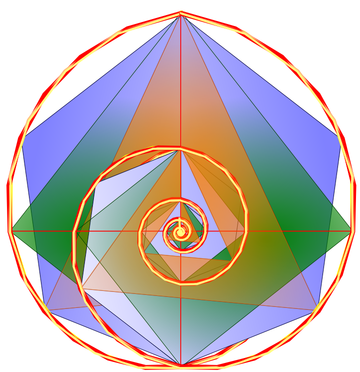
INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS



2023

PART TWO

THE PIERCE FRAMEWORK AND EXTERNAL SYSTEMS

JNH

EXTERNAL PLANETARY SYSTEMS

Preliminary remarks and initial tests

This research has made use of the Exoplanet Orbit Database and the Exoplanet Data Explorer at exoplanets.org between 2018 and the present (2022).[9] The research was placed on hold when the periods for the first and the last examples originally treated here - GJ 876 and HD 30177 - were changed drastically, which destroyed the associated analyses. These occurrences also caused a reevaluation of the available exo-planet data, resulting in a minor loss of confidence in the same, the coincidental changes notwithstanding. Nevertheless, both analyses have been omitted from the present discussion despite the further insights they provided.

What follows next is a reduced treatment presented on a take-it-or-leave it basis. I have assumed that periods of of "revolution ", "years" and "days" are just that as applied to the exoplanets and have proceeded accordingly. This is necessarily an initial attempt, but at least with standard procedures and a specific planetary framework as its basis.

INITIAL TESTS

2 : 1 and 4 : 2 Octaves

Omitting single planets and deferring multiple configurations until later, dual configurations can include alternate planet-to-planet or synodic-to-synodic pairs, adjacent planet-synodic pairs, and lastly, widely separated pairs of either kind. In particular, the periods and the 2:1 ratios of outermost pairs (Planets 1 and 2) provide base period B1, plus from relation (1) a further base period B2 (*Synodic* 2-1). Since *Synodic* 2-1 is adjacent to Planet 2 theoretical periods below the latter can be derived from the successive applications of relation (2) if required. Either way, a full theoretical framework follows from the division of the selected base period by the standard divisors. A second set of data based on the known periods and *Phi*-series/synodic relations (1), (2) and (4) permits comparison with the latter and the determination of mean and individual errors. This can be useful if uncertainties arise, *e.g.,* selection of outermost planets from either of the 2:1 or 4:2 ratios. The latter – 4:1 in terms of the theoretical framework and the fixed divisors – should not normally be present, but if detected in addition to the 2:1 ratio of the outermost pair, both can be incorporated in a possibly disturbed planetary framework, a primary example being HR 8799.

HR 8799 b-e (b, c and d detected in 2008, e in 2010)[12] Planets 1 and 2, *Synodic* 3-2 and *Synodic* 4-3, Base period B2 = 164,330 days (*Synodic* 2-1). Residual Fibonacci sequence 233-144-89-55-34-21-13-8-5-3-2-1-1 is completed at Planet 14. The restored/suggested overall planetary framework is discussed in detail later.

Period ratios other than 2 : 1

Configurations below the three outermost planets, *i.e*., with period ratios other than 2:1 initially present difficulties, but as a beginning the ratios of detected periods can be compared with those of the Pierce framework divisors and where successful the results can be applied to generate B3 base periods. Before describing the methodology one further element needs to be incorporated in the test procedures. As it turned out, the Fibonacci indicators introduced above became increasingly apparent during the inspection of planetary data, although without the Pierce planetary framework (especially relation (1) that generates the intermediate values) this association might well be dismissed as coincidence. In addition to HR 8799, examples of residual Fibonacci sequences are:

Kepler-460 c-b (2016)[11] Planet #2, adjacent *Synodic* 3-2 respectively, base period B3 = 881.5626 days (2xKepler-460 c). with residual Fibonacci sequence: 55-34-21-13-8-5-3-2-1-1 completed at Planet 8.

Kepler-321 c-b (both planets detected in 2014).[13] Kepler-321 c: 13.093921 days, Kepler-321 b: 4.915379 days. The two Kepler-321 planets detected to date have periods that round to alternate Fibonacci numbers 13 and 5. *Phi*-series/synodic relation (1) provides a synodic difference cycle between Kepler-321c and b of 7.869567 days which rounds in turn to 8 to complete the Fibonacci trio: 5-8-13. Although in years, this sequence is present in Solar System as 5 synodic periods of Venus in 8 years with 13 corresponding periods of revolution for this planet. Also, for Kepler-321, since relation (1) provides an adjacent period to that of Kepler-321b, successive applications of *Phi*-series/synodic relation (2) generate sequential periods below the latter which when rounded complete the Fibonacci sequence 13-8-5-3-2-1-1 at Planet 7. Up to this point the Pierce planetary framework and the associated divisors play no part. All that remains now are the positional assignments for the two planets and determination of the base period. This requires the *ratio* of the detected periods, assignment of the *closest ratio* from the divisor framework, and the selection of a B3 base period from the Divisor-Period products as follows:

The ratio of the Periods of Kepler-321 *c* & Kepler-321 *b* = 2.6638686.
The ratio of the Divisors for Planet 4 & Planet 5 (40/15) = 2.6666667 (Fibonacci 8/3)

The divisors for Planet 4 and Planet 5 are 15 and 40, hence the following B3 base period options with mean errors from Planets 4 through 7 the deciding factor; differences are slight with the average period (B4) a third choice.

15x Kepler-321*c* = 196.408815 days, mean error: - 0.503%.
40x Kepler-321*b* = 196.615160 days, mean error: - 0.608%.

