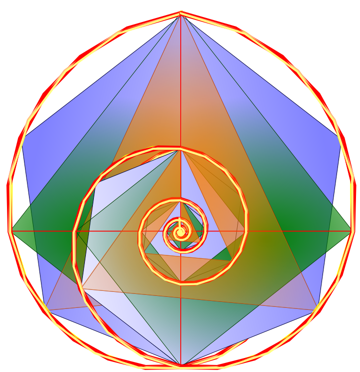
INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS



2023

PART FOUR

THE FIBONACCI SERIES, THE LUCAS SERIES AND PLATONIC TRIANGLES

JNH

HISTORICAL EXTENSIONS AND FURTHER IMPLICATIONS

Historical underpinnings

Commenting on Aristotle's *De caelo et mundo*,[1] medieval French scholar Nicole Oresme (1320–1382 CE) sought fit to include the cryptic statement "All Things are Three," augmented in part by the following quotation by Ovid: [2]

> *Said Aristotle, lord and prince of Greek philosophers and never-failing friend of Truth: All Things are three; the three-fold number is present in all things whatsoever, nor did we ourselves discover this number, but rather, nature discovers it for us*.

Antiquity and bald statement notwithstanding, the identity of the "three-fold number" and reference to "nature" in the above are abundantly clear. Indeed, since Oresme's time the latter has been variously named (*Divine Ratio*, Divine Section, Golden Section, Golden Ratio, etc.) and also discussed at length, earlier observations by Fibonacci (ca.1127–1240 CE) and series of the same name included. But, as noted earlier, it is unlikely that this intriguing series would have escaped the attention of early Greek philosophers or other inquiring minds. Nonetheless, for continuity the "Fibonacci series" has been retained as such throughout the present work, with the "three-fold number," and/or Golden Ratio defined here as the limiting value of the ratios of adjacent numbers (the larger over the lesser) of the Fibonacci series. The latter, of course, already discussed in terms of reduction ratios selected by Benjamin Pierce having now come full circle to re-embrace phyllotaxis and natural growth.

Even so, well before this the quest for enlightenment appears to have been enduring, and to some extent at least, successful. Johannes Kepler (1571–1630CE), for example, following Aristotle's use of "planes,"[3] introduced the series "1,1,2,3,5,8,13,21" and associated ratios before declaring: [4] "in the flower is displayed a pentagonal standard, so to speak." Later, N. Grew [5] surmised that "from the contemplation of Plants, men might first be invited to Mathematical Enquirys," while more widely and more recently modern mathematician Ian Stewart observed in *Nature's Numbers* (1995) that "nature leaves clues for the mathematical detectives to puzzle over."[6]

Historically, following Fibonacci inquires concerning the mathematical complexities of phyllotaxis and related matters have extended from the dimensions and forms of shells investigated by Canon Mosely[7] (1838) to the very structure of the Solar System pursued in the 19th Century by Benjamin Peirce.[8] The general inquiry was continued by Arthur H. Church[9] (1904), Sir Theodore A. Cook[10] (1914), Sir D'arcy Wentworth Thompson[11] (1917), R. C. Archibald[12] (1919), Samuel Colman[13,14] (1920) and Jay Hambidge[15] (1920) in the early part of the previous century on into the next. And significantly, the recent revelation (2021) by A. Asadi[16] that "the constant 1.618 can be seen everywhere" in the remains of the Apadama Palace in the ancient Persian city of Persepolis (ca. 550 BCE).

Lastly, to this partial list can also be added since the start of the 21st Century related material made available on on the World-Wide Web by scientists, academic institutions and interested parties.

Historical extensions

As far as Benjamin Pierce's unappreciated contribution and the stagnation that followed are concerned it is most unfortunate that the synodic component and the natural expansions were not addressed in his time. Sadly, in view of the swift rejection of his research, it is also understandable now why Benjamin Pierce in his capacity as President of *The American Association for the Advancement of Science* stressed freedom from being "helplessly exposed to the assaults of envious mediocrity," in his otherwise inspirational outgoing speech to the Association in 1853.[17] This said, the contents of the speech with its numerous erudite references to the past suggest, even without synodic components, that his understanding of the matter in both time and place was extensive. Indeed, it becomes quite apparent that the information pertaining to this matter in ancient writings is detailed, complex, comprehensive and as yet to fully enter the mainstream of modern science. Part of the difficulty lies in the oddly neglected subject of orbital velocity, particularly as treated in Plato's well-known dialogue the *Timaeus*, as Galileo (1638)[18] and more recently Harris (1989)[19] unsuccessfully attempted to impart. Why this deficiency still persists is hard to understand, but with the inclusion of velocity as applied in *Phi*-series Table 3 it can be shown that humankind has not lost as much ancient knowledge as currently thought. In fact, the underlying core seems to have survived, preserved, as Thomas Taylor (1785-1837) notes, by way of the teachings of Orpheus, Plato and Pythagoras, the first: "mystically and symbolically; by the second, enigmatically, and through images; and scientifically by the third."[20] But even so, a considerable amount of material remains to be analyzed and if anything there is too much with too little guidance. Fortunately, initial help and a narrower focus are supplied by the Neo-platonist Proclus (410–485 CE), who wrote: [21]

