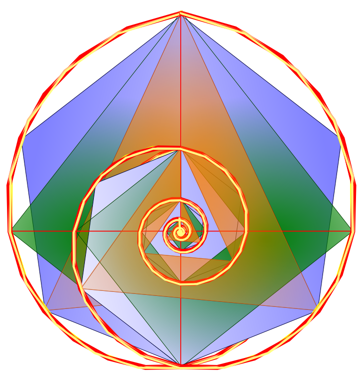
INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS



2023

PART THREE

REAL-TIME MOTION IN THE SOLAR SYSTEM
AND THE GOLDEN SECTION

JNH

REAL-TIME MOTION IN THE SOLAR SYSTEM AND THE GOLDEN SECTION

The sequential pheidian constants ϕ^{-3}, ϕ^{-2}, ϕ^{-1}, ϕ^{0}, ϕ^{1}, ϕ^{2} and the Solar System

Jupiter-Saturn mean synodic ratios and the primary constants ϕ, ϕ^2.

As seen in Table 3s both the Lucas and the Fibonacci series appear to be present, albeit with respect to the *mean* periods and motions, not the more complex *varying* velocities associated with the elliptical orbits of the planets. To include the latter what follows next initially concentrates on Jupiter and Saturn and the mean synodic data for these two planets calculated with respect to the motion of Earth (unity) as given in most modern tables. Thus for Jupiter a mean synodic period (*S*) of 1.0921105 years, during which the planet moves 33.15978173°, *i.e.*, the *mean* synodic arc (*u*). During the same interval Earth concurrently completes 360° + *u* = 393.1597817291°, *i.e.,* one year plus (*u*). The same procedure for Saturn yields in turn a mean synodic period of 1.03518213 years, 372.665567397° of sidereal motion for Earth and a corresponding mean synodic arc (*u*) of 12.665567497°.

However, the *ratio* of these two accurate *mean* synodic arcs turns out to be *of major significance*, *i.e.*,

$$33.159781729^\circ/12.665567497^\circ = 2.61810472 \text{ vs } \phi^2 = 2.61803398875 \text{ or } \textit{Phi}\text{-series relation (8)}$$

whereas the ratio of the mean synodic arcs for these two planets applied in Babylonian astronomical cuneiform texts of the of the Seleucid Era [2] (-310 to 75) of 33;8,45° [3] and 12;39,22,30° [4] produces a slightly less accurate, but nonetheless similar result, *i.e*., (as shown earlier):

$$33.1458333^{*\circ}/12.65625^\circ = 2.61893004 \text{ vs } \phi^2 = 2.61803398875 \textit{ Phi}\text{-series relation (8)}$$

Further investigation of the heliocentric methodology inherent in Babylonian astronomy next involves a parameter *P* and the following triple relation based on the mean sidereal period *T* and the mean synodic arc *u:* [5]

$$P = (360^\circ/u) = T - 1,\ T = P + 1 \qquad (13)$$

resulting in ratios between modern *T* - 1 periods for Saturn and Jupiter (Saturn *P* and Jupiter *P* respectively) of:

$$\text{Saturn}P/\text{Jupiter}P = 2.61810472 \text{ vs } 2.618033989 \text{ (relation 7, the planet-to-planet increment)} \qquad (14)$$

$$\text{Jupiter}P/\text{Saturn}P = 0.38195569 \text{ vs } 0.381966011 \text{ (limiting constant, Pierce planet-to-planet ratios)} \qquad (15)$$

Jupiter-Saturn real-time test formulas

Next, although secondary in nature, the *P*-periods are also revolutions with the general synodic formula applicable here also. Thus, with respect to the Jupiter-Saturn synodic difference cycle (hereafter *SD*1), the *P*-variant (*SD*1*P*) provides two more recognizable results:

$$\text{Saturn}P/SD1P = 1.61810472 \text{ vs } 1.618033989 \text{ (relation 8, the planet-synodic increment)} \qquad (16)$$

$$SD1P/\text{Jupiter}P = 1.61800697 \text{ vs } 1.618033989 \text{ (relation 8, the planet-synodic increment)} \qquad (17)$$

with the primary *Phi-series* constants ϕ and ϕ^2 also testable against the real-time Solar System in the form:

$$\textit{Ratios of adjacent periods, both } T_N \textit{ and } S_N = 1.61803398875 \text{ (}\textit{Phi}\text{-series relation 8)} \qquad (18)$$

$$\textit{Ratios of alternate periods, both } T_N \textit{ and } S_N = 2.61803398875 \text{ (}\textit{Phi}\text{-series relation 7)} \qquad (19)$$

Synodic difference cycles *SD*1: Jupiter - Saturn, *SD*2: Saturn - Uranus, and *SD3*: Uranus - Neptune

Finally, the increasing departures from the Solar System beyond Saturn notwithstanding, the initial tests for real-time motion can be investigated with respect to the synodic difference cycles between the four adjacent superior planets. Here initial emphasis is placed on relations (20) and (21); the *Phi*-series departures from the Peirce divisors inherent in relation (22) are discussed later:

$$\textit{Jupiter-Saturn} \text{ synodic } SD1\ f(\text{t}): \quad \frac{\textit{Saturn } f(\text{t}) \bullet \textit{Jupiter } f(\text{t})}{\textit{Saturn } f(\text{t}) - \textit{Jupiter } f(\text{t})} = SC4\text{-}3 \qquad (20)$$

$$\textit{Saturn-Uranus} \text{ synodic } SD2\ f(\text{t}): \quad \frac{\textit{Uranus } f(\text{t}) \bullet \textit{Saturn } f(\text{t})}{\textit{Uranus } f(\text{t}) - \textit{Saturn } f(\text{t})} = SC3\text{-}2 \qquad (21)$$

$$\textit{Uranus-Neptune} \text{ synodic } SD3\ f(\text{t}): \quad \frac{\textit{Neptune } f(\text{t}) \bullet \textit{Uranus } f(\text{t})}{\textit{Neptune } f(\text{t}) - \textit{Uranus } f(\text{t})} = SC2\text{-}1 \qquad (22)$$

Real-time planetary motion and period ratios in the Solar System

Methodology

In 1986, following the general availability of personal computers in the early1980s, Pierre Bretagnon and Jean-Louis Simon published *Planetary Programs and Tables from –4000 to+2800: Tables for the Motion of Uranus and Neptune from +1600 to 2800.*[1] The programs generate geocentric planetary positions for single dates by initially calculating the heliocentric radius vector *R* in a.u., the heliocentric longitude *L* in radians and heliocentric latitude *B*. Although geocentric conversions and corrections follow, the heliocentric data proves to be entirely sufficient for the present purpose since the corresponding periods and velocities can be obtained from the instantaneous heliocentric radius vectors via the harmonic law and velocity components of the same. Following a number of minor adaptations to generate heliocentric time-series outputs, the *varying* radius vectors and the corresponding sidereal and synodic periods, velocities and ratios for the superior planets were generated in sets of 36,525 data points per century, *i.e.*, intervals of one Julian day to synchronize with the methodology instituted by Bretagnon and Simon. Much shorter intervals with smaller test periods (*e.g.*, 6-hourly data) were adopted for Mars (planet #6), for Earth (synodic cycle *SC* 7-E), Venus (#7), Mercury (#8) and synodic cycle *SC* 8-7 between the latter pair.

