi_k = 10/6$, $w = 2.230040$ $\phi_k = 11/6$, $w = 2.41626$

Fig. 18. *Phylloceras (Hypophylloceras) paquieri Sayn.*[56] *Pheidian* spirals/*pentagons*: #9(-) <best fit #10>; #11(+).



Hervé Chatelier, Ammonites.fra (#0534) $\phi_k = 7/6$, $w = 1.7531$ $\phi_k = 8/6$, $w = 1.8995$ $\phi_k = 9/6$, $w = 2.0582$

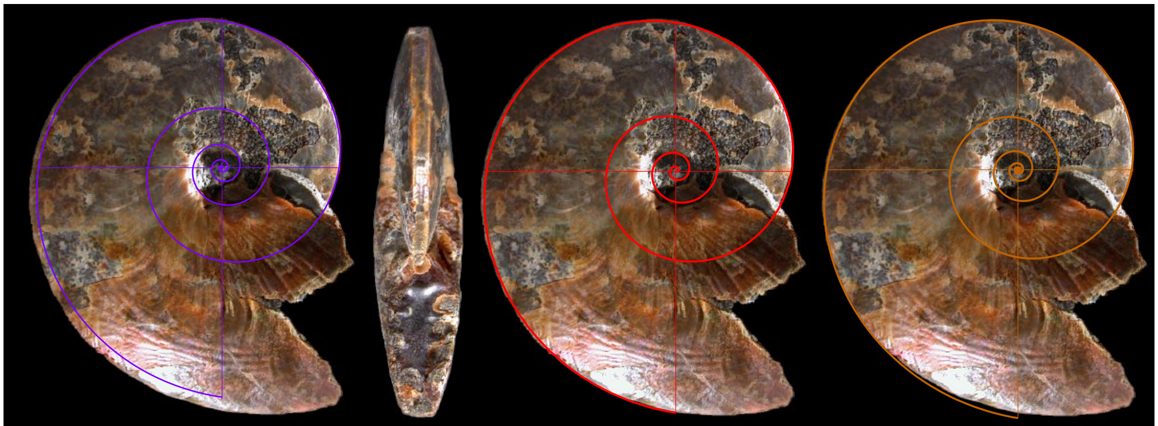
Fig. 19. *Puzosia aff. quenstedti* (Parona & Bonarelli 1897).[57] *Pheidian* spirals/*hexagons*: #7(-) <best fit #8>; #9(+).



Hervé Chatelier, Ammonites.fra (#1271) $\phi_k = 7/6$, $w = 1.75315$ $\phi_k = 8/6$, $w = 1.89954$ $\phi_k = 9/6$, $w = 2.0582$

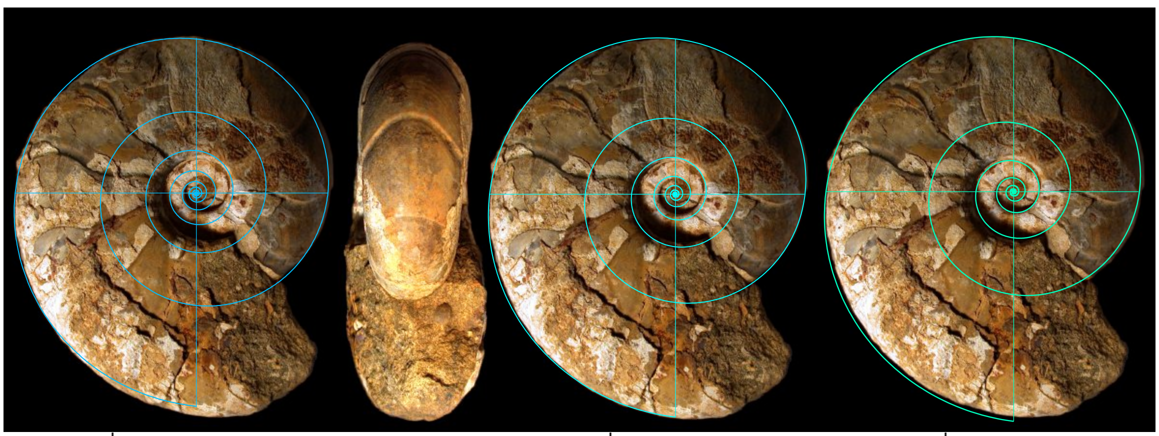
Fig. 20. *Sallfelldiella (Salfelldiella) guettardi* RASPaill 1831).5[58] *Pheidian* spirals/*0ctogons*: #7 (-) <best fit #8>; #9 (+).

Part of a larger set, the preceding assignments served well enough to expand testing to include more high quality plan and side-view photographs of ammonites for the pheidian spirals shown next in Figures 21 through 22:



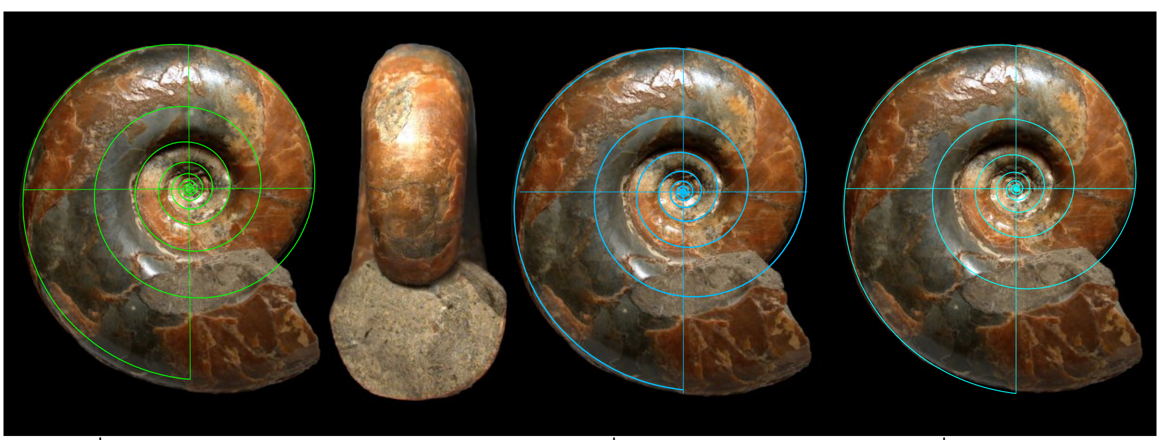
$\phi_k = 11/6$, $w = 2.41626$ Phtotos: Hervé Chatelier, (#0258) $\phi_k = 12/6$, $w = 2.618034$ $\phi_k = 13/6$, $w = 2.83663$

Fig. 21. *Metaplacenticeras subtilistriatum* (Jimbo 1894)[59] with *planorbidae* #11 (-) <best fit #12> and #13 (+).



$\phi_k = 8/6$, $w = 1.89955$ Phtotos: Hervé Chatelier, (#0838) $\phi_k = 9/6$, $w = 2.05817$, $\phi_k = 10/6$, $w = 2.23004$

Fig. 22. *Desmoceras latidorsatum* (Michelin 1938)[60] with *planorbidae* #8 (-) <best fit #9> and #10 (+).



$\phi_k = 7/6$, $w = 1.75315$ Phtotos: Hervé Chatelier, (#0250) $\phi_k = 8/6$, $w = 1.89955$ $\phi_k = 9/6$, $w = 2.05817$

Fig. 23. *Tetragonites popetensis* (Yabe 1903)[61] with *planorbidae* #7(-) <best fit #8> and #9 (+).