> *If I had it in my power, out of all the ancient books I would suffer to be current only the Oracles[22] and the Timaeus*.[23]

The latter–one of Plato's better known dialogues–is the more familiar, whereas the former still remains relatively obscure, if not entirely arcane. This is, however, the same Proclus who begins his own commentary on the *Timaeus* with an introduction which is remarkable for its scope and caustic certitude; to wit: [21]

> *That the design of the Platonic Timaeus embraces the whole of physiology, and that it pertains to the theory of the universe, discussing this from the beginning to the end, appears to me to be clearly evident to those who are not entirely illiterate.*

At which point temporal racism – that this (*whatever*) could not be known by (*whomever*) in those days (*whenever*) – likely cuts in with most modern readers. But in the final analysis it still comes down to the details and overall picture which emerges, assuming such a progression does indeed prove to be feasible.

This said, at least parts of the *Chaldean Oracles* are relatively straightforward, whereas mathematical components in Plato's *Timeaus* and related sources generally remain unclear, despite detailed examination by modern scholars, *e.g*., de Santillana (1969),[24] Cornford (1975),[25, 26] Brumbaugh (1977),[27] McClain (1978),[28] and two compendia on the "The Harmony of the Spheres" assembled by Godwin (1993).[29, 30] Cornford, however, supplies a further qualifier, for he warns that: "the *Timaeus* covers an immense field at the expense of compressing the thought into the smallest space."[31] More specifically, the two best known numerical transformations in the *Timaeus* concern firstly the role played in the construction of the "World-Soul" by the Double [1,2,4,8] and the Triple [1,3,9,27] intervals. The latter pair have already been discussed briefly with respect to the velocity components of the laws of planetary motion, but further investigations reveal that there is indeed far more to the *Timaeus,* including a return to the Pierce divisors and the associated planetary framework as explained next.

The Platonic Triangles, the rotation of the elements, and the Music of the Spheres.

Among the better-known mathematical conundrums in the *Timaeus* is the emphasis placed on two specific types of triangles, namely the *isosceles* and the *equilateral* which are described as follows: [32]

> Now, of the two triangles, the isosceles is of one type only; the scalene, of an endless number. Of this unlimited multitude we must choose the best, if we are to make a beginning on our principles. For ourselves, however, we we postulate as the best of these many triangles one kind, passing over all the rest; that, namely, a pair of which compose the equilateral triangle . . . the one isosceles (the half-square), the other having the great side triple in square of the lesser (the half-equilateral)...[54b.] We must now be more precise upon a point that was not clearly enough stated earlier. It appeared as though all the four kinds could pass through one another into one another; but this appearance is delusive; for the triangles [54c.] we selected give rise to four types, and whereas three are constructed out of the triangle with unequal sides, the fourth alone is constructed out of the isosceles. Hence it is not possible for all of them to pass into one another by resolution, many of the small forming a few of the greater and vice versa. But three of them can do this; for these are all composed on one triangle, and when the larger bodies are broken up several small ones will be formed of the same triangles, taking on their proper figures; and again when several of the smaller bodies are dispersed into their triangles, the total [54d.] number made up by them will produce a single new figure of larger size, belonging to a single body. So much for their passing into one another. (*Timaeus*, 53d - 54d, translation by Francis MacDonald Cornford).

It remains to show that ***the isosceles triangle not only pertains to the Fibonacci series*** but also ***the Pierce ratios in their resonant triple form***. Furthermore, in a like manner it can be demonstrated that ***the equilateral triangle produces the Lucas series***, and here ***once again in resonant form***.

Either way, insights concerning this material are provided by a Babylonian mathematical tablet (YBC 7289) from the Old Babylonian period in the shape of an "ellipsoid" that circumscribes a square with both the diagonals and value for the side of the square obtained from the Babylonian estimate for root of two: 1;24,51,10 (1.4142129).[33] Thus the diagram has four isosceles triangles with the hypotenuse obtained from the radius and the Pythagorean theorem some 1500 hundred years before the time of Pythagoras. This is nothing new, and nor apparently, is the application in the present context, perhaps best described as "The Rotation of the Elements," as described by John Opsopaus (1995),[34] who included the following line from Alchemist George Ripley (d.1490): "*When thou hast made the quadrangle round, Then is all the secret found*." Opsopaus adds later that "The rotation of the elements is a key alchemical procedure, the principal means by which the purified essence of a substance is extracted and raised to its most sublime state."