Thus the provisional assignments for Kepler 321*c* and *b* are Planets 4 and 5 with a B3 base period of 15x Kepler 321*c* = 196.40882days. Thereafter the Peirce planetary framework follows from the application of the standard divisors with relations (1) and (2) completing the residual Fibonacci sequence: 13-8-5-3-2-1-1 at planet 7.

PLANETS N *Synodics* #	DIVISOR (added)	PERIODS 1 Base/Divisor	PERIODS 2 B3:Restored	PERIODS 3 B3: Integers	EXOPLANET Kepler-321	%Error (Div:B3)
PLANET 1	1	196.40882	196.40882	196	15x Kepler-321c	Base B3
Synodic 2-1	1	196.40882	202.90383	203	Relation (1)	3.31%
PLANET 2	2	98.204407	99.801751	100	Rel (4F + 3)	1.63%
Synodic 3-2	4	49.102204	50.712538	51	Rel (4E ± 3)	3.28%
PLANET 3	6	32.734803	31.071269	31	Rel (4E ± 4)	-5.08%
Synodic 4-3	9	21.823202	22.630965	23	Relation (1)	3.70%
PLANET 4	15	13.093921	13.093921	13	Kepler-321 c	0.00%
Synodic 5-4	25	7.8563526	7.8695670	8	Relation (1)	0.17%
PLANET 5	40	4.9102204	4.9153790	5	Kepler-321 b	0.11%
Synodic 6-5	64	3.0688877	3.0255822	3	Relation (2)	-1.14%
PLANET 6	104	1.8885463	1.8728064	2	" "	-0.83%
Synodic 7-6	169	1.1621823	1.1567742	1	" "	-0.47%
PLANET 7	273	0.7194462	0.7150872	1	" "	-0.61%
					Mean error:	-0.292%

Table 1. The Divisor framework, Kepler-321 Planets 4 and 5, Base period B3c.

HD 37605 *b-c* (both planets detected in 2014).[4] HD 37605 *c* : 2,720 days. HD 37605 *b* : 55.01307 days.

The suggestion of a Fibonacci presence provided by the near 13-day period of Kepler-321 *c* does not appear to be a coincidence, nor does it appear to be an isolated occurrence. For example, a similar presence is suggested by the lower period of two-planet HD37605, specifically, the approximate 55-day period of HD37650 *b* of 55.01307days. In this system the two detected periods are widely separated and both also occupy *synodic* rather than planetary positions. Nevertheless, though a variation from the successive planetary periods of Kepler-321, relations (1), (2) and (4) are equally applicable to both *Planet* and *Synodic* locations in the *Phi*-series planetary framework. Therefore sequential, multiple applications of these three relations permit the restoration of the seven periods between the two detected planets as shown in Table 7.

ASSIGNMENTS
The ratio of the periods of HD 37605 *c* and HD 37605 *b* = 49.442796.
The ratio of the divisors for *Synodic* 4-3 and *Synodic* 8-7 = 49 (441/9).
The provisional base period (B3) is 9x HD 37605 *c* = 24,480 days.

PLANETS N Synodics #	DIVISORS (added)	PERIODS 1 B3/Divisors	PERIODS 2 B3r restoration	PERIODS 3 B3r (rounded)	EXOPLANETS HD 37605	%Error (B1: B3)
PLANET 1	1	24480	24480	24480	9x HD 37605 c	Base B3c
Synodic 2-1	1	24480	25405.81	25406	Relation (1) 12th	3.78%
PLANET 2	2	12240	12467.16	12467	Rel (4F + 3) 11th	1.86%
Synodic 3-2	4	6120	6349.265	6349	Rel (4E ± 3) 10th	3.75%
PLANET 3	6	4080	3899.066	3899	Rel (4E ± 4) 9th	-4.43%
Synodic 4-3	9	2720	2720	2720	HD 37605 c	0.00%
PLANET 4	15	1632	1646.780	1647	Relation (1) 8th	0.91%
Synodic 5-4	25	979.2	1025.754	1026	Rel (4E ± 2) 2nd	4.75%
PLANET 5	40	612	621.0260	621	Relation (1) 7th	1.47%
Synodic 6-5	64	382.5	386.8275	387	Rel (4E ± 4) 1ST	1.13%
PLANET 6	104	235.3846	234.1985	234	Relation (1) 6th	-0.50%
Synodic 7-6	169	144.8521	145.8786	146	Rel (4E ± 2) 3rd	0.71%
PLANET 7	273	89.67033	88.31984	88	Relation (1) 4th	-1.51%
Synodic 8-7	441	55.51020	55.01307	55	HD 37605 b	-0.90%
PLANET 8	714	34.28571	33.89832	34	Relation (2) 5th	-1.13%
Synodic 9-8	1156	21.17647	20.97426	21	" "	-0.95%
PLANET 9	1870	13.09091	12.95715	13	" "	-1.02%
Synodic 10-9	3025	8.092562	8.009294	8	" "	-1.03%
PLANET 10	4895	5.001021	4.949701	5	" "	-1.03%
Synodic 11-10	7921	3.090519	3.059158	3	" "	-1.01%
PLANET 11	12816	1.910112	1.890646	2	" "	-1.02%
Synodic 12-11	20736	1.180556	1.168488	1	" "	-1.02%
PLANET 12	33552	0.729614	0.722164	1	" "	-1.02%
					Mean value	0.212%

Table 2.The Divisor planetary framework and HD 37605, *Synodic* 4-3 & *Synodic* 8-7, Base B3c