Plus, it would seem, even when there are larger overlaps for the thicker ammonites, the overlaps nevertheless appear to maintain the same pheidan form as the parent spiral, *e.g.,* Figures 23 and 24:

$\phi_k = 7/6$, $w = 1.75315$ Hervé Chatelier (#0866v, 0866) $\phi_k = 8/6$, $w = 1.89955$ $\phi_k = 9/6$, $w = 2.05817$

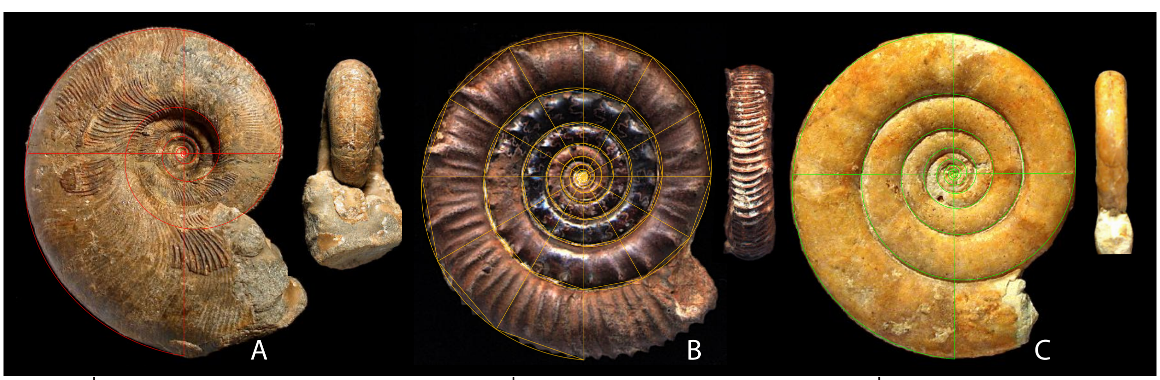
Fig. 24. *Hauericeras gardeni* [62] with *planorbidae* #7(-), <best fit #8> and #9(+).

Where ammonites are damaged or showing pronounced changes at the outermost septa it is possible to find a provisional best-of-three fit for certain examples from the inner-displayed spiral alone, *e.g.*, for *Stephanoceras*:

$\phi_k = 7/6$, $w = 1.75315$ Hervé Chatelier (#0863v, 0863) $\phi_k = 8/6$, $w = 1.89955$ $\phi_k = 9/6$, $w = 2.05817$

Fig. 25. *Stephanoceras.* sp.[63] Inner spirals: *planorbidae* #7 (-) <best fit #8> with #9 slightly too large (+).

And finally, best-fit assignments that include *Spirasolars* (#12), the missing *Pheidias* (#6), and missing #7:



$\phi_k = 12/6$, $w = 2.618034$ $\phi_k = 6/6$, $w = 1.618034$ $\phi_k = 7/6$, $w = 1.75315$

Fig. 26A-C. A: Outline, *Spirasolaris*, B: Whorl-to-whorl spiral, *Pheidias*,#6 C: Whorl-to-whorl spiral, #7.

A. *Gaudryceras denmanense* (Whiteaves 1901).[64] Photographs: Hervé Chatelier, Ammonites.fra (#0702; 0702v reduced).
B. *Septimaniceras zittel* (Oppel 1862)(M).[65] Photographs: Hervé Chatelier, Ammonites.fra (#0147; 0147v reduced).
C.. *Nannolytoceras pygmaeum* (d'Orbigny 1845.[66] Photographs: Hervé Chatelier, Ammonites.fra (#0603; 0603v reduced).

At which point, using the above methodology, back to the growth factors (*w)* for four adjacent positions: 8/6 9/6 and Solar System substitutes for 10/6 and 12/6, *i.e.*, for Jupiter and Jupiter-Saturn *SD*1 *V*i = Spirasolaris.

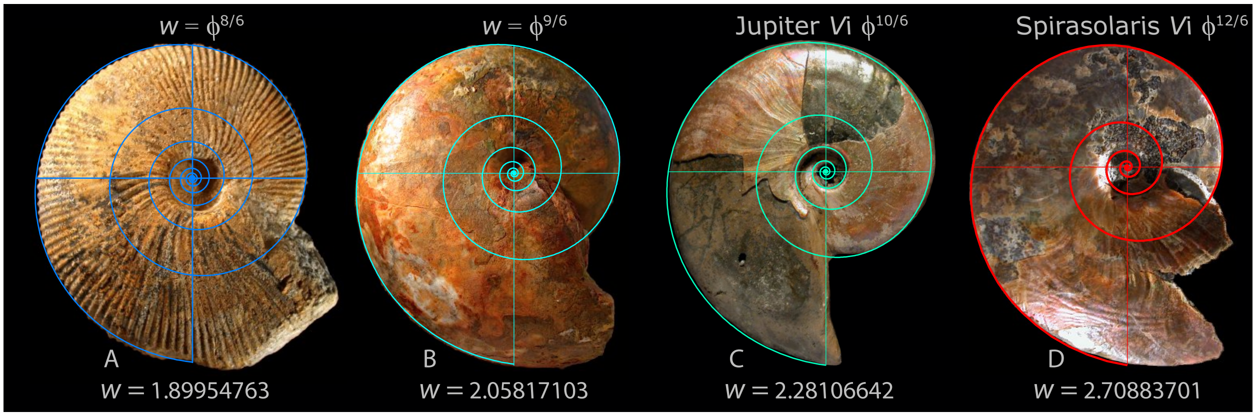
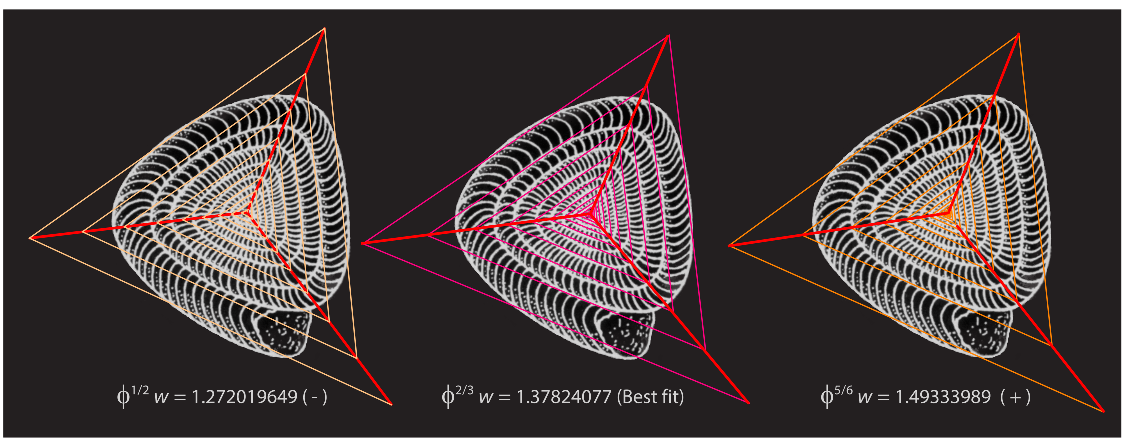


Fig. 27A-D. Ammonites, best fit Sixths (A & B); Solar System *Vi* for Jupiter(C) and Jupiter-Saturn *Vi* (D) *SD*1 = Spirasolaris.

A. *Macrocephalities verus Buckman 1922.* [67] Photograph: Hervé Chatelier, Ammonites.fra (#0026).
B. *Paracladiscites* [68] Photograph: Hervé Chatelier, Ammonites.fra (#0766).
C. *Beudanticeras laevigatum* (Sowerby 1827).[69] Photograph: Hervé Chatelier, Ammonites.fra (#0766).
D. *Metaplacenticeras subtilistriatum* (Jimbo 1894).[70] Photograph: Hervé Chatelier, Ammonites.fra (#0258).

"Triangular" Ammonites, Radiolarians and Diodom(s)

Despite their rarity, a small sample of triangular ammonites are nonetheless included in the *Treatise on invertebrate paleontology*, *e.g.*, line drawings of *Soliclymenia*, which, with a lightened grey-scale (but otherwise unchanged) can checked against transparent, scalable and rotatable overlays of the three closest equiangular spirals and associated equi-triangles.



Soliclymenia, $\phi^{1/2}$ equi-triangles. *Soliclymenia,* $\phi^{2/3}$ equi-triangles. *Soliclymenia,* $\phi^{5/6}$ equi-triangles.

Fig. 28. Triangular ammonite *Soliclymenia;* [71] limited fit equiangular triangles for $\phi_k = 2/3$ (4/6), $w = 1.37824077$ between $\phi_k = 1/2$ (3/6), $w = 1.272019649$ and $\phi_k = 5/6$, $w = 1.49333989$ (parent equiangular spirals omitted).