The Isosceles Triangle, the Equilateral Triangle, and the Rotation of the Elements

As for the "rotations," they proceed in 90-degree stages, thus four "rotations" per revolution commencing with the parameters of the two specified right triangles. Initial values for the *isosceles* triangle are Base =1, Perpendicular=1, Hypotenuse = $\surd 2$. For the first 90° rotation the initial base of 1 is retained but the Perpendicular (1) is replaced by the original hypotenuse ($\surd 2$) with the new hypotenuse ($\surd 3$) obtained from the Pythagorean theorem, and so on. Shown in Table 1, the successive *squares* of the sides of the first triangle are in due order immediately recognizable as the resonant Fibonacci Triples (RZT) in company with the Peirce period divisors *en route* to the limiting triangle:

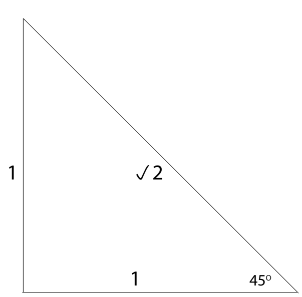
GROWTH/90° (¼ Rotations)	ISOSCELES Sides: B, P & H	EXPANDING Rt. Triangles	SIDES² Fib.Triple	B. PIERCE Reduction	RATIO OF SIDES P/B, H/B (Ref B)	ANGLES (∠) Degrees (B_{60})
Initial Triangle	BASE B1 = 1	1	1	NEPTUNE	1	Initial Angle
Timaeus (54a)	PERP. P1 = 1	1	1	1/2	1	(ISOSCELES)
$H = \sqrt{(B^2 + P^2)}$	HYP. H1 = √2	1.4142136	2	Uranus	1.41421356237	45°
Quadrant #1	BASE B2 = P1	1	1		1	
¼ Cycle	PERP P2 = H1	1.4142136	2	1/3	1.41421356237	
90°	HYP H2 (calc.)	1.7320508	3	Saturn	1.73205080757	54;44,8,12
Quadrant #2	BASE B3 = P2	1.4142136	2		1	
½ Cycle	PERP P3 = H2	1.7320508	3	2/5	1.22474487139	
180°	HYP H3 (calc.)	2.2360679	5	Jupiter	1.58113883008	50;46,6,33
Quadrant #3	BASE B4 = P3	1.7320508	3		1	
¾ Cycle	PERP P4 = H3	2.2360679	5	3/8	1.29099444874	
270°	HYP H4 (calc.)	2.8284271	8	(M-J Gap)	1.63299316186	52;14,19,30
Quadrant #4	BASE B5 = P4	2.2360679	5		1	
Revolution #1	PERP P5 = H4	2.8284271	8	5/13	1.26491106407	
360°	HYP H5 (calc.)	3.6055513	13	Mars	1.61245154966	51;40,16,15
Quadrant #5	BASE B6 = P5	2.8284271	8		1	
¼ Cycle	PERP P6 = H5	3.6055513	13	8/21	1.27475487839	
450°	HYP H6 (calc.)	4.5825757	21	Venus	1.62018517460	51;53,13,28
Quadrant #6	BASE B7 = P6	3.6055513	13		1	
½ Cycle	PERP P7 = H6	4.5825757	21	13/34	1.27097781860	
540°	HYP H7 (calc.)	5.8309519	34	Mercury	1.61721508013	51;48,16,8
Quadrant #7	BASE B8 = P7	4.5825757	21		1	
¾ Cycle	PERP P8 = H7	5.8309519	34	(21/55)	1.27241802057	
630°	HYP H8 (calc.)	7.4161985	55	(IMO #1)	1.61834718743	51;50,9,38
Quadrant #8	BASE B9 = P8	5.8309519	34		1	
Revolution #2	PERP P9 = H8	7.4161985	55	(34/89)	1.27186754767	
720°	HYP H9 (calc.)	9.4339811	89	(IMO #2)	1.61791441641	51;49,26,16
		{ Quadrants 9 through 27 omitted }				
Quadrant #28	BASE B29 = P28	717.09762	514229	B29 = 1	1	Limiting
Revolution #7	PERP P29 = H28	912.16227	832040	P29 ≈ √ϕ	1.27201964951	Angle:
2520°	HYP H29 (calc.)	1160.2883	1346269	H29 ≈ ϕ	1.61803398875	51;49,38,15

Table 1. The Isosceles triangle, the "Rotation of the Elements" I and the Pierce reduction ratios.