Initial testing

Relations 20 through 22 involve the major Jupiter-Saturn synodic difference cycle (*SD*1), the secondary Saturn-Uranus synodic cycle (*SD*2) and the third major synodic difference cycle between Uranus and Neptune (*SD*3). In terms of real-time motions these relations and variants can be tested against the *Phi*-series as follows:

Relative motions of Jupiter, Synodic cycle *SD*1, Saturn and the constant ϕ = 1.61803398875.

Commencing on Julian day JD2451544.5 (January 1st 2000) and ending on Julian day JD488069.5 (January 1st 2100) both the *P* and *T* ratios of interest were plotted against *Phi*-series relation (8), *i.e.*, the constant ϕ (1.61803398875). Thus data for the ratios (Saturn*P*/*SD*1*P*), (*SD*1*P*/Jupiter*P*) and (Saturn*P*/Jupiter*P*) were generated along with a second control set from the unmodified ratios, (Saturn*T*/*SD*1), (*SD*1/Jupiter*T*) and (Saturn*T*/Jupiter*T*). Initial tests show that despite consistent differences in the amplitudes and times the intercepts are nevertheless *pheidian*. In other words, both sets intersect at the target constant ϕ (1.61803398875) with finer definition available if required. The initial results also simplified matters considerably since further testing could proceed using real-time parameters as shown in Figure 1a (both sets) and thereafter standard *T*-values for the configurations shown in Figures 1b through 6c.

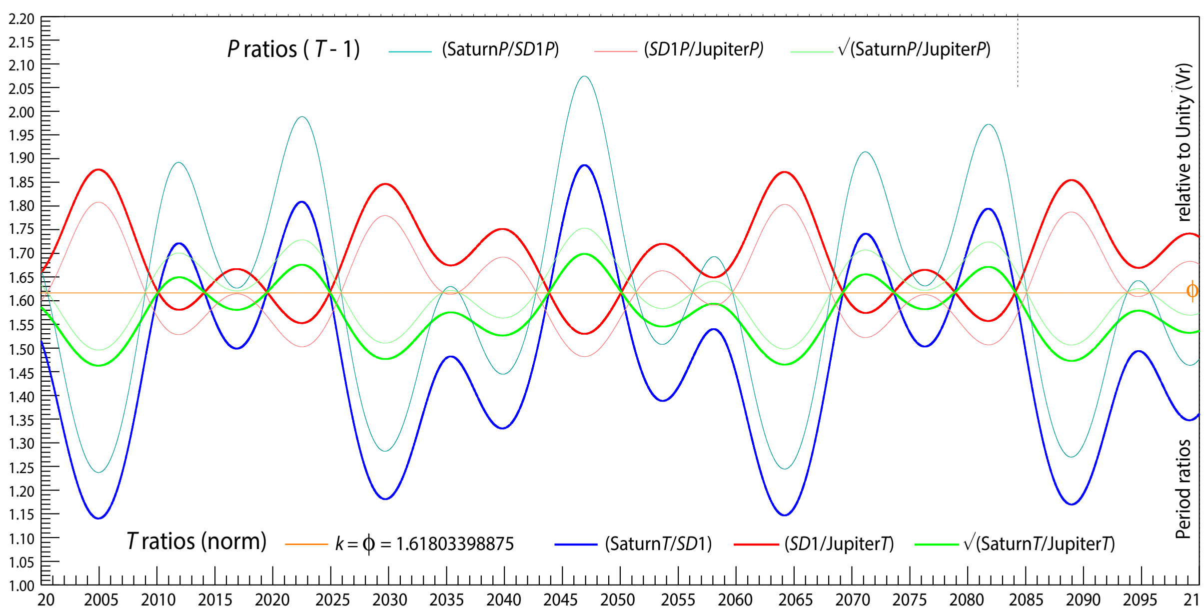


Fig. 1a. Daily *T* and *P ratios* for Saturn, *SD*1 and Jupiter, 2000 CE to 2100 CE, *k* = ϕ = 1.61803398875.

Figure 1b also includes the corresponding waveform for the sixth root of the generating *Phi*-series exponent (6) of the Jupiter-Saturn synodic difference cycle *SD*1. The similar waveforms for Jupiter and Saturn with generating exponents of 5 and 7 respectively have been omitted for clarity.

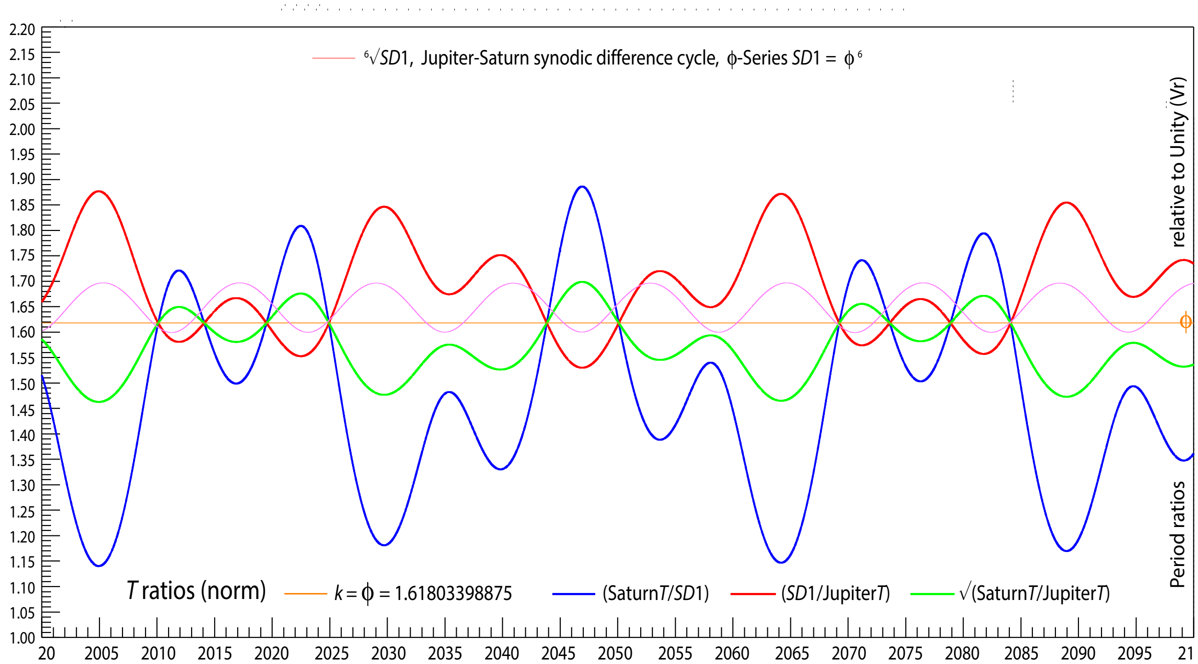


Fig. 1b. Daily *T ratios* for Saturn, *SD*1and Jupiter, plus $SD1^{1/6}$ from 2000 CE to 2100 CE, $k = \phi = 1.61803398875$.