Radiolarians and Diodoms

Radiolarians

It is at this point, however, in view of the scarcity of triangular ammonites and similarities in shape elsewhere that the inquiry now expands to include microscopic organisms and added complexity associated with natural growth. Fortunately, however, the configuration of one example in particular lends itself readily to the task (as shown next in Figure 29) by the fit for a micro-photograph of Late Triassic radioarian *Sarla.* Here the exponential growth is more readily apparent from the angle and scale of the left side extremity, with the smaller side on the right and largest vertical component together providing the orientation for a plan-view of anti-clockwise growth.

All that remains is the determination of a centre from the intersection point of the three lines (B) dropped from the extremities with the resulting spiral and triangular figures again the growth rate of $\phi^{2/3}$, $w = 1.37824077$.

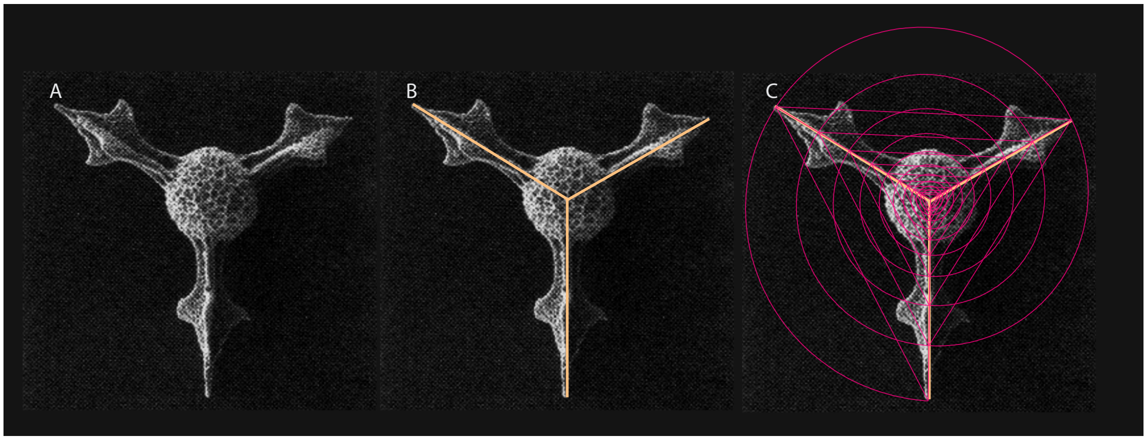


Fig. 29. **Radiolarian** *Sarla*. (Late Triassic).[72] Equiangular *Planorbidae* spiral & triangles $\phi_k = 2/3$, $w = 1.37824077$.

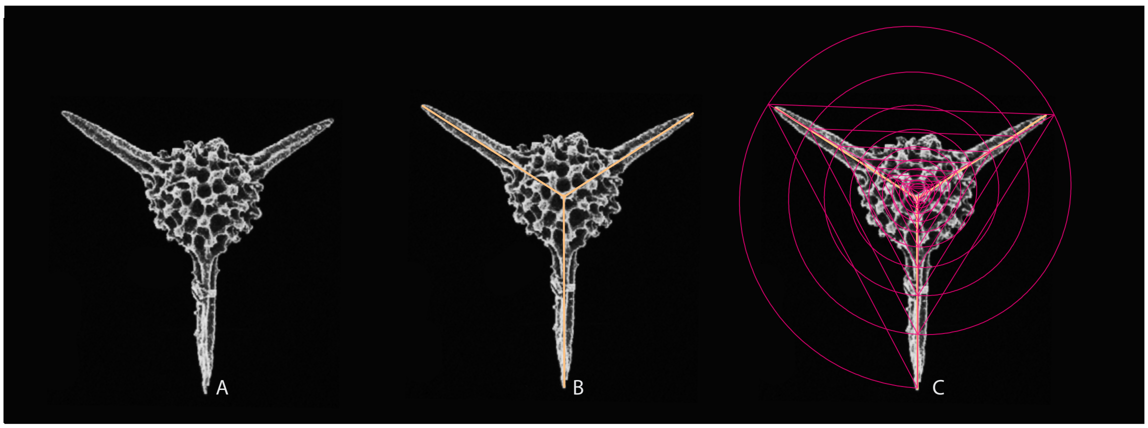


Fig. 30. **Radiolarian** *Chariottea amurensis*.[73] Equiangular *Planorbidae* spiral & triangles $\phi_k = 2/3$, $w = 1.37824077$.

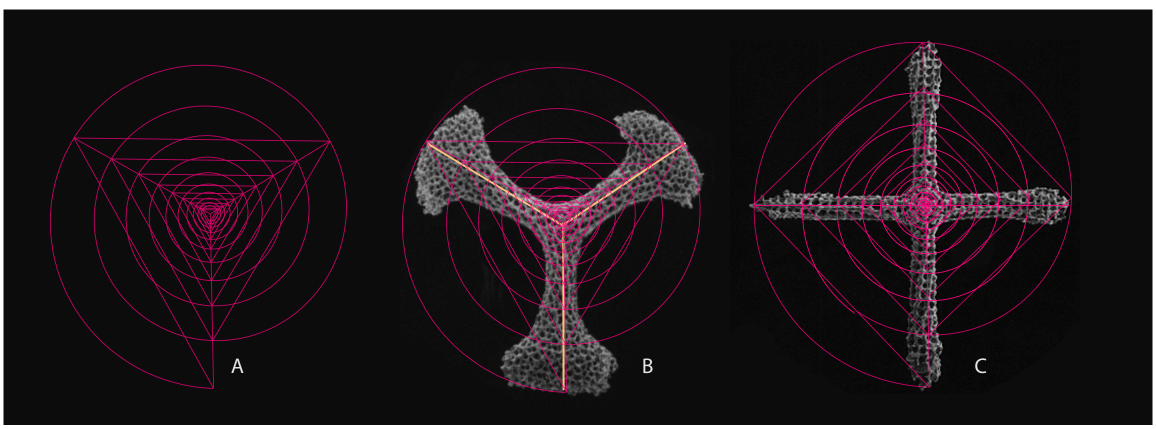


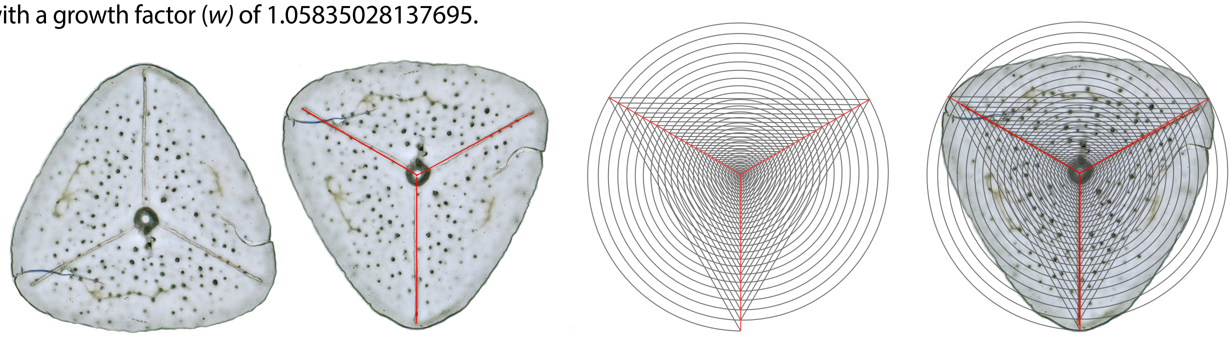
Fig. 31A-C. Radiolarians [74,75] *and* Equiangular triangle/square/spirals, $\phi_k = 2/3$, $w = 1.37824077$.

Thus, despite wide variations in size and shape, certain microscopic radiolarians and diodoms can be assigned equiangular spirals replete with integral equiangular figures. Accordingly, the four micro-photographs shown here were re-oriented to synchronise with anticlockwise motion and growth (w) per revolution for best-fit spirals and equiangular figures in the same manner used for ammonites. Therefore they also require a center, *provisionally* the junction point of lines extended inwards from each extremity towards their common meeting point as applied in Figures 29 and 30. The "square" configuration in Figure 31c was simpler to test since the centre is provided by the horizontal and vertical axes of the test spirals whi9ch were included for such purposes.