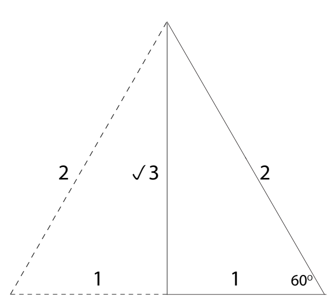
Limits, with angles given to the third sexagesimal place, dimensions of the fourth dual triangle and growth per quadrant/per revolution are as follows:

LIMIT: Base angle: 51;49,38,15.° Vertex: 38;10,21,45.° LIMIT: Proportion: 1, √ϕ, ϕ^1 (The Half-*Phi*-Series).
Double vertex: 76;20,43,30. Double Triangle: Base 2, Height 1.2720196495, both sides 1.61803398875.

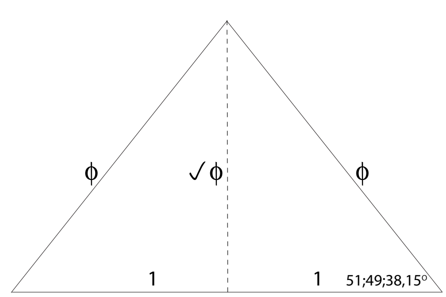
Growth per quadrant: √ϕ (1.2720196495). Growth per Revolution: (*w*) = ϕ^2 (2.61803398875), Relation (8).



THE INITIAL ISOSCELES TRIANGLE
BASE 1, HEIGHT 1, SIDE (H) = 1.4142



THE INITIAL & HALF EQUILATERAL TRIANGLES
HALF: BASE 1, HEIGHT √3, SIDE (H) = 2



THE LIMITING TRIANGLE
BASES 1, 2, HEIGHT √ϕ, EACH SIDE (H) = ϕ

THE ISOSCELES TRIANGLE FULL/HALF EQUILATERAL TRIANGLES ROTATION OF THE ELEMENTS (*Timaeus*, 54a)

The Half-equilateral triangle and the Rotation of the Elements

Even so, there is far more to this approach, for the same procedure commencing with the half-equilateral triangle uses the Half *Phi*-series as opposed to the *Phi*-series introduced earlier in Table 3. Here, however, "rotations" *proper* commence at Mars and move *outwards* towards the Half *Phi*-series as shown for the twelve quadrants and three revolutions between the latter planet and Saturn in Table 2. Included here, albeit as *pheidian* exponents, are the "Fourth," (**4:3**), "Fifth" (**3:2**), "Major Six" (**5:3**) and "Octave" (**2:1**) with the Fourth and the Octave also two primary constants for the increases in planetary distances and periods of revolution respectively, thus *Phi*-series relations (9) and (7).