The relative motions of Jupiter (*T*), Synodic cycle *SD*1, Saturn (*T*), and constant ϕ^{-2} (0.38196601125).
Next, the two *SD*1-associated *T*-ratios were reduced by the exponent -2 for comparison with the limiting value of Peirce's planet-to-planet reduction ratios ($k = \phi^{-2} = 0.38196601125$). The Saturn/Jupiter ratios were reversed *sans* exponentiation since the Jupiter *T* / Saturn *T* ratio is also the mean synodic velocity of Jupiter with respect to Saturn for the *Phi*-series planetary framework (*SD*1*Vr*). Relative orbital velocities (*Vr*) were included for both planets, and ϕ^{-2} (0.38196601125) examined in terms of the parameters associated with the Inferior planets Mercury and Venus.

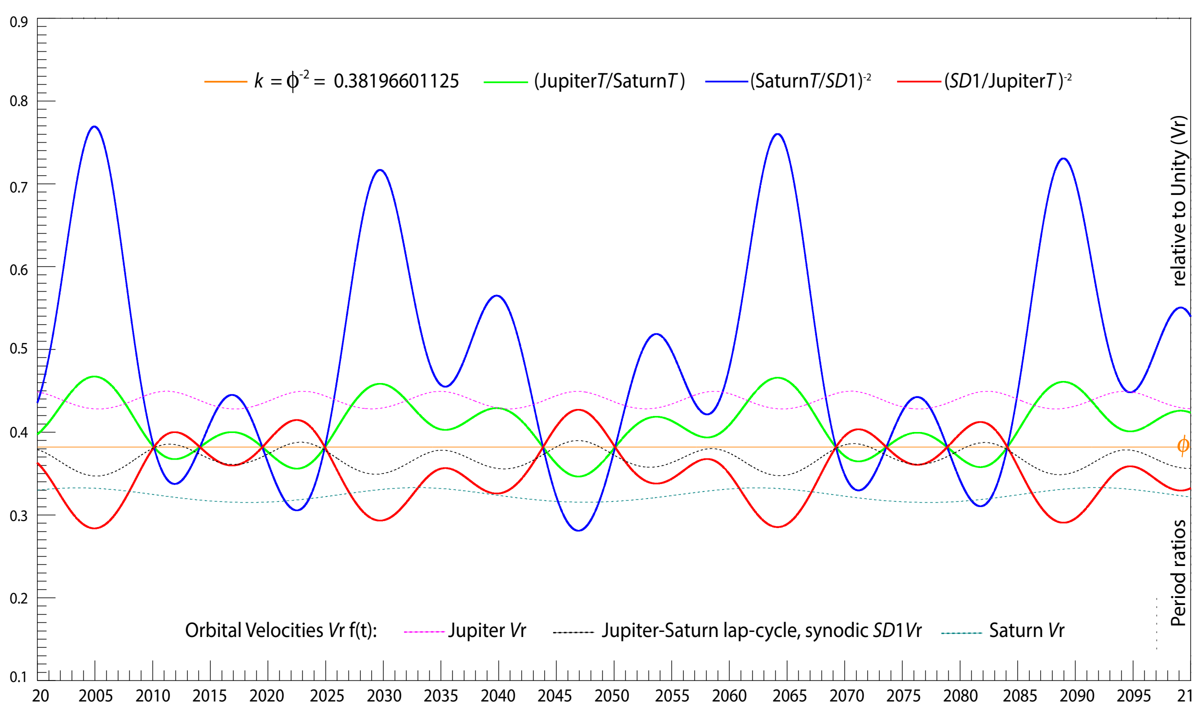


Fig. 2. Daily period ratios & orbital velocities (*Vr*): Jupiter *T*, *SD*1 & Saturn *T*, 2000−2100 CE, $k = \phi^{-2} = 0.38196601125$.

The Terrestrial planets and synodic location of Earth between Mars and Venus

Although Earth appears to be occupying the synodic location SC7-6 between Mars and Venus, the synodic period *SC*7-E between the latter and Earth is atypically greater than that of Earth itself. In other words, the intermediate, synodic period has a mean value that is greater than that of the outermost planet, *i.e.*,1.5986495 years, thus close to Fibonacci 8/5 = 1.6. Also of interest is the occurrence of ϕ^{-2} for the mean *heliocentric distance (R*) of Mercury, the mean synodic *period* (*S*) of Mercury with respect to Venus (*SC*8-7) and also *relative velocity* (*Vr*) of Jupiter-Saturn *SD*1.

PLANETS N Synodics #	ϕ^x x	= *Phi*-series (*T*) (Years)	*Phi*-series (*R*) Distance (a.u.)	*Phi*-series (*Vi*) Inverse Velocity	*Phi*-series (*Vr*) Velocity (ref.1)
Mars 6	1	1.618033989	1.378240772	1.173984997	0.851799642
Earth/*Syn* 7-E	0	1.000000000	1.000000000	1.000000000	1.000000000
Venus 7	-1	0.618033989	0.725562630	0.851799642	1.173984997
Syn *SC* 8-7	-2	0.381966011	0.526441130	0.725562630	1.378240772
Mercury 8	-3	0.236067978	0.381966011	0.618033989	1.618033989

Table 1t. Limited *Phi*-series, periods *T, S, Distance R*, Velocity *Vi* (Inverse) & *Vr* (ref. unity).

All of which are readily available for real-time testing augmented by the occurrence of *Phi* as the mean *period* of revolution (*T*) of Mars and the mean orbital *velocity* (*Vr*) of Mercury by way of the following *Phi*-series relations:

$$\textit{Orbital velocity of Mercury } Vr\, f(t) = \textit{Period of Mars } T\, f(t), \quad \text{mean: } \phi\ (1.61803398875) \tag{23}$$

$$\textit{Mercury-Venus Syn SC7-8 } S\, f(t) = \frac{\textit{Venus } f(t) \bullet \textit{Mercury } f(t)}{\textit{Venus } f(t) - \textit{Mercury } f(t)}, \quad \text{mean: } \phi^{-2}\ (0.38196601125) \tag{24}$$

$$\textit{Mercury, Heliocentric distance } R\, f(t) \textit{ from Mercury } f(t), \quad \text{mean: } \phi^{-2}\ (0.38196601125) \tag{25}$$

The associated parameters can be extended in range and complexity by the inclusion of Mars and and Earth, *e.g.*, over the four years extending from 2045 to 2049 CE:

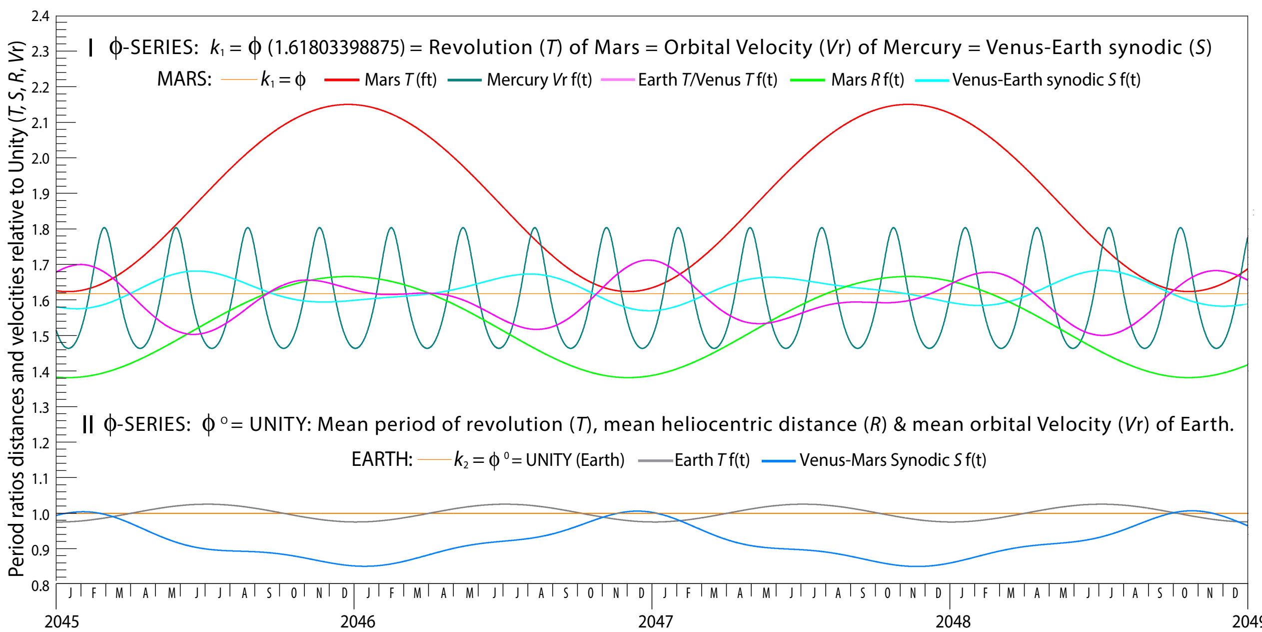


Fig 3a. Six-hourly data for Mars, Earth, Venus and Mercury, 2045 - 2049 CE ($k_1 = \phi$, $k_2 = \phi^0$ = Unity (Earth, *T*, *R*, *Vr*).

Figure 3a I- II waveforms are primarily concerned with *Phi* equated with (1): the period of revolution (*T*) of Mars, (2): the relative velocity (*Vr*) of Mercury, (3): the varying distance (*R*) of Mars, (4): the Venus / Earth synodic (*S*), and lastly, (5): the Earth (*T*) / Venus (*T*) ratio. In addition to the various intersections, the generally balanced position of the Mercury velocity (*Vr*) about the central constant (*Phi*) is of interest, as is the periodic grazing of the latter by the *perihelion* periods/positions of Mars. Also apparent for the longer waveforms are the maxima and minima around the end of 2046 and/or the beginning of the year 2047.

Figure 4b III. In order to examine the Venus waveforms in the same manner a different vertical scale was required. In addition to the standard waveform for Venus *T* f(t), the ratios below were selected because they provide close, unequivocal maximum and minimum dates for the four-year test interval. Also, one of the chosen ratios utilising unity ($\phi^0 = 1$ instead of the real-time periods of Earth about this mean) provided *pheidian* intercepts with the Venus waveform as follows:

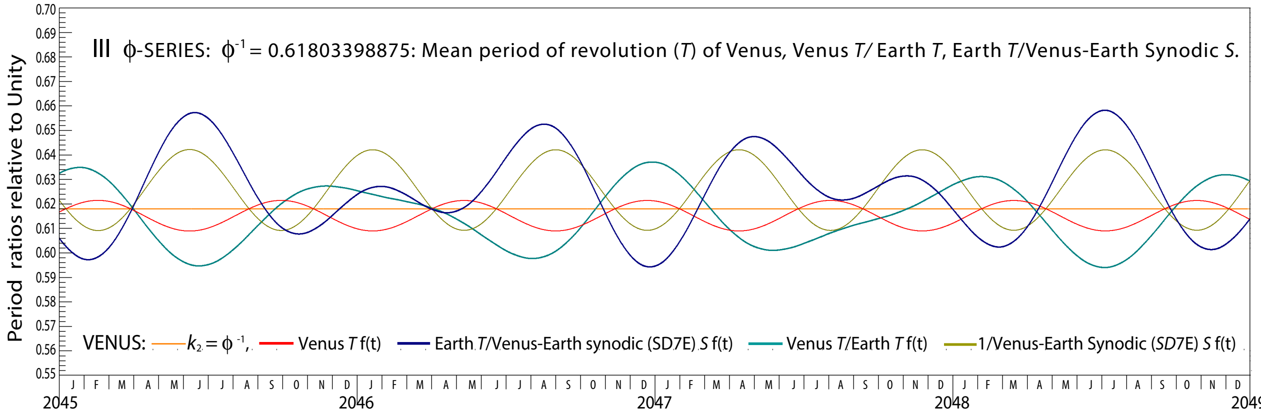


Fig. 4b. Six-hourly data for Venus and Earth, 2045 to 2049. $k_1 = \phi^{-1} = 0.61803398875$. ($k_2 = \phi^0$ = Unity, Earth, *T*, *R*, *Vr*).

In fact, the dates are similar to the maxima and minima around the middle of the present century seen in Figures 4a, 4b and 5 concerned with the motions of Jupiter and Saturn. It remains now to include the motions of Uranus and Neptune over the same initial interval (2000-2100 CE).

Waveforms for Saturn-*SD*2-Neptune, plus the outermost triple Uranus-*SD*3-Neptune.

The real-time planetary data for Saturn, *SD*2 and Uranus from 2000 to 2100 CE with $k = 1.61803398875$ resulted in only *two pheidian* intercepts, once again around the middle of the present century (Figure 5). Also, as in the cases for Jupiter and Saturn over the same period the waveforms were similarly symmetrical. Whereas, in keeping with increasing departures from the *Phi*-series periods beyond Uranus and the first resonant triple for the outermost pair of planets supplied by the Peirce Divisors [1, 1, 2] no intercepts occur for the Uranus : *SD*3 : Neptune trio.