The sudden shift to invoke equiangular spirals with such a small growth factor requires explanation, whatever its its origins might be. Clearly, any equiangular spiral would now have a much smaller growth rate than the *Pheidian* Thirds and the Sixths. Even so, for continuity *Pheidian* growth was retained with test spirals (plus triangular figures) initially involving the *Twelfths*, namely $\phi^{1/24}$ (w = 1.034969827), $\phi^{1/12}$ (w = 1.040915886), $\phi^{1/16}$ (w = 1.030532583) and $\phi^{1/15}$ (w =1.032600924) then finally rounded out by $\phi^{1/9}$ (w = 1.054923213). Plus, from Plato's *Timaeus* and for good measure, the growth factor which results from the ratio 256/243 (w = 1.053497942).

Diodom(s)

The third example concerns the semi-related shape of a Diodom which already has a distinct center, but no obvious suggestion that a spiral of any kind is necessarily involved in its structure. Apart, that is, from the slight difference in lengths for the indicated 120° divisions, which prove to be sufficient to at least reorient the diodom as done for the radiolarians. Plus, by way of the *geometric mean,* an equiangular spiral based on $\phi_k = 1/\sqrt{72}$ (0.117851130197758) with a growth factor (w) of 1.05835028137695.



Original orientation & center. Test configuration $\phi_k = 1/(\sqrt{72})$ Spiral, w = 1.0583502814 Diodem/w $\phi_k = 1/(\sqrt{72})$ Spiral.

Fig. **32**. **Diodom** *Symbolophorus amblyoceros*[76] and equiangular spiral/triangles, $\phi_k = 1/(\sqrt{72})$, w = 1.0583502814.

In wider natural contexts, however, two diverse configurations immediately come to mind, namely individual snowflakes and the hexagonal cells grouped together in honeycombs. Both have pluses and minuses in terms of investigation, and so nearly perfect are their structures that neither suggest a spiral is necessarily present at all. Or perhaps more to the point, such spirals would be nearly circular with extremely small values for the growth factor per revolution. This is where the discussion returns to microscopic organisms, beginning with radiolarians, where it was perhaps fortuitous that the *Sarla* example suggests both logarithmic expansion and a possible spiral form among a bewilderingly wide range of other configurations. To a lesser extent the same may be said of the second and third examples, with the equiangular "squares" usable, although somewhat irregular.

Bi-Polar Figures

Clearly, this expansion has moved far beyond the initial investigation into the fit of sequential *pheidian* parameters to planispiral ammonites. Moreover, in spite of the suitability of generally complete and symmetrical radiolarians examined here, many examples in GSC 496 and the *Treatise on invertebrate paleontology* are unusable for this task due to damaged extremities or relatively unsymmetrical forms. However, there are still "square" and "hexagonal" radiolarians plus other "triadic" examples including relatively robust "bi-polar" forms, *e.g.,* the radiolarian *Chariottea harbridgensis*[74] similar to that discussed next.

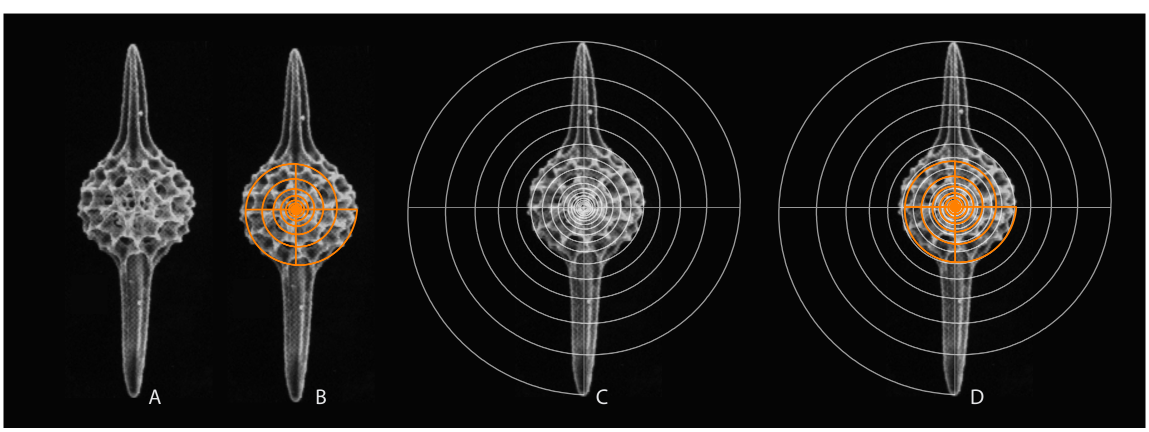


Fig. **33**. ***Radiolarian*** *Chariottea harbridgensis*[77] Equiangular *Planorbidae* spiral (B) ϕ_k = 5/6, w = 1.4933898 and main spiral (C and D) from the *Pheidian Twelfths* - ϕ_k = 5/12, w = 1.2220196.

The micro-photographs of radiolarians tested so far are from *GEOLOGICAL SURVEY OF CANADA BULLETIN* 496 (E. S. Carter, P. A. Whalen, and J. Guex, 1998) which also includes a number of bipolar figures. But even if the bipolar formats also employ *pheidian* growth with resulting equiangular spirals, there would appear to be no immediate way to determine the center of the spiral or the growth factor (*w*). Nevertheless, the provisional selection of three equiangular spirals: 1. Primary, $\phi_k = 1/3$, $w = 1.173984997$ (error: 5.26%), 2. Width B - X, X - B′ (common center), ϕ_k 5/12, $w = 1.222019633$ (error: -0.537%, and 3. Internal at 45°, $\phi_k = 9/12$, $w = 1.434632715$ (error: 0.767%) obtained from *measurements* of a bi-polar figure included in Fig. 34A by Pessagno, E.A. Jr., and Blome, C.D. 1980:

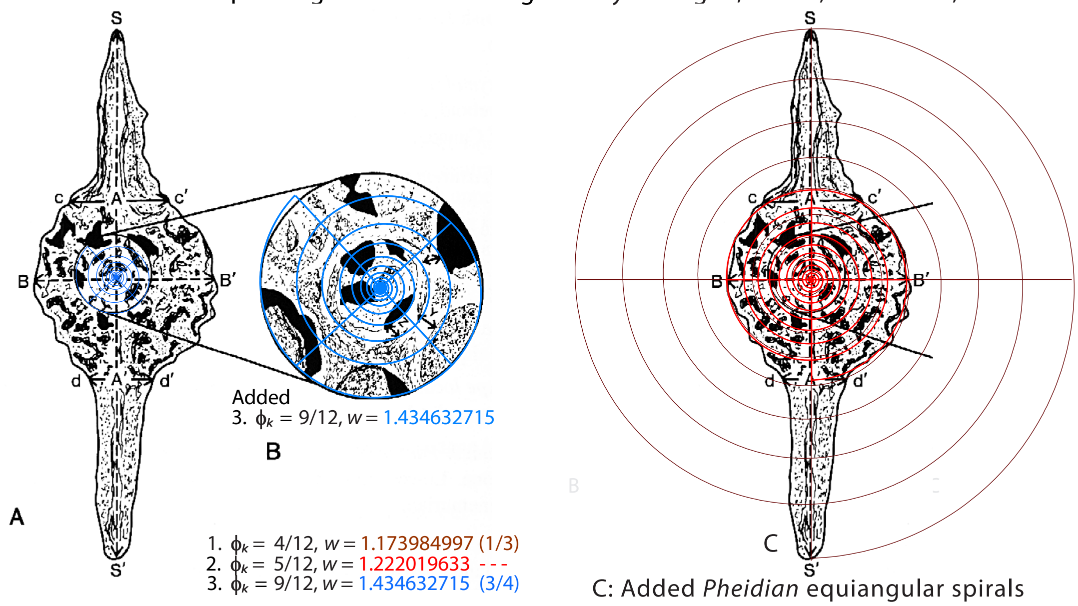


Fig. 34. *Added* best-fit equiangular pheidian spirals based on three sets of accurate, intersecting measurements and markers for Upper Triassic *Pantanelliinae* determined by Pessagno, E. A. Jr., and Blome, C.D. 1980.[78] The larger spiral has been flipped 180° about the vertical axis to improve the fit.