The two widely separated planets HD 37605 *c* and *b* with a period ratio of 49.442796 (2720 days/55.01307 days) are readily equated with the Peirce planetary framework and the ratio of the divisors for *Synodic* 4-3 and *Synodic* 8-7 (441 and 9 respectively). The lowest mean error is obtained from product of the period of HD 37605 c and the the divisor for *Synodic* 4-3 (9) = 24,480 days. The odd number of periods between the two detected planets permits the theoretical restoration of all periods between HD37605 *c* and *b* plus all those below the latter. This is feasible since relation (4) – the extended geometric mean – can be applied three times, first at the midpoint between the two planets (at *Synodic* 6-5) then twice more between two new mid-points to obtain the periods for *Synodic* 5-4 and *Synodic* 7-6. This fills three of the seven positions with those remaining determined by relation (1), including the period adjacent to HD 37605 b, thus permitting the generation of the periods for Planets 4, 5 and 6, and finally the completion of the residual Fibonacci sequence: 55-34-21-13-8-5-3-2-1-1 at Planet 12 by use of Relation (2). Another example involving both a Fibonacci indicator (5 versus 5.41608 days) and multiple applications of relation (4) and (2) is given below *sans* table.

HATS-59 *b-c* [5] HATS-59 *c*: 1422 days, HATS-59 *b*: 5.41608 days.
Here the more widely separated periods of HATS-59 *c* and *b* (1422 and 5.41608 days respectively) can be assigned to Planet 1 and Planet 7 with a separation of eleven intervening periods. Again, the odd number permits multiple applications of relation (4) to determine the period of mid-point Planet 4 followed by mid-point periods on either side belonging to *Synodic* 3-2 and *Synodic* 6-5. In this instance there are two positions between the determined periods, not one, therefore relation (1) is not applicable. Instead, with the Pierce planetary framework available, the period of Planet 2 of 711 days can be introduced above *Synodic* 3-2 to allow relation (2) to end at Planet 9.

ASSIGNMENTS

The ratio of the periods of HATS-59 *c* and HATS-59 *b* = 262.551465.
The ratio of the divisors for Planet 1 & Planet 7 (273/1) = 273 (3.83%).
The provisional base period is B1, Planet 1, HATS-59 *c* (1422 days).
The residual Fibonacci presence 13-8-5-3-2-1-1 including HATS-59 *b* is completed at Planet 9.

Variations and Additions

Applying the above procedures to other systems brought to light additional numerical sequences including the double-Fibonacci series,. *i.e.,* instead of Fibonacci 13-8-5, the sequence 26-16-10, etc. The second occurrence, the replacement of the residual Fibonacci series by the Lucas series (1-3-4-7-11-18-29-47-76-123,... etc.,) featured one common departure, namely the inclusion of the number 2 below the sequence 7-4-3. Examples of residual Lucas series present among the available external systems are:

Kapteyn's *c-b* (2014)[6] Kapteyn's *c:* 124.54 days, Kapteyn's *b*: 48.616 days.
No series is initially indicated with the Lucas sequence only becoming apparent after the assignment of the base period and the derivation of the divisor framework. The two periods are again consecutive synodic locations, *i.e*., *Synodic* 5-4 and *Synodic* 6-5 respectively.

ASSIGNMENTS

The ratio of the divisors for *Synodic* 5-4 and *Synodic* 6-5 = 2.56. (square of 1.6, or Fibonacci 5/3)
The ratio of the periods of Kapteyn's *c* and Kapteyn's *b* = 2.5617081.
The provisional base period is B3 = 25x Kapteyn's c = 3113.5 days.
The residual Lucas series 18-11-7-4-3- {2}-1-1 is complete at Planet 10 with {2} anomalous.

Other external systems featuring Lucas sequences include the following, all with {2} anomalous:

Nu Ophiuci *c-b* (2010)[7] Planets 1 and 3 respectively, Base period B1 = 3,186 days (as detected).
Residual Lucas sequence: 18-11-7-4-3- {2}-1-1 completed at Planet 10.
XO-2s *b-c* (2014)[8] Planets 3 and 5 respectively, Base Period B3 = 724.8 days (6x XO-2s c).
Residual Lucas sequence: 18-11-7-4-3-{2}-1-1 completed at *Synodic* 9-8.
Kepler-49 *c-b* (2012)[9] *Synodic* 3-2 and Planet 3 respectively, Base period B3 = 43.6517372 days (4x Kepler-49 c).
Residual Lucas sequence: 11-7-4-3- {2}-1-1 completed at *Synodic* 8-7
Kepler-198 *c-b* (2014)[10] Planets 3 and 4 respectively, Base period B3 = 273.404496 days (6x Kepler-198 c).
Residual Lucas sequence: 29-18-11-7-4-3- {2}-1-1 completed at Synodic 8-7.
Kepler-396 *c-b* (2014)[11] Planet 2 and *Synodic* 3-2 respectively, Base period B3 = 177.01 days (2x Kepler-396 c).
Residual Lucas sequence: 29-{17}-11-7-4-3- {2}-1-1 completed at Planet 7 with {17} for Lucas number 18.
HD 60532 *b-c* (2008)[12] Planets 2 and 3 respectively, Base period B3 = 1214.12 days (2x HD 60532 c).
Residual Lucas sequence: 76-47-29-18-11-7-4-3-{2}-1-1 completed at Planet 9.
HD 163607 *b-c* (2011)[13] Planets 3 and 6 respectively, Base Period B3 = 7,884 days (6x HD 163607 c).
Residual Lucas sequence: 76-47-29-18-11-7-4-3-{2}-1-1 completed at Planet 11.