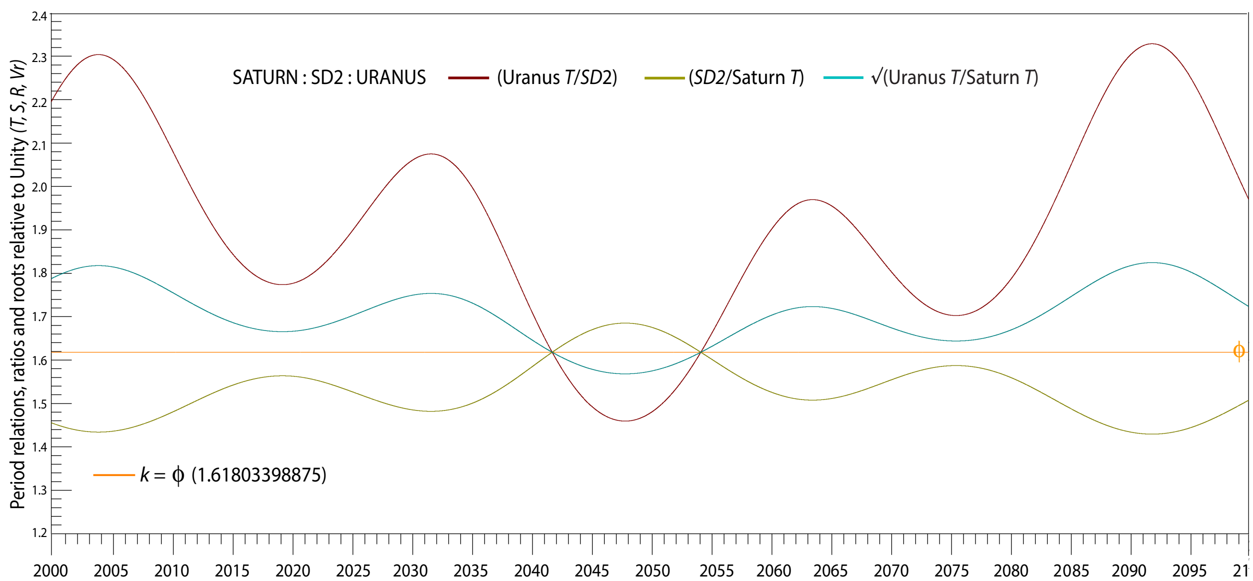


Fig. 5. Daily real-time period ratios & roots for Saturn, *SD*2 and Uranus from 2000 to 2100 CE, $k = \phi = 1.61803398875$.

The associated time-series waveforms for the same formulas do, however, provide both *phase* and *amplitude*. As such the latter are included with the Jupiter-Saturn and Saturn-Uranus data in the last two intervals, *i.e.*, Figure 6a2 from 2000 to 2100 CE and Figure 6e from 2415 to 2515 CE.

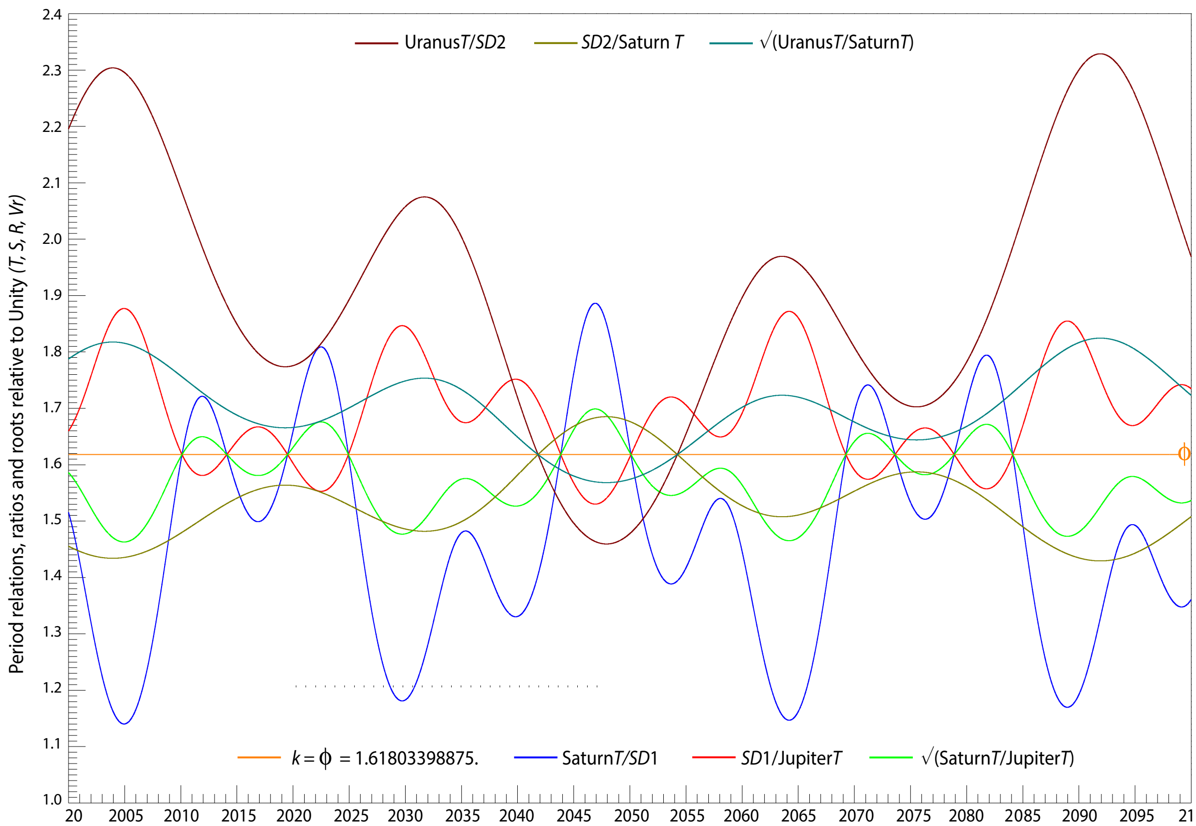


Fig 6a. Daily period ratios: Jupiter *T*, *SD*1, Saturn *T*, *SD*2 and Uranus *T* from 2000 to 2100 CE, $k = \phi = 1.61803398875$.