The luxury of having a clearly defined, accurate center for this bi-polar figure also provides an opportunity to apply the *pheidian planorbidae* best of three fit for adjacent or close known examples, which in the present case involves the lower Sixths: $\phi_k = 1/6$, $\phi_k = 2/6$ (1/3) and $\phi_k = 3/6$ (1/2). Scaled to fit in each instance from S′ to S only one of the latter will also match or come close to the center at X, as indeed is the case for $\phi_k = 2/6$ (1/3), as seen in Figure 35 where the differences on either side are both relatively small:

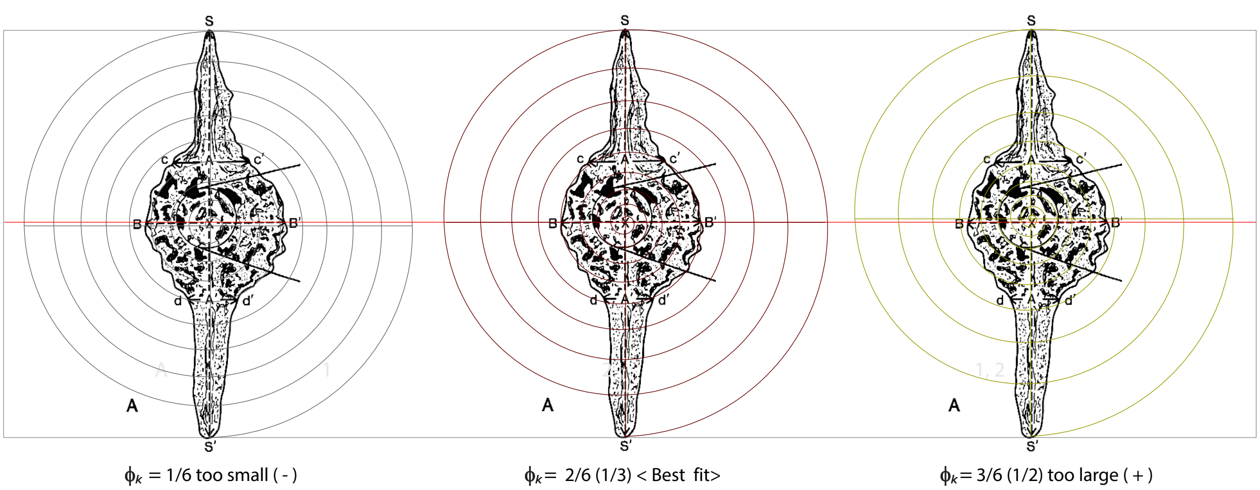


Fig. 35. Best fit test for the Pheidian Sixths, $\phi_k = 1/6$, 2/6 (best fit) and $\phi_k = 3/6$ based on the measurements for Upper Triassic *Pantanelliinae* determined by Pessagno, E. A. Jr., and Blome, C.D. The spirals are again flipped 180° about the vertical axis.

At which, having already moved beyond original intentions there remains the matter of hexagons and octagons among ammonites and the most likely *Pheidian* Planorbidae of interest in this context, ($\phi^{4/3}$, $w = 1.8995476$) .

The Pheidian *Sixths* as Periods (*T*) in Julian years and days

Even though equiangular squares, pentagons, hexagons and octagons are discernable in Figures 17 to 20, they are not clearly defined over a full revolution. Furthermore, recalling the complications raised by Peter Ward concerning ammonite growth and the outermost *septa*, what correlation there is invariably deteriorates as the last quadrant is reached. Then again, this region is more likely to suffer damage over time in addition, whereas the actual age of the examples may also be crucial, *i.e.*, pertaining to *juvenile* stages of development during which rapid changes may or not have taken taken place. Exactly *how* rapid and precisely *what* changes may have occurred may be debatable, with the suggestion that progression between figures - the triangle, the square, the hexagon and beyond - purely speculative. Nevertheless - as shown earlier - there *does* indeed exist an ammonite which is *triangular* in form.[67]

Either way, it now becomes necessary to include time and distance with velocity in terms of pheidian growth in the present astronomical context. In short, to produce the same planorbidae and same separation for Periods (*T*) and (*S*) from the generation of the Pheidian *Sixths* from Earth outwards for 1 to 3.073532624 years and also their corresponding radius vectors for the distances (*R*) and the velocities (Vi) with respect to unity.

Before more detailed examination, in terms of the possible *pheidian* nature of the Solar System it is necessary to recall that the four major relationships which pertain to increases in the mean sidereal periods of revolution (*T*) and mean heliocentric distances (*R*) - in years and a.u. respectively - are also members of the *planorbidae* Sixths, *i.e.*, numbers 4, 6, 8 and 12:

#4: *Phi*-series Planet-Synodic distance (*R*) increase: $k = \phi^{2/3}$ (1.37824077249), Outwards. Relation (11)
#6: *Phi*-series Planet-Synodic periods (*S*) increase: $k = \phi^{1}$ (1.61803398875), Outwards. Relation (8)
#8: *Phi*-series Mean heliocentric distance (*R*) increase: $k = \phi^{4/3}$ (1.89954762695), Planets, Outwards. Relation (9)
#12: *Phi*-series Mean periods of revolution (*T*) increase: $k = \phi^{2}$ (2.61803398875), Planets, Outwards. Relation (7)

All four are of continuing interest, especially ϕ^1 and ϕ^2 underlying the spiral of *Pheidias* and *Spirasolaris*. Moreover, relation (9) also concerns mean distance (*R*) with $w = 1.89954762695$ again associated with *both* the hexagon and the octagon in Figures 19 and 20. Thus, with the Pheidian Thirds as *Phi*-series *periods* derived from successive values from 1 through 7 a new test set including the latter range in years and days is as shown in the following table:

Relation/resonance	Period *T* (Years)		Period *T* (Days)	Distance *R*	Velocity (*Vi*)	Velocity (*Vr*)
EARTH: Synodic	THIRDS	#	365.25	1	1	1
Relation 12 (*Vi*)	1.172984997	1	428.79802004	1.1128629857	1.054923213	0.947936293
Relation 11 (*R,S*)	1.378240772	2	464.60517698	1.2384640249	1.112862986	0.898583215
Relation 8 (*T,S*)	1.618033989	3	503.40244215	1.3782407725	1.173984997	0.851799642
Rel. (9) FOURTH 4 : 3	1.899547627	4	693.80977074	1.5337931411	1.238464025	0.807451795
MAJOR SIX 5 : 3	2.230040415	5	814.52226142	1.7069016144	1.306484449	0.765412862
Rel. (7) OCTAVE 2 : 1	2.618033989	6	956.23691439	1.8995476269	1.378240773	0.725562630
	3.073532624	7	1122.6077907	2.1139362436	1.453938184	0.687787150

Table. 7. The Pheidian planorbidae *T*, *Thirds*, #1 to #7 in Julian years and days

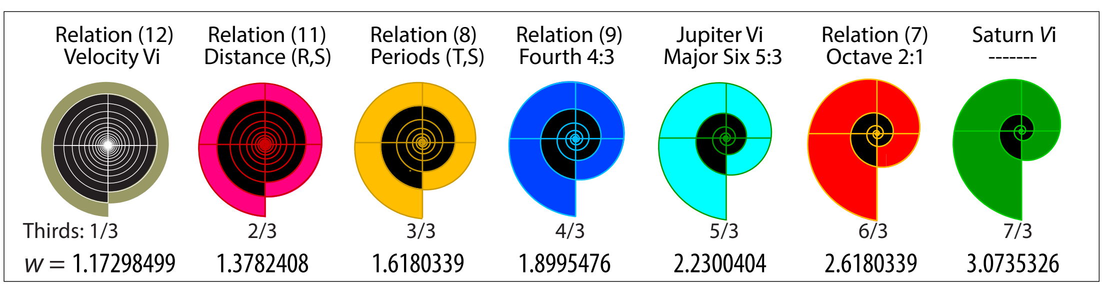


Fig. 3y. *The Pheidian Planorbidae* in Years. *Thirds*, $w = \phi^{1/3}$ through $\phi^{7/3}$ (not to scale; $w = \phi^{N/3}$)