Finally, partial residual sequences, *i.e.*, confined to three consecutive values occur in some instances, while other systems, *e.g.,* TRAPPIST-1 treated next have no immediately discernable sequence.

TRAPPIST-1 (*b* - *g* detected in 2016,[14] TRAPPIST-1 *h* detected in 2017).[15]
TRAPPIST-1*h* : 18.767 days[26]
TRAPPIST-1*g* : 12.35294 days
TRAPPIST-1*f* : 9.206690 days
TRAPPIST-1*e* : 6.099615 days
TRAPPIST-1*d* : 4.049610 days
TRAPPIST-1*c* : 2.4218233 days
TRAPPIST-1*b* : 1.51087081 days
Base period B3, Planet #1: 36.82876 days (4x TRAPPIST-1f)

Initially the period of TRAPPIST-1h was thought to range between14 and 35 days with the absence of a precise base period preventing generation of a divisor-based planetary framework. Nevertheless the latter still appears to be present in the structure of TRAPPIST-1 as indicated by the sequential twinned reduction ratios for both two-third ratios and also the upper three-fifth ratio. Thus the Pierce ratios have a key role to play in the present example. In more detail, commencing with the 9.206690-day period of TRAPPIST-1*f* the successive reduction ratios generate ordered approximations for the periods of the remaining four TRAPPIST-1planets1*e* through1*b*. Furthermore, the applicationof *Phi*-series relation(4) to the periods of TRAPPIST-1*f* and TRAPPIST-1*d* results in a period of 6.081193 days versus the 6.0992672-period of TRAPPIST-I *e*.

Although the twinned Pierce ratios appear to be reflected in a substantial part of the structure of TRAPPIST-1 it appears that with respect to the theoretical framework the five adjacent planets TRAPPIST-1*f* through 1b occupy consecutive planetary/synodic positions. Thus while the Solar System has one planet (Earth) in a synodic location between adjacent planets, TRAPPIST-1 appears to have at least three in ordered succession, two of which can be approximated (if not confirmed) by relation(1). In short, the general synodic formula can be applied to the mean periods of revolution of TRAPPIST-1*c* and 1*e* to approximate the difference period in the current location of 1*d*, and and similarly applied to TRAPPIST 1*b* and 1*d* for the difference period in the current location of 1*c*.

PLANETS Synodics	N #	RATIOS (Pierce)	DIVISOR (added)	DIVISOR RATIO Results/Ratios	B1-RATIOS B1s/Divisors	PERIODS 1 B3/Divisors	PERIODS 2 B1s(actual)	EXOPLANETS TRAPPIST-1	%ERR (B1: B3)
PLANET	1	1/1	1	(1/1)	.	36.826760	.	4x TRAPPIST-1f	Base 3
Synodic	?	1/1	2	2 (2/1)	1.9623147	18.413380	18.767	TRAPPIST-1h	1.92%
PLANET	?	1/2	3	3 (3/1)	2.8912142	12.275587	12.35294	TRAPPIST-1g	0.63%
Synodic	?	1/2	4	4 (4/1)	4	9.20669	9.20669	TRAPPIST-1f	0.00%
PLANET	3	2/3	6	3 (6/2)	6.0378991	6.1377933	6.0992672	TRAPPIST-1e	-0.63%
Synodic	4-3	2/3	9	2.25 (9/4)	9.1683135	4.0918622	4.0167431	TRAPPIST-1d	-1.84%
PLANET	4	3/5	15	2.5 (15/6)	15.206213	2.4551173	2.4218233	TRAPPIST-1c	-1.36%
Synodic	5-4	3/5	25	2.777* (25/9)	24.374526	1.4730704	1.5108708	TRAPPIST-1b	2.57%
PLANET	5	5/8	40	2.666*(40/15)	39.580739	0.9206690	0.9304212	(Rel.2)	1.06%
Synodic	6-5	5/8	64	2.56 (64/25)	63.955265	0.5754181	0.5758206	" "	0.07%
PLANET	6	8/13	104	2.6 (104/40)	103.53600	0.3541035	0.3556904	" "	0.45%
Synodic	7-6	8/13	169	2.6406 (169/64)	167.49127	0.2179098	0.2198727	" "	0.90%
PLANET	7	13/21	273	2.625 (273/104	271.02727	0.1348966	0.1358784	" "	0.73%
Synodic	8-7	13/21	441	2.609 (441/169)	438.51854	0.0835074	0.0839799	" "	0.57%
PLANET	8	21/34	714	2.615 (714/273)	709.54581	0.0515781	0.0519019	" "	0.63%
								Mean error:	0.407%

Table 3. The Pierce planetary framework, TRAPPIST-1, compressed adjacent planets, Base period B3

Irrespective of additional complications that arise from the dual occurrence of TRAPPIST-1*d* and 1*c* in both synodic computations the first two-thirds reduction ratio nonetheless serves to synchronize TRAPPIST-1*e* with divisor planet #3 and therefore all the remaining positions. At which point the simplest option for a theoretical base period is to reverse standard procedures and use the products of the known periods and their associated divisors to generate B3 estimates, with TRAPPIST-1*f* (4x = 36.82676 days) the closest to the mean value.