GROWTH/90° (¼ Rotations)	½ EQUILATERAL Sides: B, P & H	SIDES² Lucas RZT	LUCAS-BASED Right triangles	HALF ϕ-SERIES Periods (Years)	½ ϕT Exp.	PLANORBIDAE $T^{1/3}$ = Velocity (*Vi*)
Initial Triangle	BASE B1 = 1	1	1	(1)	(0)	*PHEIDIAN* SIXTHS
Rev. 0 MARS	PERP. P1 = √3	3	1.7320508076	1.6180339887	1	Exponents: **6ths**
H = √(B² + P²)	HYP. H1 = 2	4	2	2.0581710273	1.5	[1.083505882] **1**
Quadrant #1	BASE B2 = P1	3	1.7320508076	1.6180339887	1	1.1739849967 **2**
¼ cycle	PERP P2 = H1	4	2	2.0581710273	1.5	1.2720196495 **3**
90°	HYP H2 (calc.)	7	2.6457513111	2.6180339887	2	
Quadrant #2	BASE B3 = P2	4	2	2.0581710273	1.5	
½ cycle (Syn.)	PERP P3 = H2	7	2.6457513111	2.6180339887	2	1.3782407725 **4**
180°	HYP H3 (calc.)	11	3.3166247904	3.3301906768	2.5	($\phi^{4/6}$ Relation 10)
Quadrant #3	BASE B4 = P3	7	2.6457513111	2.6180339887	2	
¾ cycle	PERP P4 = H3	11	3.3166247904	3.3301906768	2.5	1.4933319840 **5**
270°	HYP H4 (calc.)	18	4.2426406871	4.2360679775	3	
Quadrant #4	BASE B5 = P4	11	3.3166247904	3.3301906768	2.5	
Rev. 1 (MJ-Gap)	PERP P5 = H4	18	4.2426406871	4.2360679775	3	1.6180339887 **6**
360°	HYP H5 (calc.)	29	5.3851648071	5.3883617041	3.5	($\phi^{6/6}$ Relation 8)
Quadrant #5	BASE B6 = P5	18	4.2426406871	4.2360679775	3	
¼ Cycle	PERP P6 = H5	29	5.3851648071	5.3883617041	3.5	1.7531493444 **7**
450°	HYP H6 (calc.)	47	6.8556546004	6.8541019663	4	
Quadrant #6	BASE B7 = P6	29	5.3851648071	5.3883617041	3.5	**Fourth 4:3** (8/6)
½ cycle (Syn.)	PERP P7 = H6	47	6.8556546004	6.8541019663	4	1.8995476269 **8**
540°	HYP H7 (calc.)	76	8.7177978871	8.7185523808	4.5	($\phi^{4/3}$ Relation 9)
Quadrant #7	BASE B8 = P7	47	6.8556546004	6.8541019663	4	**Fifth 3 : 2** (9/6)
¾ cycle	PERP P8 = H7	76	8.7177978871	8.7185523808	4.5	2.0581710273 **9**
630°	HYP H8 (calc.)	123	11.090536506	11.090169944	5	
Quadrant #8	BASE B9 = P8	76	8.7177978871	8.7185523808	4.5	**Major Six 5:3** (10/6)
Rev. 2 JUPITER	PERP P9 = H8	123	11.090536506	11.090169944	5	2.2300404146 **10**
720°	HYP H9 (calc.)	199	14.106735980	14.106914085	5.5	($\phi^{10/6}$ **Jupiter** *Vi*)
Quadrant #9	BASE B10 = P9	123	11.090536506	11.090169944	5	
¼ cycle	PERP P10 = H9	199	14.106735980	14.106914085	5.5	2.4162619067 **11**
810°	HYP H9 (calc.)	322	17.944358445	17.944271910	6	
Quadrant #10	BASE B11 = P10	199	14.106735980	14.106914085	5.5	**Octave 2:1** (12/6)
½ cycle (Syn.)	PERP P11 = H10	322	17.944358445	17.944271910	6	2.6180339887 **12**
900°	HYP H11 (calc.)	521	22.825424421	22.825466466	6.5	($\phi^{12/6}$ Relation 7)
Quadrant #11	BASE B12 = P11	322	17.944358445	17.944271910	6	
¾ cycle	PERP P12 = H11	521	22.825424421	22.825466466	6.5	2.8366552265 **13**
990°	HYP H12 (calc.)	843	29.034462282	29.034441854	7	
Quadrant #12	BASE B13 = P12	521	22.825424421	22.825466466	6.5	
Rev. 3 SATURN	PERP P13 = H12	843	29.034462282	29.034441854	7	3.0735326237 **14**
1080°	HYP H13 (calc.)	1364	36.932370625	36.932380550	7.5	($\phi^{14/6}$ **Saturn** *Vi*)

Table 2. "Rotation of the Elements" II. Lucas and Half-*Phi* series, the Fourth, the Fifth, the Major Six and Octave. The Sixths (*Vi*) are color-coded growth rates (*w* per revolution) for *Pheidian* test spirals ($w = \phi^{1/6}$ to $\phi^{14/6}$) from Mars to Saturn. The above range brackets the majority of equi-angular spirals found among ammonites and numerous (but not all) shells.

Thus, while emphasizing the Fibonacci series the first set generates the Pierce reduction ratios and the associated resonant triples that underly the present survey of the Solar System and Systems further afield. Above all else, the rotations pertain to the divisors for the *periods of revolution* and *synodic cycles* commencing with the outermost as later adopted by Pierce.

The half-equilateral triangle on the other hand, although again understood in terms of periods of revolution and synodic cycles involves the *Lucas* series, begins at Mars and proceeds *outwards* towards the Half-*Phi*-series with correspondence increasing with distance. However, there is now something else to consider, for the addition of quarter-cycle periods generates a rectangular ("square") spiral with a quadrantal growth rate of $\sqrt{\phi}$ (1.2720196495 and ϕ^2 (2.61803398875) per revolution, thus relation (8), the *Phi*-series planet-to-planet increment for the periods of revolution. The inclusion of the pheidian exponents 2/1, 5/3, 3/2 and 4/3 for *Vi* follows from these periods, but whether this is the "Harmony of the Spheres" *per se* is far from secure, general correspondence notwithstanding.