This provides a useful line of inquiry, *e.g.*, using relation (9) with $T = \phi^{4/3} = 1.89954762695$ and the Julian year of 365.25 days the correspondingly period is 693.8097707 days per revolution with the mean periods for each of the *six* or *eight* partitions obtained by simple division:

Hexagon (6/side): 115.634961791 days per septum (3)
Octagon (8/side): 86.726221343 days per septum (4)

These results are recognisable as periods associated with the combined motions of both Earth and Mercury, *i.e.*, the Julian year, synodic relation (1) and the modern period (T) for Mercury produce the accurate synodic period (S)

Mercury-Earth synodic (S): 115.876693996 days (5)
Synodic relation (1): 365.249999988 days

Mercury (mean period T): 87.968435362 days (6)
with 693.80977074/365.25 = 1.889547627… ($\phi^{4/3}$)

The value for Mercury in relation (4) is on the low side, but that said, it is also closer to the comparable *phi*-series period for this position (86.726221343 days versus 86.22382878 days obtained from the product ϕ^{-3} JYR). Whether these two periods per section/septum are valid remains an unresolved issue at this stage of the investigation.

Whereas, despite their minor differences, the identical procedures for relations (3) and (4) produces a significant result recognisable as a close approximation for the luni-solar eclipse cycle: ECY of 346.620107 days (modern) and 346.619576 days (Babylonian) obtained from the mean synodic (MSM) and mean draconic month (MDRA):

Hexagon (per side): 115.634961791 days per septum (3)
Synodic relation (1): 346.904885372 days per cycle (8)
Octagon (per side): 86.726221343 days per septum (4)

As for associated *planorbidae* #8 for the two figures (ϕ_k = (8/6), 4/3, w = 1.8995476269), a half of 693.80977074 days per revolution is also 346.9048854 days. More immediately, however, despite relatively small masses, the positions of Earth, Venus, Mercury, and below the latter theoretical IMO 1 (of unknown mass and uncertain presence) can all be included. Thus for the *planorbidae* of primary interest - #2 and #4 from the *Thirds* (#6 and #8 from the *Sixths*) - in the present context, remaining with the product of of the latter and the Julian year, the following table of *Fibonacci* divisors applied to the 693-day period with modern estimates for comparison is:

Divisor	Periods (days)	Modern (T,S)	Assignments (T,S), Cycles
Base	**693.8097707**	Julian Yr 365.25 =	**Earth**, Revolution T (days)
2	346.9048854	346.604554 ~	**Eclipse Cycle** EYC (days)
3	231.2699236	224.695434 ~	**Venus**, Sidereal T (days)
5	138.7619541	144.566223 ~	**Mercury-Venus** Synodic S
(6)	**115.6349618**	115.876694 ~	**Mercury-Earth** Synodic S
8	**86.72622134**	87.9684354 ~	**Mercury,** Sidereal T (days)
13	53.36998236	52.8177728 ~	**IMO1-Mercury** Synodic S
(18)	38.54498726	38.6843214 ~	**IMO1-Venus** Synodic S
--	33.99877856	33.9988938 ~	**MSM-Venus** Synodic Si
21	33.03856051	33.0025 =	**IMO1**, Revolution T (days)
--	29.53059414	29.5305895 =	**MSM,** Mean synodic month

Table 26. *Triangular, pentagonal, hexagonal, octagonal divisors* and the Inferior planets plus IMO1.Base period of 693.809770744 days from #8: ϕ_k = 8/6 or (4/3), w = 1.89954762698 •JYR (365.25 days).

Technically, the inclusion of the mean synodic month is *positionally* incorrect, yet despite the disparity and further complexity which arises from its concurrent motions about Earth, in terms of *order the period is numerically* correct. In other words, the numerical positions from Earth *inwards* according to the *periods* are Earth-Venus-Mercury-Moon.

Thus Earth, Venus and Mercury are joined by IMO1 plus, not unsurprisingly, the motion of the moon, albeit initially the approximate 29.5-day mean synodic month (MSM; Babylonian 29;31,50,78,20 days) and the shortest associated month, the draconic (MDRA). Furthermore, the 411-day anomalistic cycle involving MSM and anomalistic month (MAN) can be similarly investigated by reversing the period process.

The fact is, however, that almost all the present attempts to expand on the occurrences of spirals - pheidian or otherwise - are hampered by orientation, and above all, periodicity. Plus another area of complexity arising from three-dimensional form and growth. On the other hand, at least, certain growth factors seem to occur more often than others, *e. g.,* ϕ_k = (2/3), w = 1.37824077 among radiolarians, and ϕ_k = (4/3), w = 1.899547627 discussed above and earlier with respect to hexagonal and octagonal features among ammonites.

Pheidian Thirds and Spirals among particle tracks in bubble chambers

In view of the relatively small numbers of examples tested to date, additional searches for spiral growth based on ϕ_k = (4/3), w = 1.899547627 were undertaken. The searches more recently included recognizable examples of spirals encountered among particle tracks in bubble chambers, with those investigated by Syed Waqar Ahmed[79] of the

National University of Computer and Emerging Sciences, Karachi, Pakistan of interest. Augmented by Wolfram[80], this research was made freely available on the Internet, which in the present instance included a description of the experiment and the methodology applied to the fitting of spirals in this context. The results as given in the text are shown below in Figure 36.

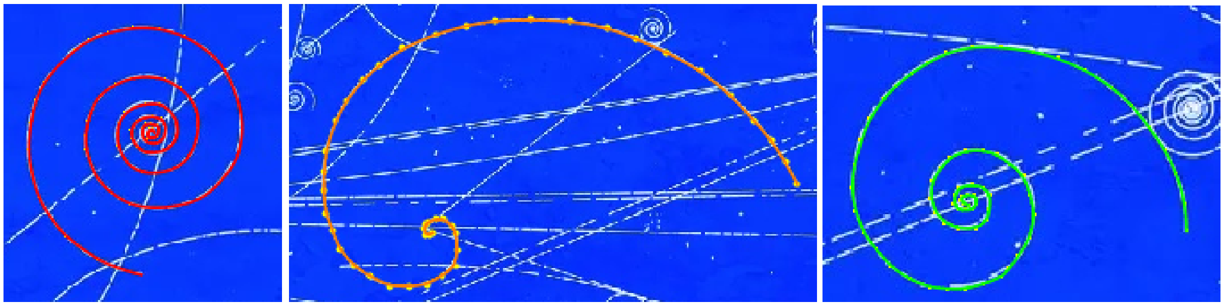


Fig. 36. The fitting of spirals to those in bubble tracks described and demonstrated by Syed Waqar Ahmed.

Next, for comparison with the standard colours used for the *Pheidian Planorbidae*, the blue background of the original figure has been replaced by black with inner titles A, B, C, the orange spiral, and three-spiral inset in Figure 37AC added to aid the following commentary.