More recently,[16] a period of 18.767 days has been deduced for TRAPPIST-1*h* with a resulting *Synodic* 2-1 interval of 36.14366 days between the latter and the 12.35294-day period of TRAPPIST-1*g*. The last period could be applied as a provisional base period B2. Lastly, though the12.35294-day period of TRAPPIST-1*g* appears to be anomalous, the synodic period between this value and any of the 36-day estimates for base period B3 results in values between 18.356 and 18.767 days. All of which raises the possibility that if disruptions of TRAPPIST-1 may have occurred, that they might have involved the two outermost planets. If so there would be no perceptible gap *per se*, but absence of the outermost planet (or both) and possible readjustments by the others. Whether this scenario would be drastic enough to cause TRAPPIST-1*b* thru 1*f* to occupy adjacent sidereal/synodic locations *en masse* or cause the anomalous period of 1*g* is another matter. Then again, this scenario may also be a more natural occurrence with compression a component of later phases in the life-cycle of this particular System itself.

Further candidates for compressed systems

Additional systems with apparent planet/synodic compression and residual Fibonacci/Lucas sequences include:
YZ Cet *d-c-b* (2017) [17] Synodic 3-2, Planet 3, *Synodic* 4-3. Base B3 = 18.62508 days
(4x YZ Cet *d*). Residual Fibonacci sequence: 5-3-2-1-1 completed at *Synodic* 5-4.
Kepler-23 *d-c-b* (*b-c* 2012, *d* 2014) [18],*Synodic* 3-2, Planet 3, *Synodic* 4-3, Planet 4. Base B3 = 61.097216 days
(4xKepler-23 *d*). Residual Lucas sequence: 11-7-4-3- {2}-1-1 completed at Planet 6.
Kepler-37 *d-c-b* (2016)[19] *Synodic* 4-3, Planet 4, *Synodic* 5-4. Base B3 = 358.129683 days.
(9x Kepler-37 *d*). Residual Fibonacci sequence: 21-13-8-5-3-2-1-1 completed at *Synodic* 8-7.
Kepler-107 *e-d-c-b* (2014)[20] Planet 2, *Synodic* 3-2, Planet 3, *Synodic* 4-3. Base B3 = 29.498352 days
(2x Kepler-107 *e*). Residual Fibonacci sequence: 8-5-3-2-1-1 completed at Planet 5.
Kepler-184 *d-c-b* (2014)[21] *Synodic* 5-4, Planet 5, *Synodic* 6-5. Base B3 = 203.304324 days
(25x Kepler-184 *d*). Residual Lucas sequence: 11-7-4-3- {2}-1-1 completed at *Synodic* 9-8.

Kepler-208 *e-d-c-b* (2014)[22] *Synodic* 3-2, Planet 3, *Synodic* 4-3, Planet 4. Base B3 = 65.0783 days (4x Kepler-208 *d*). Residual Lucas sequence: 11-7-4-3- {2}-1-1 completed at Planet 6.
Kepler-295 *d-c-b* (2014) [23] Planet 3, *Synodic* 4-3, Planet 4. Base B3 = 203.304324 days (6x Kepler-295 *d*). Residual Fibonacci sequence: 34-{22}-13-8-5-3-2-1-1 completed at Planet 7.
Kepler-374 *d-c-b* (2014)[24] Planet 3, *Synodic* 4-3, Planet 4. Base B3 = 30.169314 days (6x Kepler-374 *d*). Residual Fibonacci sequence: 5-3-2-1-1 completed at Planet 5.
Kepler-446 *d-c-b* (2014)[25] *Synodic* 4-3, Planet 4, *Synodic* 5-4. Base B3 = 46.340289 days (9x Kepler-446 *d*). Residual Fibonacci sequence: 5-3-2-1-1 completed at Synodic 6-5.
Kepler-758 *e-d-c-b* (2016)[26], Planet 4, *Synodic* 5-4, Planet 5, *Synodic* 6-5. Base B3 = 307.4493 days (15xKepler-758 *d*). Residual Fibonacci sequence: 8-5-3-2-1-1 completed at *Synodic* 8-7.

HR 8799 (HR 8799 *b, c* and *d* detected in 2008, HR 8799 *e* detected in 2010.[2]
Initially detected as a three-planet system, HR 8799 has known [3] 1:2 and 1:4 resonances already subject to detailed analyses.[27, 28] The theoretical planetary framework is augmented by a possible 1:9 resonance following the discovery in 2010 of a fourth planet (HR 8799 *e*) with a period of 18,000 days[38] versus 18,250 days for the associated resonance.

ASSIGNMENTS (MEAN PERIODS: Days/Julian Years)
HR 8799 *b* : 164,250 days/449.691991786 years. HR 8799 *c* : 82,145 days/224.900752909 years.
HR 8799 *d* : 41,054 days/112.399726215 years. HR 8799 *e* : 18,000 days/49.2813141684 years.