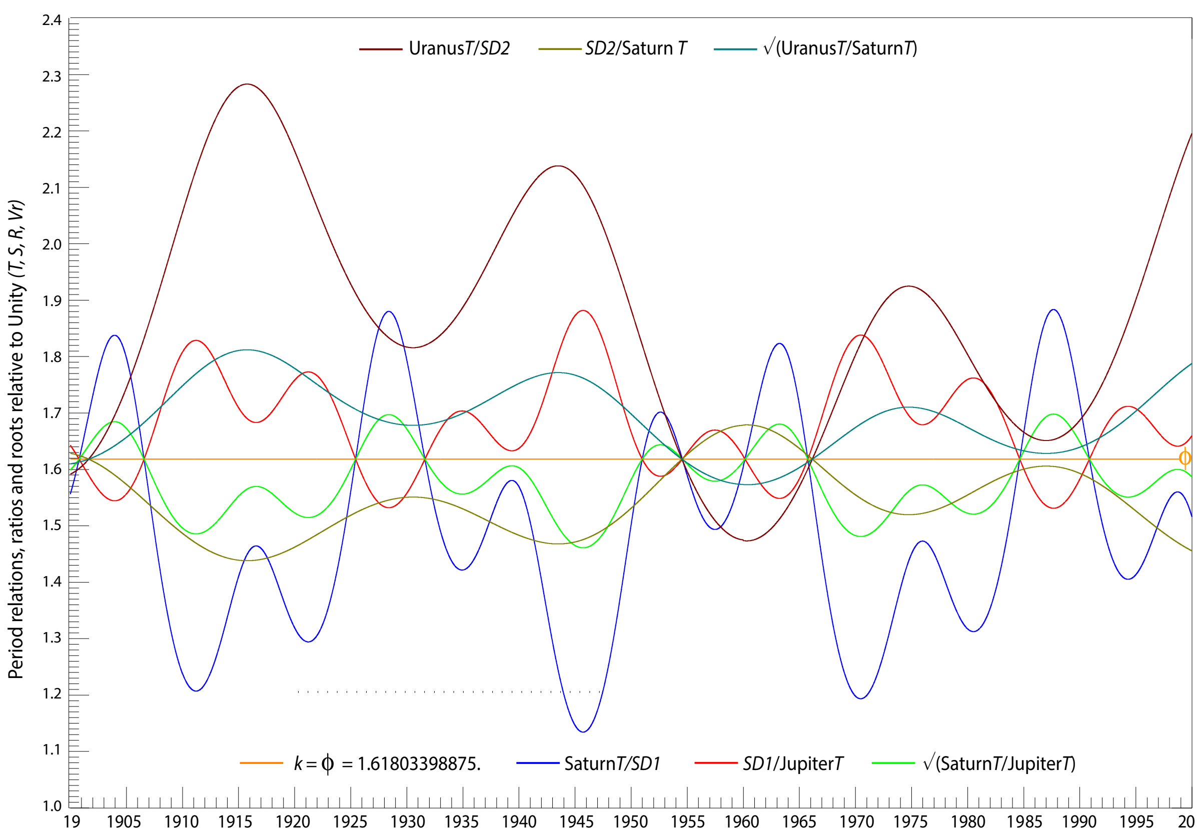


Fig 6b. Daily period ratios: Jupiter *T*, *SD*1, Saturn *T*, *SD*2 and Uranus *T* from 1900 to 2000 CE, $k = \phi = 1.61803398875$.

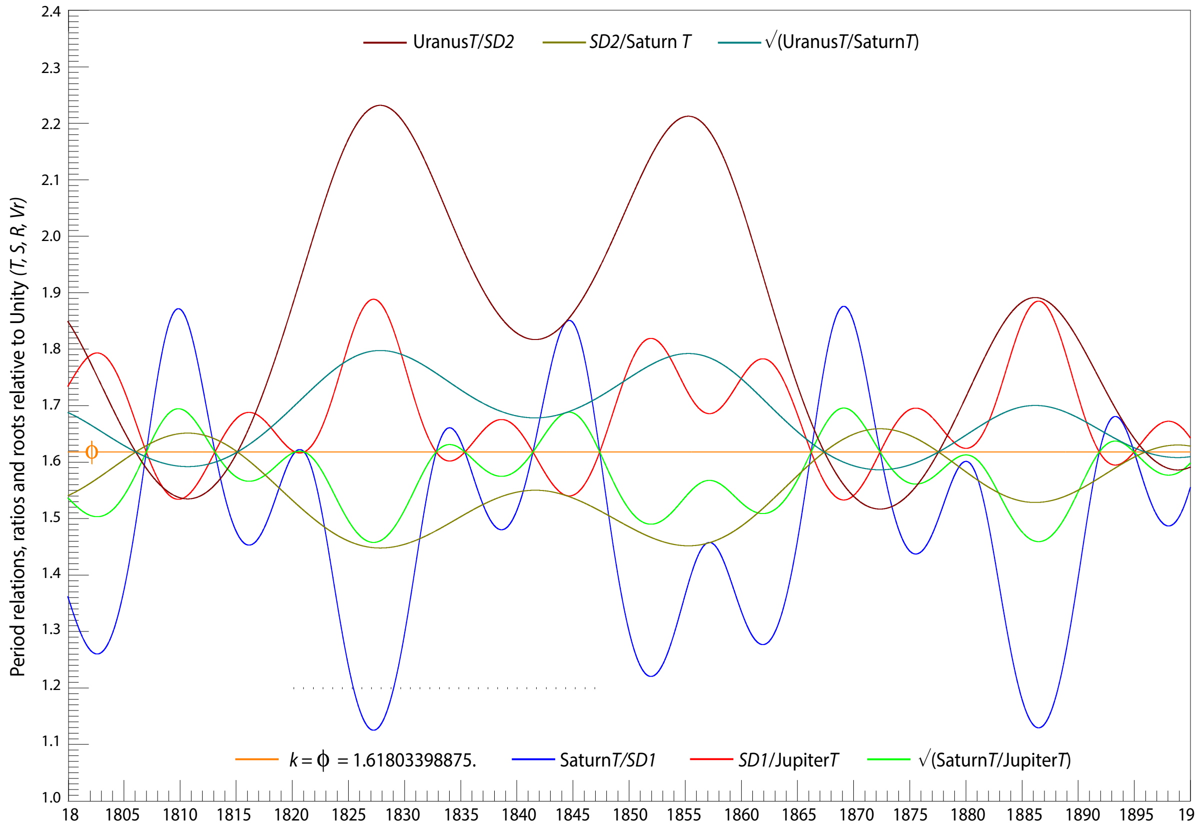


Fig 6c. Daily period ratios: Jupiter *T*, *SD*1, Saturn *T*, *SD*2 and Uranus *T* from 1800 to 1900 CE, *k* = ϕ = 1.61803398875.