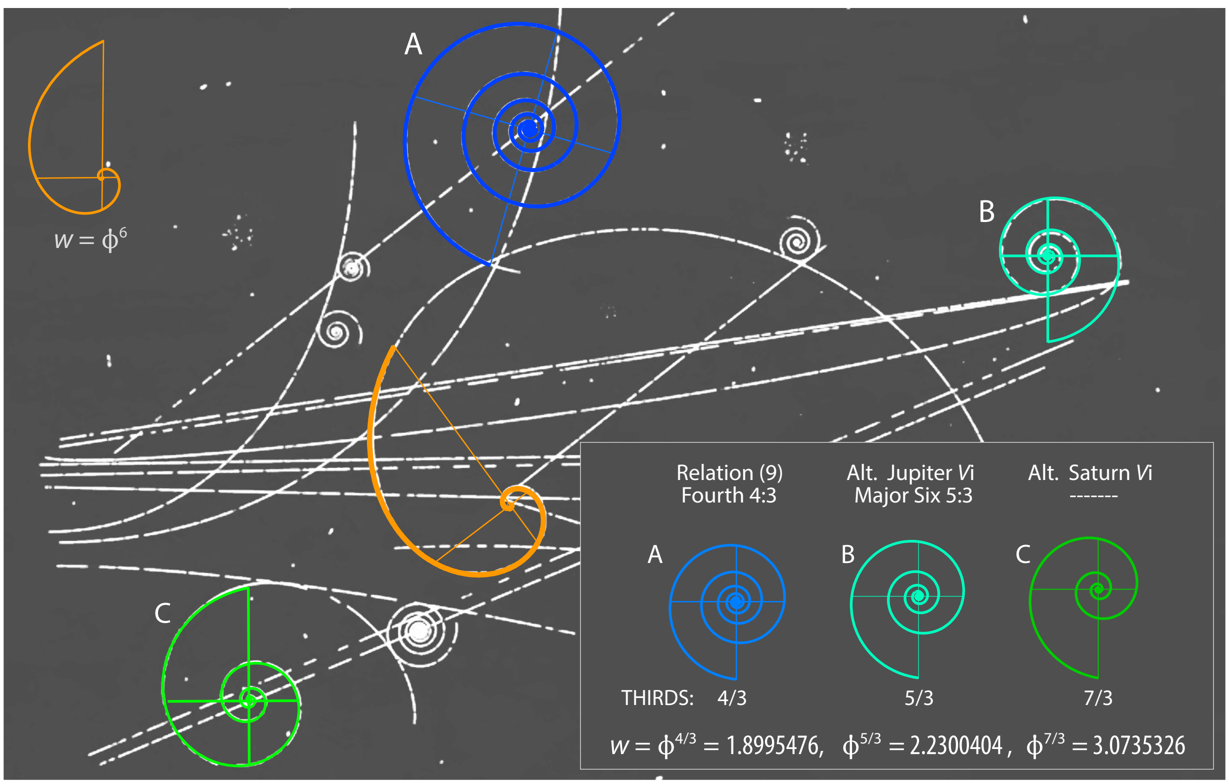


Fig. 37AC. *Pheidian Planorbidae* and spirals in bubble tracks. *Thirds*, $w = \phi^{4/3}$, $\phi^{5/3}$ and $\phi^{7/3}$ (not to scale; $w = \phi^{N/3}$)
(The larger spiral was provisionally assigned a growth rate of $w = \phi^6$ {17.94427191 : 1} per revolution but not included in the inset as such).

Remarks

First of all, the blue spiral (A) is clearly recognizable as the spiral of current interest - $\phi^{4/3}$ with $w = 1.8995476$.
Secondly, the blue-green spiral (B) is found to be the next *Third*, *i.e.*, following $\phi^{4/3}$ is $\phi^{5/3}$ with $w = 2.2300404$.
Thirdly, the green spiral(C) also belongs to the *Thirds*, but no longer sequential, *i.e.*, $\phi^{7/3}$ with $w = 3.0735326$.
Fourthly, the orange spiral (top left addition) is given an uncertain assignment of ϕ^6 with $w = 17.944272$.

Although not a problem *per se,* there is nevertheless something puzzling about the inclusion of equiangular spirals without identification in modern works, especially those of scientific or mathematical nature. Yet as shown next, there are three examples of the spiral under consideration ($\phi^{4/3}$, $w = 1.8995476$) in an "artistic" format published in 2001 by mathematician Ian Stewart.

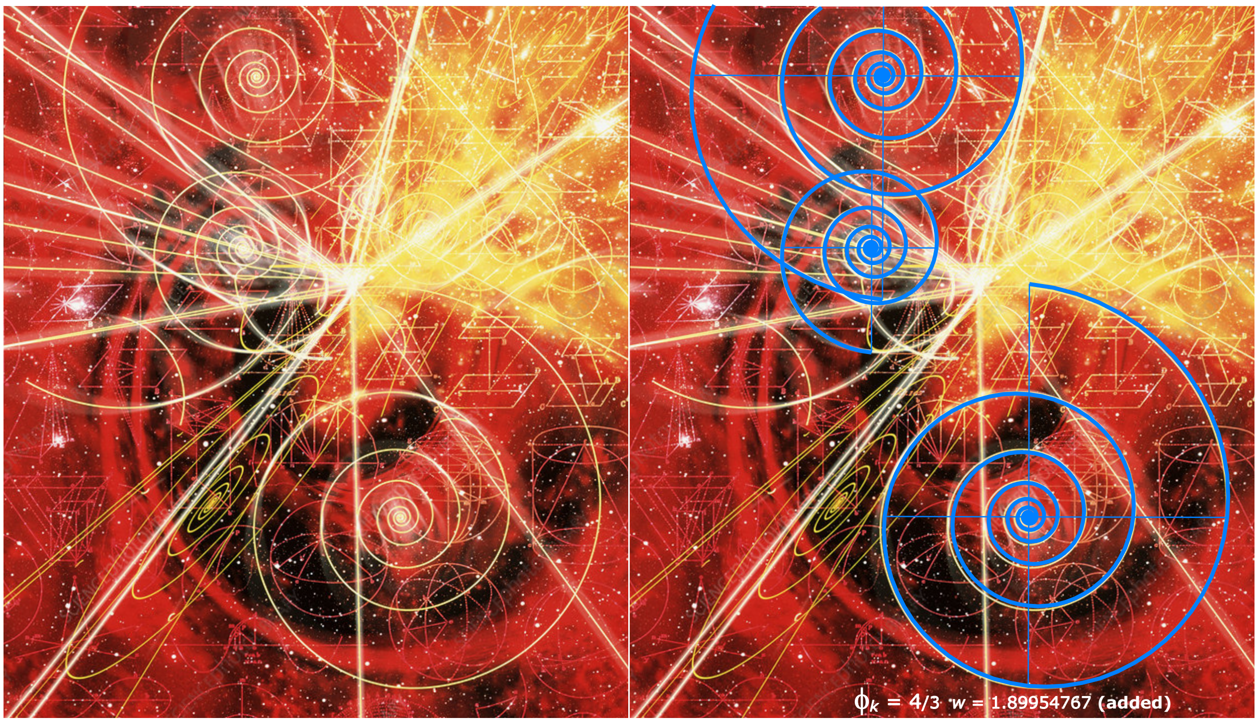


Fig. 38. The *Pheidian spiral* $\phi^{4/3}$, w = 1.8995476 and spirals with geometric patterns in *What Shape is a Snow Flake? Magical Numbers in Nature* included by Ian Stewart.[81]

Nor do such additions stop here either; with the self-same spiral again in a similar artistic representation [82] which might suggest - intentionally or otherwise - that the ratio between the two outermost whorls of these major spirals may be of interest in addition to the esoteric beauty of the images, and from such a start it is just possible that the Golden Ratio itself might ultimately emerge in a setting already geared towards scientific expansion.

A fanciful notion perhaps, but given how close David Raup got to the *Phi*-series planorbidae shown earlier in the Figure 11 comparisons ("Test spirals within the Raup range w = 1.25 to 3.00. . . "), and the 22 years since Stewart's inclusion of this fundamental spiral, we might well benefit from an occasional hint or prod now and again.

Then again, in the last month of the year 2023 matters are becoming increasingly disturbing on a global scale although this is not the time for anger or despair against the self-serving excesses enacted by nations, religions, institutions, networks or members at large. Better to follow the cautions offered by Confucius about murmuring against God or raging at Mankind, and continue with the business at hand. Which in the present context was the intimations of Benjamin Pierce's *Fibonacci, Lucas* and *Phi*-based planetary framework. Followed n turn by natural expansions generated by the apparent presence of the *Fibonacci* series and *Phi* itself in Babylonian astronomical cuneiform texts of the Seleucid Era. Which, although not originally intended, precipitated the need for an optional Excursus on the matter included at the end of this final section, our increasingly troubled times notwithstanding.