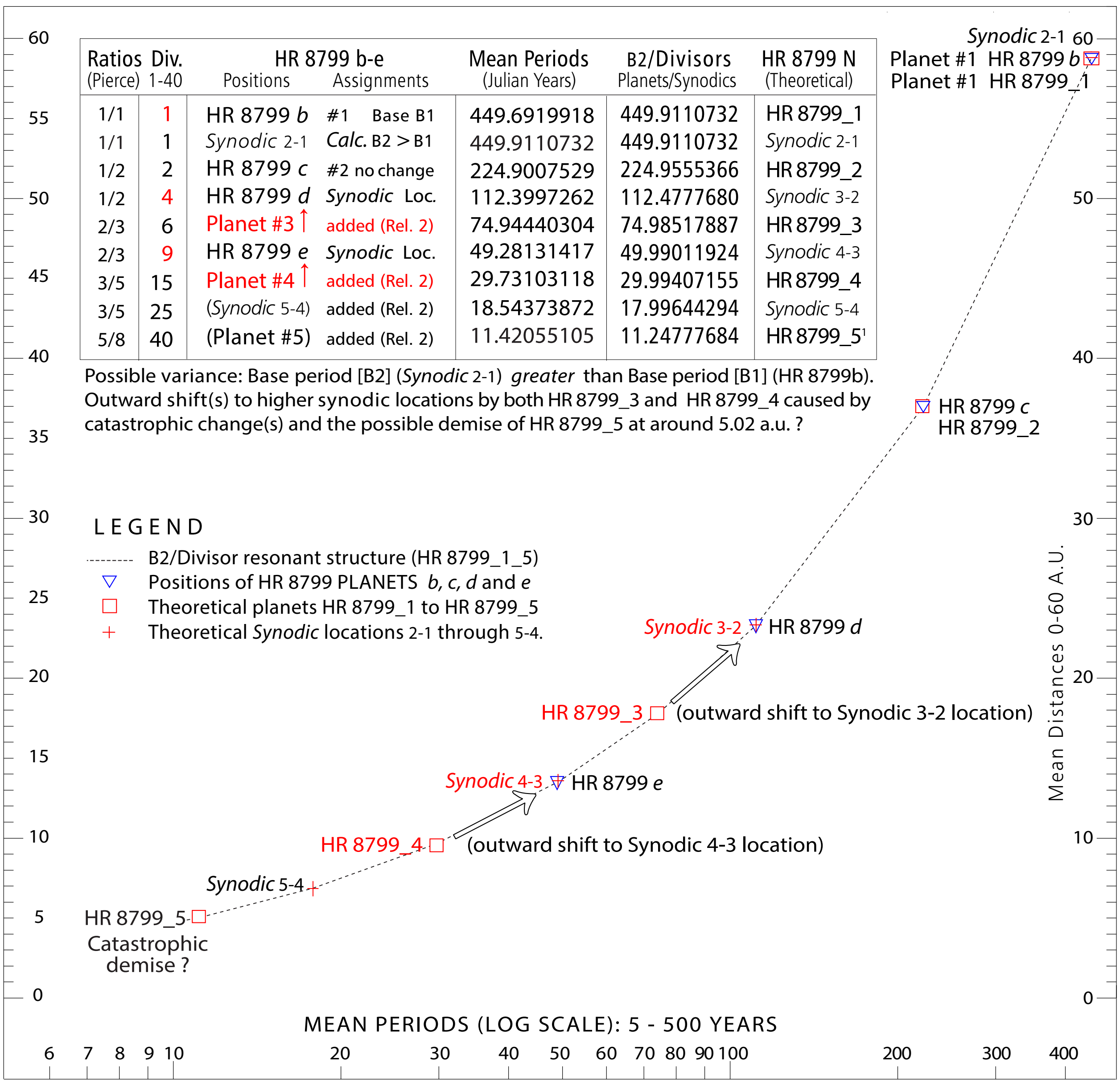
The ratio of the Divisors for Planet 1 and Planet 2 (2/1) = 2.00000; second octave (HR 8799 *c* and *d*) is also present. Ratio of the periods of HR 8799 *b* and c is 1.999613; the 4/2 ratio of the second octave HR 8799 *c* and *d* = 2.000901. The provisional base period (B2) is *Synodic* 2-1(164330.019 days) between HR 8799 *b* and *c* (lowest mean error).

PLANETS N Synodics #	RATIOS (Pierce)	DIVISORS (added)	PERIODS 1 B2/Divisors	PERIODS 2 B1 Actual	PERIODS 3 B1/(Rounded	EXOPLANETS HR 8799	%ERROR (B1: B2)
PLANET 1	1/1	1	164330.019	164250	164250	HR 8799 *b*	-0.049%
Synodic 2-1	1/1	1	164330.019	164330.019	164330	(Rel. 1)	0.000%
PLANET 2	1/2	2	82165.0097	82145	82145	HR 8799 *c*	-0.024%
Synodic 3-2	1/2	4	41082.5049	41054	41054	HR 8799 *d*	-0.069%
PLANET 4	2/3	6	27388.3366	27373.443	27373	(Rel. 2)	-0.054%
Synodic 4-3	2/3	9	18258.8911	18000	18000	HR 8799 *e*	-1.418%
PLANET 4	3/5	15	10955.3346	10859.2591	10859	(Rel. 2)	-0.877%
Synodic 5-4	3/5	25	6573.20078	6733.10057	6773	" "	3.041%
PLANET 5	5/8	40	4108.25049	4171.35627	4171	" "	1.536%
Synodic 6-5	5/8	64	2567.65655	2581.49088	2581	" "	0.539%
PLANET 6	8/13	104	1580.09634	1594.63378	1595	" "	0.920%
Earth/*Syn* 7-6	8/13	169	972.366979	985.730287	986	" "	1.374%
PLANET 7	13/21	273	601.941463	609.169393	609	" "	1.201%
Synodic 8-7	13/21	441	372.630430	376.498114	376	" "	1.038%
PLANET 8	21/34	714	230.154089	232.686099	233	" "	1.100%
Synodic 9-8	21/34	1,156	142.153996	143.808516	144	" "	1.164%
PLANET 9	34/55	1,870	87.8770158	88.8784095	88	" "	1.140%
Synodic 10-9	34/55	3,025	54.3239734	54.9299112	55	" "	1.115%
PLANET 10	55/89	4,895	33.5709948	33.9485443	34	" "	1.125%
Synodic 11-10	55/89	7,921	20.7461204	20.9813561	21	" "	1.134%
PLANET 11	89/144	12,816	12.8222550	12.9671908	13	" "	1.130%
Synodic 12-11	89/144	20,736	7.92486591	8.01416473	8	" "	1.127%
PLANET 12	144/233	33,552	4.89777121	4.95302617	5	" "	1.128%
Synodic 13-12	144/233	54,289	3.02694873	3.06113853	3	" "	1.130%
PLANET 13	233/377	87,841	1.87076672	1.89188765	2	" "	1.129%
Synodic 14-13	233/377	142,129	1.15620330	1.16925087	1	" "	1.128%
PLANET 14	377/610	229,970	0.71457155	0.72263678	1	" "	1.129%
						Mean error:	0.809%

Table 4. The Pierce planetary framework and HR 8799; Planets 1 and 2, Synodics 3-2 and 4-3. Bases B1 and B2.