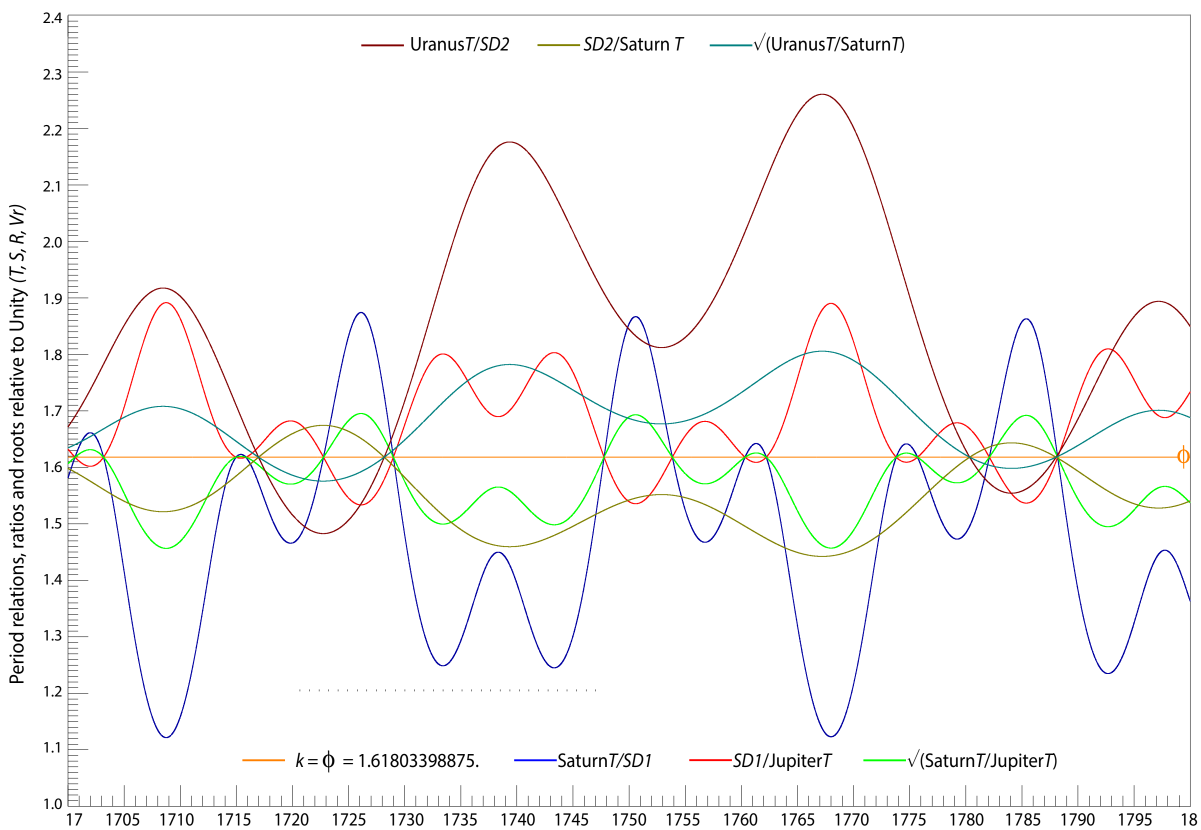


Fig 6d. Daily period ratios: Jupiter *T*, *SD*1, Saturn *T*, *SD*2 and Uranus *T* from 1700 to 1800 CE, *k* = ϕ = 1.61803398875.

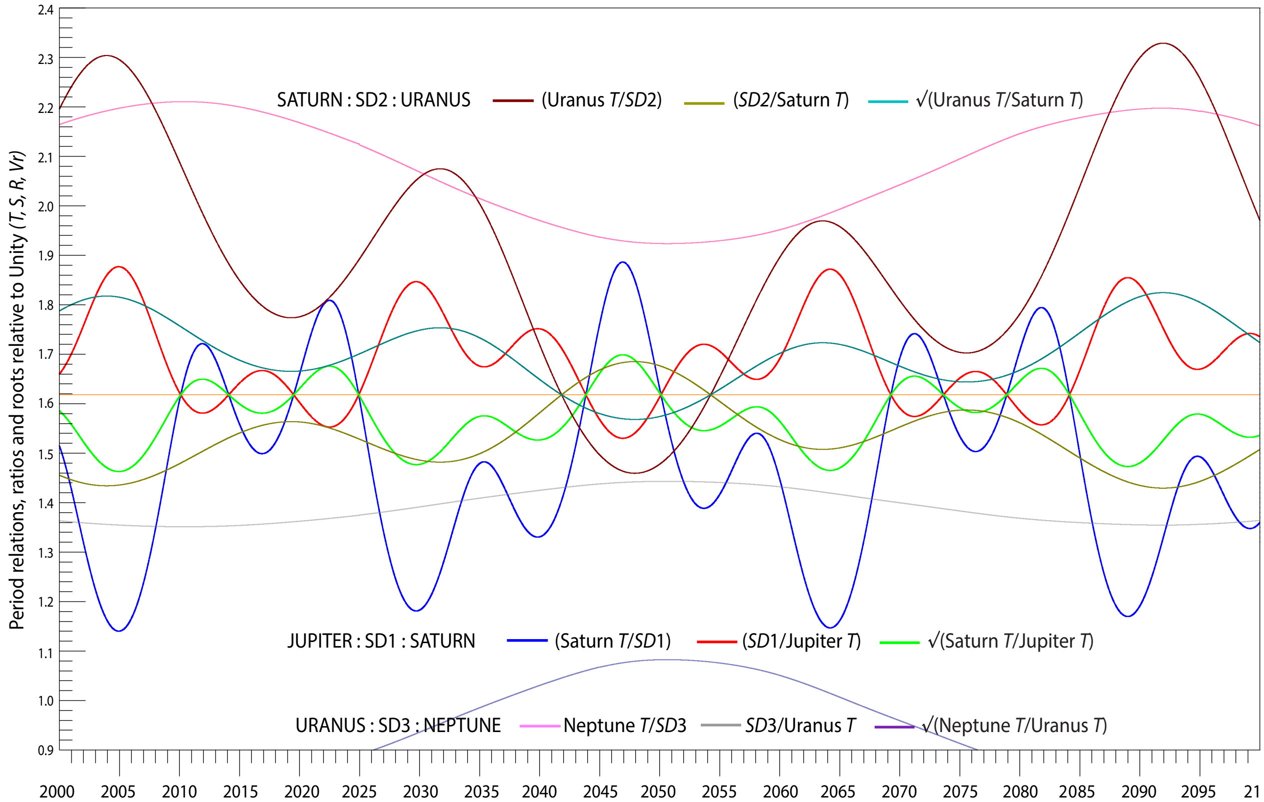


Fig. 6a2. Daily period ratios & roots: Jupiter, *SD*1, Saturn, *SD*2, Uranus, *SD*3 & Neptune, 1800 - 1900 CE, $k = \phi = 1.61803398875$.