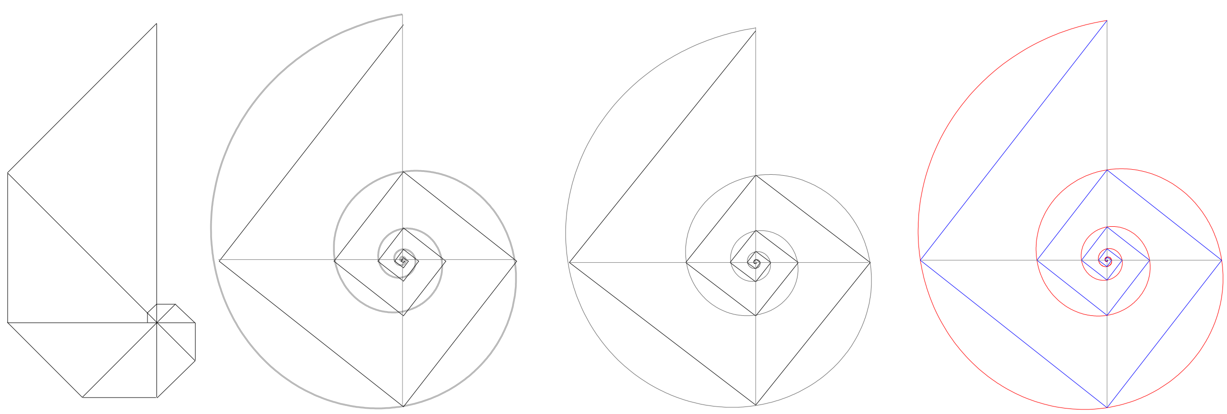
Further considerations

It must be acknowledged that it helps to return to this material with details of the Pierce Divisor framework already in place. And also, of the two analyses presented here the latter is perhaps the more controversial. Even so, neither is entirely out of place, especially in light of the material analyzed by Jöran Friberg in *A REMARKABLE COLLECTION of BABYLONIAN MATHEMATICAL TEXTS* published in 2007.[35] Particularly relevant here is the combination of both Greek and Babylonian texts dealing with something akin to the "rotation of the elements" just discussed.

Given below is the "spiral" expansion of the isosceles triangle obtained from continued 90° rotations of the same with the last hypotenuse the base of the next, and so on. Rotated here, the figure is otherwise as given in Friberg's analysis of Old Babylonian mathematical text MCL 7028 originally treated by Neugebauer and Sachs[36] as a table of logarithms and and exponents. Friberg's analysis is quite different, dealing with a "spiral chain algorithm," and the fixed rotation of the isosceles triangle as shown. Which in itself is not only of conceptual interest, but also a helpful exercise for the somewhat more complex rotations involving the Pythagorean expansions of the sides.

On the other hand, rotating the Fibonacci sides in Table 1 generates a tighter line spiral, which in turn leads to an a near equi-angular spiral with a growth rate per revolution (*w*) of approximately 2.666:1, and similarly 2.765:1 from Lucas data rotations. Neither of the two rectangular expansions quite matches the equi-angular spiral based on ϕ^2 (2.61803398875), but they are nonetheless still close, the Fibonacci variant especially.

Not to scale, the fixed format line spiral, the latter pair plus the *Phi*-series spiral based on the last constant (ϕ^2) with its uniform expansion rate of $\phi^{1/2}$ (1.27201964951 : 1) *per* (90°) *quadrant* are:



(a) (b) (c) (d)

Fig. 1. (a) Rotations of Fixed Isosceles triangles, (b) Lucas rotations, (c) Fibonacci rotations, and (d) the *Phi*-series.

But what is the underlying purpose of such extensions, and how might they relate to obscure statements such as that already noted, namely Alchemist George Ripley's "*When thou hast made the quadrangle round, Then is all the secret found*." ? Perhaps - despite their arcane nature - further obscure explanations in later alchemical works, *e.g.*, section 83 in the *Hermetic Arcanum* (1623) may be of assistance since this matter appears to involve *two* spirals in complex association with three circles. With, apparently, such "Circulations" considered to be "Nature's instruments, whereby the elements are prepared." Thus in full, section #83 (of 138) from the *Hermetic Arcanum*: [37]

> 83. The Circulation of the Elements is performed by a double Whorl, by the greater or extended and the less or contracted. The Whorl extended fixeth all the Elements of the Earth, and its circle is not finished unless the work of Sulphur be perfected. The revolution of the minor Whorl is terminated by the extraction and preparation of every Element. Now in this Whorl there are three Circles placed, which always and variously move the Matter, by an Erratic and Intricate Motion, and do often (seven times at least) drive about every Element, in order succeeding one another, and so agreeable, that if one shall be wanting the labour of the rest is made void. These Circulations are Nature's Instruments, whereby the Elements are prepared. Let the Philosopher therefore consider the progress of Nature in the Physical Tract, more fully described for this very end.

Although still difficult, the above quotation provides some understanding of the materials already assembled here, particularly the two "whorls" (spirals), with the "greater or extended" reasonably the ever-extending *Phi*-series spiral ($w = \phi^2$), with the "lesser" confined, perhaps, to Solar System equivalents, or Fibonacci variants of the same.