HR8799 AS A DISTURBED PLANETARY SYSTEM

As in the case of the Solar System, base period B2 (164330.019 days, *Synodic* 2-1 between HR 8799 *c* and HR 8799 *b*) is marginally greater than base period B1 (164,250 days). This, allied with 1:4 and 1:9 resonances recognizable as squares belonging to two successive synodic positions in the divisor framework suggests that HR8799 may also be a disrupted system as seen in Table 4 and Figure 1 below. If so, it is perhaps possible that HR 8799 *d* and HR 8799 *e* may currently be occupying *Synodic* 4-3 and *Synodic* 3-2 locations resulting from outward orbital shifts of divisor planets #3 and #4. In which case theoretical mean periods of revolution for the latter pair can be approximated by either successive applications of relation (2) to HR 8799 *c* and HR 8799 *d,* and (or) the application of divisors 1 to 40 to base period B2. Whether a theoretical planet at or near position #5 (~ 4.5 HR 8799 standard mass?) suffered a catastrophic demise is hypothetical, but still a possibility which can be considered further in terms of the debris field in the inner region of HR 8799 discussed by Moore and Quillen (2013),[29] Contro *et al*, (2014)[30] and Contro *et al,* (2016) [31]. In particular, the theoretical distance for possibly defunct HR_8799_5 at ~5.02 a.u. is situated reasonably close to the " inner and outer edges, located at ~6 and ~8 au," of the debris belt discussed by the latter authors.