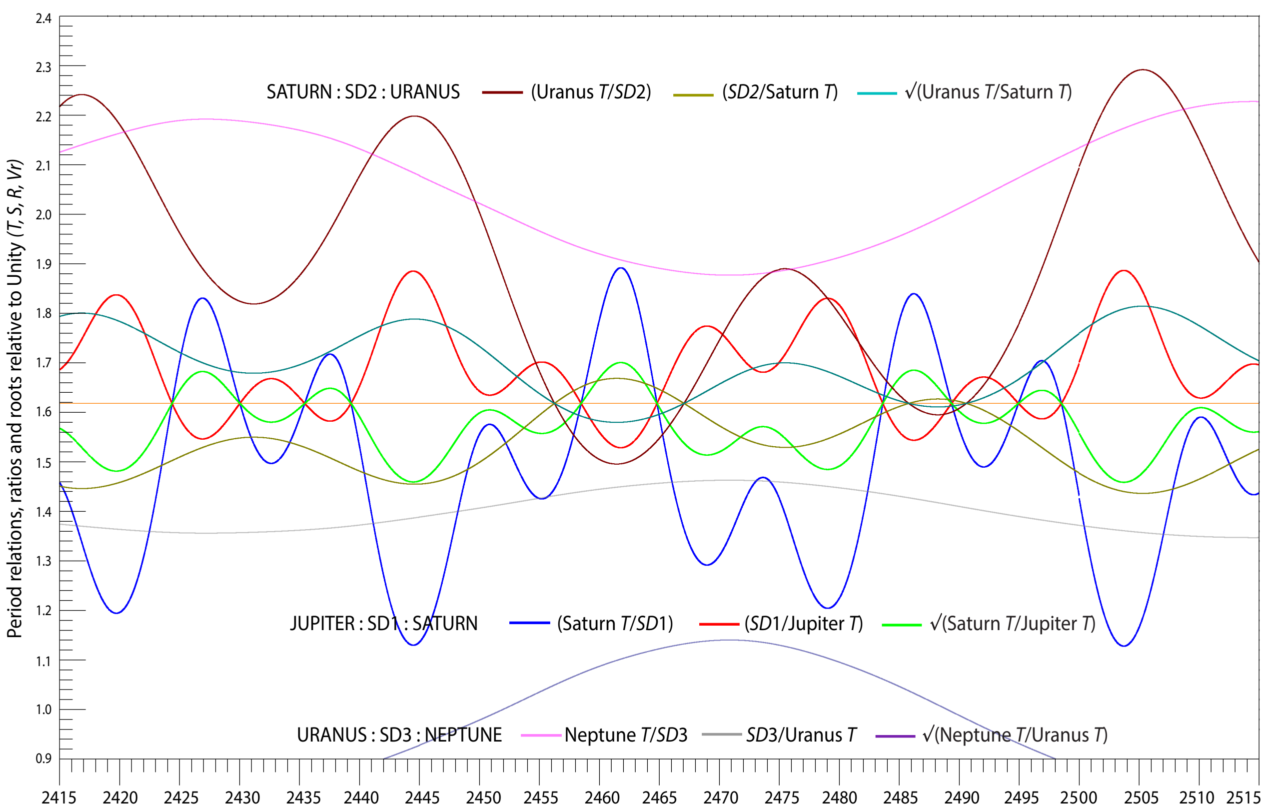


Fig. 6e. Daily period ratios & roots: Jupiter, *SD*1, Saturn, *SD*2, Uranus, *SD*3 & Neptune, 1900 - 2000 CE, $k = \phi = 1.61803398875$.

Dates for Figure 6e are ***adjusted*** for comparison with the waveforms of Figure 6a2. No earlier/similar adjustments are possible since the generating power-series data commences on January 1, 1600 CE. Nonetheless, it seems more complex intersections about the constant ϕ occur during this earlier period, especially near the end points of the interval 1640 to 1695. Further investigation reveals a similar occurrence to the former *circa* 2542 - 2557 CE, but at this point, remaining with Figures 6a through 6e, it is uncertain to what extent these waveforms provide sufficient information to confirm Kepler's interest [6] in an 800-year cycle between Jupiter and Saturn. Or, from the tests here, an approximate 795-year cycle with a possible maximum before the middle of the present century, *i.e.*, December 2046. Then again, there are the Jupiter-Saturn grand alignments that occur some 25 years *earlier* than the dates of interest (March 1226, July 1623 and December 2020). Then again, 795-year maxima before 2047 (1252, 457, -338, -1133, -1928, 2723, -3518, -4313, -5108, -5903) and/or the 397.5-year half-cycles are not necessarily of immediate historical significance in terms of cyclic causes-and-effects, etc. The current concerns with global warming on the other hand may not lie so much in such cycles *per se*, but the troubling possibility that present industrial activity could conceivably tip the balance beyond the "norm" with potentially disastrous consequences.

In any event, such theorising lies well beyond the present examination of the consecutive Pheidian constants ϕ^{-3}, ϕ^{-2}, ϕ^{-1}, ϕ^{0}, ϕ^{1}, ϕ^{2} and the preliminary real-time survey of the planetary waveforms of the Superior and Inferior planets tentatively explored here.

As for the generation of the data applied in Figures 1 through 6, it could be suggested that methods developed by Bretagnon and Simon are truly excellent, but however intriguing, that the results are nonetheless still subject to a potentially troublesome Korzibskian qualifier, namely, that:

"The Map is *not* the Territory."

Nevertheless, whether embedded in the methodology, the Solar System, or indeed both, the Golden Ratio is to a certain degree undoubtedly present, enough at least, to justify Benjamin Peirce's research and his conclusions.

This said, and bearing in mind the observation made earlier concerning the reduction of the base period(s) for HR 8799 by Pierce's limiting constant ϕ^{-2}, a matter of considerable import remains.

The Peirce planetary framework and *Phyllotaxis*

In addition to the limiting value of Pierce's planet-to-planet reduction ratios, the constant ϕ^{-2} (0.38196601125) has long been associated with phyllotaxis and the ideal growth angle of 137.50776 degrees obtained from a variety of viewpoints that include revolution-associated multiplication or division. Thus either from $360 \cdot \phi^{-2}$ or the reciprocal constant 2.61803398875 ($360/\phi^{2}$) [7] and also the more convenient Fibonacci ratio 55/144 (0.3819444*) =137.5/360°.

What do phyllotaxis and the natural growth angle have to do with the present discourse ?

Firstly, the title of Benjamin Pierce's initial paper [8] was "Mathematical Investigations of the Fractions Which Occur in in Phyllotaxis." Secondly, his hypothesis in the present astronomical context was precisely summarized in terms of phyllotaxis by Louis Agassiz in the *Essay on Classification*, the entire set of the planet-to-planet Fibonacci reduction ratios included. Thus Agassiz states: [9]

> It is well known that the arrangement of the leaves in plants may be expressed by very simple series of fractions, all of which are gradual approximations to, or the natural means between 1/2 or 1/3, which two fractions are themselves the the maximum and the minimum divergence between two single successive leaves. The normal series of fractions which expresses the various combinations most frequently observed among the leaves of plants is as follows: *1/2,1/3, 2/5,3/8, 5/13, 8/21,13/34, 21/55, etc. Now upon comparing this arrangement of the leaves in plants with the revolutions of the members of our Solar System, Peirce has discovered the most perfect identity between the fundamental laws which regulate both*. (italics supplied).

There are a number of reasons why this quotation is given here rather than in the Introduction. The first stems from the negative reception of Benjamin Pierce's hypothesis and the manner that it was cast aside. The second reason is related to the first in so much as it is almost certain that the synodic component would have been incorporated far earlier had the work received a favorable reception. Here readers may decide for themselves. The third reason is more pragmatic; even with the inclusion of the *Phi*-series it was felt that refinements to Pierce's planetary model might still be required before embarking on standard tests beyond the Solar System.

This said, a short introduction and an acknowledgment of both earlier and recent contributions appears to be in order, beginning with the "three-fold number," now that readers are acquainted with both the number and the significance assigned to the matter by Benjamin Pierce and Louis Agassiz.

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