The "three Circles" on the other hand, are simpler and less controversial in so much as the previous analyses of the real-time motions of the Jupiter, Synodic SD1 and Saturn included - although not shown - plan views of both the *real-time* orbits and *mean circular* orbits of Jupiter and Saturn with the "orbit" of SD1 necessarily in between. Hence *three circles* in a readily recognized configuration. For this purpose, however, the same mean sidereal periods used earlier for the mean synodic arcs are preferred since there is no ambiguity concerning their values.

As presented in ACT (1955) by Neugebauer, Babylonian mean periods were obtained from the following "simple" integer period relationships: [38]

Superior planets: N years : Π Synodic arcs/periods : Z Rotations
Inferior planets: N years : Π Synodic arcs/periods. (only)

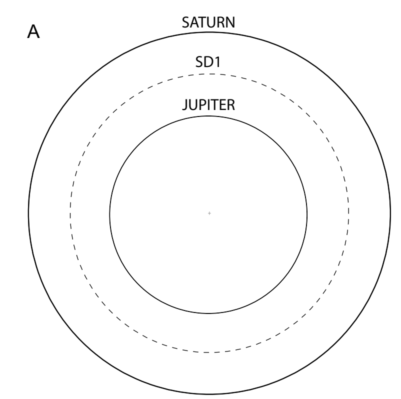
which, with *revolutions* substituted for Neugebauer's *rotations* and periods of revolution (Z) added for Venus and Mercury, are:

SATURN:	265 years (N),	256 Synodic arcs (Π),	9 Revolutions (Z)
JUPITER:	427 years (N),	391 Synodic arcs (Π),	36 Revolutions (Z)
MARS:	284 years (N),	133 Synodic arcs (Π),	151 Revolutions (Z)
VENUS:	1151 years (N),	720 Synodic arcs (Π),	1871 Revolutions (Z *added)*
MERCURY:	46 years (N),	145 Synodic arcs (Π),	191 Revolutions (Z *added)*

yielding by simple division mean periods of revolution which, by way of the Harmonic law give the corresponding mean distances for Saturn, SD1 and Jupiter shown in Figure 2A.

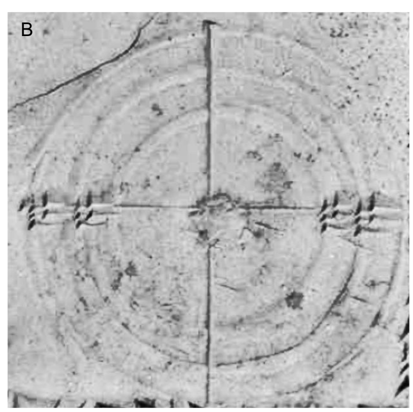
SATURN: 265 years / 9 Revolutions = 11.86111* (11;51,40) years. R = 9.535532
(Added: Synodic *SD*1 [Relation 1] = 19.86220818 years. "R" = 7.334182)
JUPITER: 427 years / 36 Revolutions = 29.4444* (29;26,40) years. R = 5.200961

The mean distance orbits are comparable with a figure in a "babylonian-clay-tablet-with-geometrical-problems."[39] Where, as seen in Figures 2B and 2C, an astronomical indicator is suggested, *i.e.*, an off-set vertical center line in so much as Solar System planetary orbits with their small eccentricities do indeed resemble off-set circles when shown in planview. Then again, the text may perhaps refer to a more mundane application, though just what that might be is unclear as opposed to the present provisional suggestion.

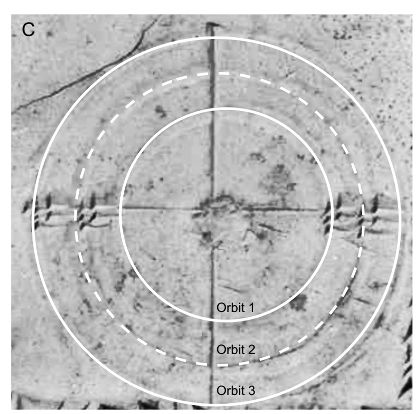


SATURN: 9.535532, SD1: 7.334181, JUPITER: 5.200961

Fig. 2A. Mean orbits for Jupiter, SD1 & Saturn based on the attested Babylonian "Long" periods.



2B. Three circles in a Babylonian tablet with an off-set vertical center line.



2C. Figure with three off-set orbits for Jupiter (#1), SD1 (orbit #2), and Saturn (orbit 3) added.