As for the past on a wider scale, the extensive range and influence of Babylonian mathematics are discussed in detail by Jorän Friberg in works published in 2005, 2006 and 2007.[83-85] More recently, inroads into complexities inherent in Babylonian astronomical texts of the Seleucid Era have been made by Mathieu Ossendrijver (2016)[86] concerning the *velocity of Jupite*r and an unexplained *trapezoid* in Babylonian texts for this planet. Furthermore, additional complexities arise from the analysis by Daniel F.Mansfield and N.J.Wildberger (2017) [87] of Plimpton 322 (a complex Babylonian mathematical text) as an exercise in "Babylonian exact sexagesimal trigonometry."

On a more mundane level it should be explained that the name *planorbidae* did not stem from the astronomical nature of the two *Phi*-series under discussion, but earlier attempts to fit *planorbidae* spirals to shells, ammonites and in particular the configuration of the earliest ammonite, *Psiloceras Planorbis.* Subject to further refinement, the best fit for the latter is a pheidian growth rate of 1.8995476279 per revolution, and thus (perhaps coincidentally), *Phi*-series relation (9), the planet-to-planet increase in heliocentric distance ($\phi^{4/3}$). This remains to be confirmed or redefined, as do further attempts to fit scalable *planorbidae* to various organisms and mechanisms including *radiolara*, *diodoms*, *foramina*, shells and ammonites. This said, there are now sophisticated modern studies such as the "Ammon" Database by Paul L. Smith (1986)[88] and later work by Liang and Smith (1997).[89]

On a far wider level, who knows how extensive and how complex "pheidian order" might ultimately be, the very structures of tornadoes, hurricanes and spiral galaxies included at one extreme and micro particles at the other. Which are, after all, natural and logical expansions of Aristotle's "all things whatsoever," are they not?

Another purpose for including ammonites in the present discussion is the apparent retention of the underlying planetary structures despite omissions and departures from the theoretical model both locally and further afield. This, coupled with the reappearance of " Lazarus " ammonites [90] eons after extinction suggests that some form of ordering mechanism may be involved. Although not, of course, with respect to this occurrence, the possibility of relevant knowledge in the past comes to fore, augmented by both the Fibonacci and Lucas series in the present study. It remains, however, unclear how much relevance and verity unduly obscure statements in the *Chaldean Oracles* concerning the "monad," the "dyad," and "Intellectual sections to govern all things, and to order all things not ordered." Or, in the same place, "Fountain of fountains, and of all fountains, the matrix containing all things.[91]

Certainly Plato and Proclus were held in high regard centuries after their passing, with the former's *Timaeus* also given special prominence despite its scale and complexity. In this respect the opinions of later commentators are also illuminating. Macrobius, for example, in his *Commentary on the Dream of Scipio*, states that "numbers preceded the World-Soul, being interwoven in it, according to the majestic account in the *Timaeus*, which understood and expounded *Nature* herself." [92] Whereas Vitruvius (ca. 75-25 BCE) sort fit to include the number 216 (*i.e.*, 6^3 from the senary interval) in the *Ten Books on Architecture* in an obscure reference to Pythagoras and rules "founded on the analogy of *nature*." [93] This is a recurring theme; according to Theon of Smyrna, the *Tetractys* referred to earlier was "not only principally honored by the Pythagoreans, because all symphonies are found to exist within it, but also because it appears to contain the nature of all things." [94] Thus all-encompassing assessments embraced by scholars that seem to have continued up to and including the Reformation, but thereafter less publicly.

As for earlier times, consider the insights provided by Archytas [95] who was reputedly: "The first who methodically applied the principles of mathematics to mechanics: who imparted *an organic motion* to *a geometric figure*." Italics are supplied here for one primary reason, which is the linking of mathematics, motion, and ultimately, "Nature," in an *organic* sense. What follows from this awareness, or if preferred, this theoretical premise, remains conjectural whereas Plato provides both encouragement and a positive conclusion towards the end of the *Epinomis,* [96] thus:

> To the man who pursues his studies in the proper way, all geometric constructions, all systems of numbers, all duly constituted melodic progressions, the single ordered scheme of all celestial revolutions, should disclose themselves, and disclose themselves they will, if, as I say, a man pursues his studies aright with his mind's eye fixed on their single end. As such a man reflects, he will receive the revelation of a single bond of natural interconnection between all these problems. (Plato, *Epinomis* 991-992).

Whatever the single bond and the natural interconnection might be, from the contents of many ancient works some clarity is gained by the time of the Reformation and the Enlightenment. First of all, there seems little doubt that the fundamental premise of "The Doctrine of the Timeaus" was wholeheartedly embraced by many scholars, who in turn nurtured it and labored to pass it on, dangers and troubled times notwithstanding. Secondly, in view of statements in ancient, medieval and later sources, it begins to becomes apparent, to some extent at least, what was being handed down. Taken together, the implications of "organic motion" introduced by Archytas, reference by Proclus to "physiology," the scope of the Pythagorean *Tetractys,* the texts, canons and aphorisms of Alchemists, and the bald reference to *Phyllotaxis* by Peirce and Agassiz all point to an active, extensive view of life itself, and ultimately, one might suggest, *something akin to a living universe* as the ancients perceived it.

Whether comprehension of an all-inclusive, all-pervading living universe is in line with the scale and the intent of Plato's *Timaeus* as Proclus understood it is likely to remain uncertain. But either way, given the turmoil and strife in the Middle Ages and long thereafter, it is understandable that this all-encompassing viewpoint could not become generally known or widely propagated. The intent, however, may still have flourished, albeit guardedly, and from this viewpoint, it might well be that Sir Isaac Newton, renowned scientist but also an alchemist, was correct in his day when he offered the following conclusion to his *Mathematical Principles of Natural Philosophy*: [97]

> And now we might add something concerning a certain most subtle spirit which pervades and lies hid in all gross bodies; by the force and action of which spirit the particles of bodies attract one another at near distances, and cohere, if contiguous; and electric bodies operate to greater distances, as well repelling as attracting the neighboring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this spirit, mutually propagated along the solid filaments of the nerves, from the outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explained in few words, nor are we furnished with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates.
> Sir Isaac Newton, *Mathematical Principles of Natural Philosophy*.

To what degree this assessment implies the acceptance of a complex, living universe is also unknown, but in view of the massive outpouring of obscure yet highly detailed literature from the Alchemists before and after Newton's time, it can be surmised that the Enlightenment was disappointingly incomplete without this all-encompassing viewpoint. As for this failure, if that is what it was, this is not a matter for recrimination or blame, but in retrospect it was in all likelihood simply the wrong time and the wrong place.

As for the present, one would like to think that we are more than ready now. In fact four centuries have elapsed since Johannes Kepler and the year 1618. So, though long overdue, perhaps the *Enlightenment* can continue and the concept of the living universe, or whatever name is appropriate, be permitted to run unfettered alongside both *Science* and *Religion*.

Though resistance to change is almost universal, for religions the concept of God is generally synonymous with an all-pervading entity, thus the living universe is essentially one more name for the *Almighty* according to numerous cultural perspectives. Science, on the other hand, will still retain the "Big Bang Theory" as a primary hypothesis, possibly influenced by "living" aspects of the matter, and quite possibly not. But either way, one might also hope this awareness may eventually help to establish common ground between science, religion and atheism, and all persuasions and passions in between.

In any event, in these troubled times, what might be needed above all else is a sense of belonging accompanied by a related sense of responsibility towards others, all forms of life, and not least of all, our immediate environment. As for myself, these fundamental issues are aptly indexed by *NUMEN LUMEN, the* motto of the University of Wisconsin in Madison, USA where this inquiry began in the darkness before the dawn more than fifty wandering years ago:

The Divine within the Universe, however manifested, is my Light.

And possibly, *the Light that fills the World . . .*

John Nicholas Harris, spirasolaris.ca
West Vancouver, British Columbia, Canada. 20 December, 2023

Only One Great Thing

I think over again My Small Adventures,
My Fears.
Those Small ones that seemed so Big.
Of all the Vital Things
I Had to Get and Reach.
And yet there is only One Great Thing,
The Only Thing,
To Live to see the Great Day that Dawns,
And the Light that fills the World.*

* Old Inuit Song

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INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS

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