As for the priority of Jupiter and Saturn in the *Hermetic Arcanum*, the final paragraph is "The Times of the Stone," in which, after mentioning Capricorn and Aquarius with respect to Saturn, and later also Scorpio and Sagittarius, the *Arcanum* states: "And thus the Philosophers' admirable offspring taketh its beginning in the Reign of Saturn, and its end and perfection in the Dominion of Jupiter." [40] Thus, not only the relative motions of these two planets, but also specific locations, which brings to mind what was discussed in the Part I Excursus concerning Saturn and Jupiter at the junction points of their twin sectors associated with their lines of apsides, *i.e.,* approximations for ϕ and ϕ^2.

As for the present context, the "Rotation of the Elements" as suggested here can be at least represented by the following diagram:

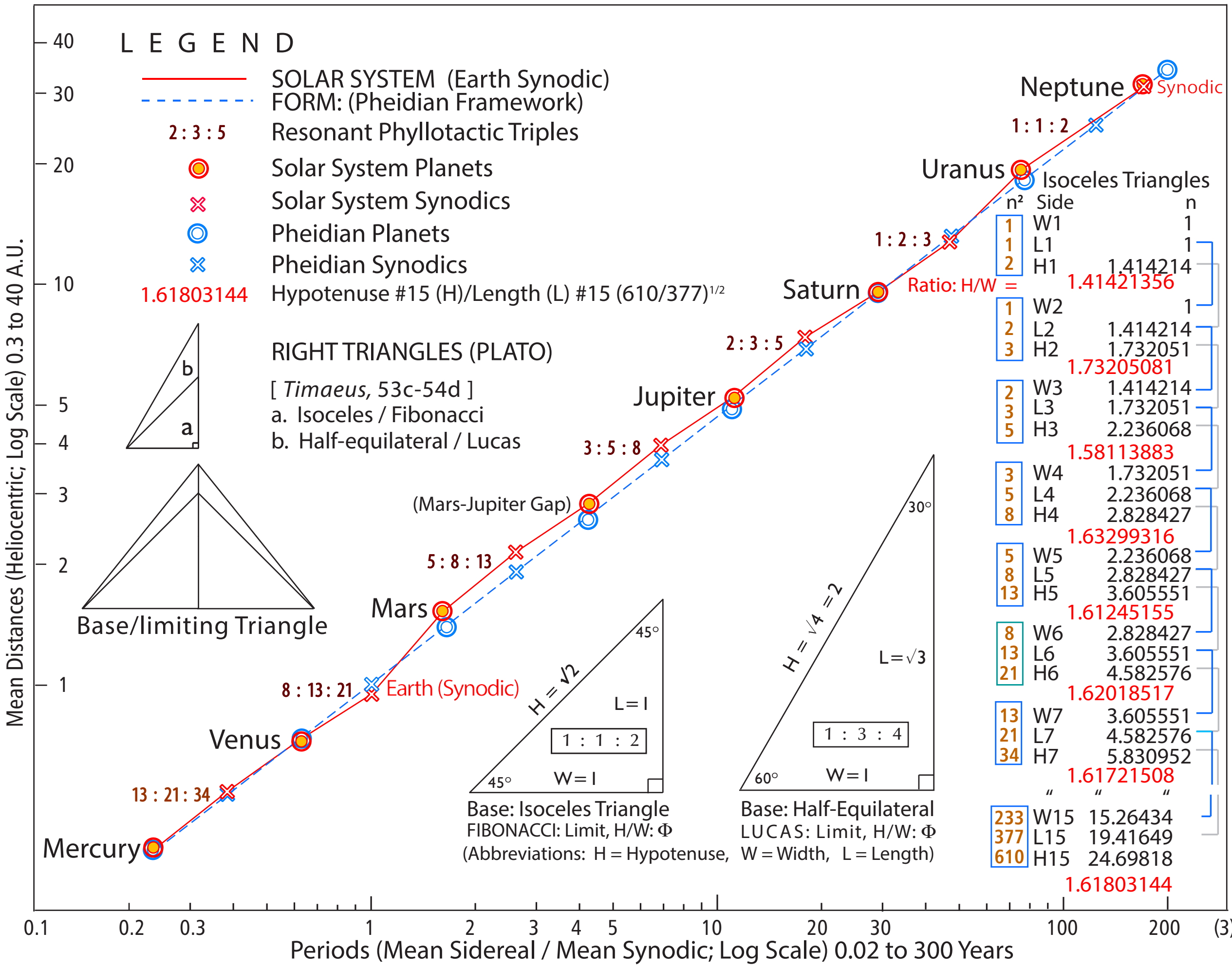


Fig. 3. The Resonant Triples and the *Isoceles / Half-Equilateral* Right Triangles (*Timaeus* 53c-54d).

subject to corrections and/or further refinements, provided (perhaps), by modern "Alchemists," or perhaps not.

* * *

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INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS

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