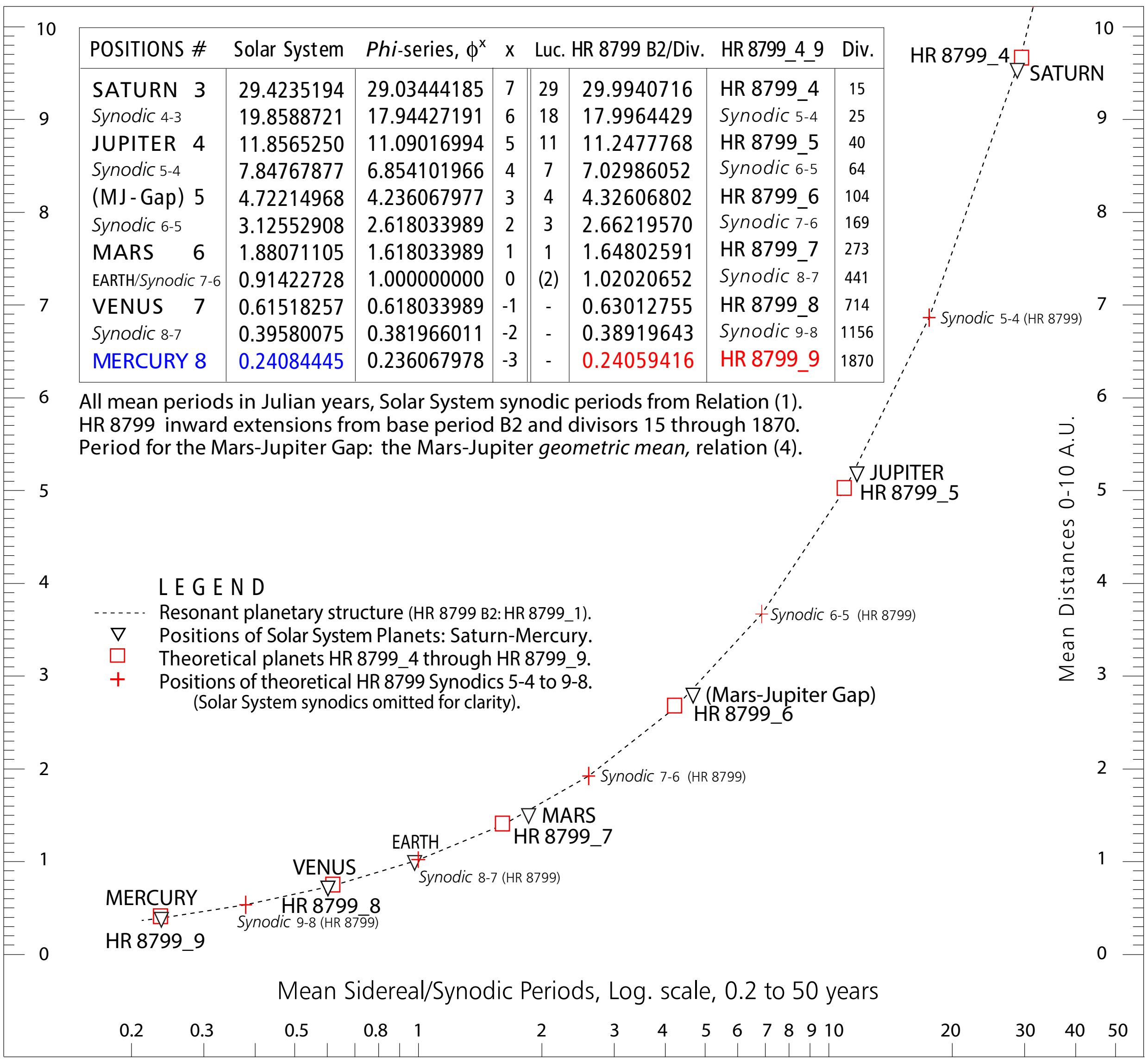


Ratios (Pierce)	Div. 1-40	HR 8799 b-e Positions	Assignments	Mean Periods (Julian Years)	B2/Divisors Planets/Synodics	HR 8799 N (Theoretical)
1/1	1	HR 8799 *b*	#1 Base B1	449.6919918	449.9110732	HR 8799_1
1/1	1	*Synodic* 2-1	*Calc.* B2 > B1	449.9110732	449.9110732	*Synodic* 2-1
1/2	2	HR 8799 *c*	#2 no change	224.9007529	224.9555366	HR 8799_2
1/2	4	HR 8799 *d*	*Synodic* Loc.	112.3997262	112.4777680	*Synodic* 3-2
2/3	6	Planet #3 ↑	added (Rel. 2)	74.94440304	74.98517887	HR 8799_3
2/3	9	HR 8799 *e*	*Synodic* Loc.	49.28131417	49.99011924	*Synodic* 4-3
3/5	15	Planet #4 ↑	added (Rel. 2)	29.73103118	29.99407155	HR 8799_4
3/5	25	(*Synodic* 5-4)	added (Rel. 2)	18.54373872	17.99644294	*Synodic* 5-4
5/8	40	(Planet #5)	added (Rel. 2)	11.42055105	11.24777684	HR 8799_5[1]

Fig 1. HR 8799 *b* to *e* and departures from the Pierce planetary framework for planets 1 through 5.

In any event, if this scenario is valid there is a distinct similarity between HR 8799 and the Solar System, since–for whatever reasons–the fifth planet in both systems (counting inwards) can be considered to be absent. Also, in addition to the known planetary resonances, similarities between the two systems, especially with respect to the gas giants Jupiter and Saturn have already been noted by Fabrycky and Murray-Clay (2010).[31] In fact, the similarity between the two may be greater than already suspected, as shown in log-linear Figure 2 with the Solar System in an eight-planet configuration and HR 8799_1_9 as a theoretical nine-planet system. Or, as a substitute, depending on what might have taken place and the original mass of planet HR 8799_5, major compensatory adjustments that may have occurred in another eight-planet system.

The linkage between the divisors from the Fibonacci-based Peirce approach, the *Phi*-series planetary framework and the Lucas series – all with respect to unity and the mean parameters of Earth – is also shown in Figure 2. Here the eight-planet Solar System from Saturn to Mercury is compared to a theoretical inward extension of HR 8799 (HR 8799_4 thru HR 8799_9) with the latter represented as a nine-planet system.



POSITIONS #	Solar System	*Phi*-series, ϕ^x	x	Luc.	HR 8799 B2/Div.	HR 8799_4_9	Div.
SATURN 3	29.4235194	29.03444185	7	29	29.9940716	HR 8799_4	15
Synodic 4-3	19.8588721	17.94427191	6	18	17.9964429	*Synodic* 5-4	25
JUPITER 4	11.8565250	11.09016994	5	11	11.2477768	HR 8799_5	40
Synodic 5-4	7.84767877	6.854101966	4	7	7.02986052	*Synodic* 6-5	64
(MJ-Gap) 5	4.72214968	4.236067977	3	4	4.32606802	HR 8799_6	104
Synodic 6-5	3.12552908	2.618033989	2	3	2.66219570	*Synodic* 7-6	169
MARS 6	1.88071105	1.618033989	1	1	1.64802591	HR 8799_7	273
EARTH/*Synodic* 7-6	0.91422728	1.000000000	0	(2)	1.02020652	*Synodic* 8-7	441
VENUS 7	0.61518257	0.618033989	-1	-	0.63012755	HR 8799_8	714
Synodic 8-7	0.39580075	0.381966011	-2	-	0.38919643	*Synodic* 9-8	1156
MERCURY 8	0.24084445	0.236067978	-3	-	0.24059416	HR 8799_9	1870

Fig. 2. HR 8799_4 to HR8799_9, extended Pierce resonances and the Solar System from Saturn to Mercury.

In the above configuration Solar System base B2 is 171.4442890 years, HR 8799 base B2 is 449.9110736 years and HR 8799 base B1 is 449.69199818 years. The similarity between the two Systems, the Lucas Series and lower *Phi*-series gives rise to the following relations involving the limiting Pierce reduction ratio ϕ^{-2} (0.38196601125) and reciprocal ϕ^{2} (2.61803398875), the outward *Pheidian* constant for the periods of revolution, thus:

Solar System, B2 = 171.4442890 years
Base B1, HR 8799•ϕ^{-2} = 171.8507380 years
Base B2, HR 8799•ϕ^{-2} = 171.7670564 years

ϕ^2• B2, Solar System = 448.8469772 years
Base B1, HR 8799 b = 449.6919918 years
Base B2, HR 8799 = 449.9110736 years

and the following comparison:

PLANETS N Synodics #	DIVISORS 1- 714	Sol. System II B2/Div. 1-714	HR 8799 B2/ϕ^2	LUCAS N	Phi-Series ϕ^x (Periods/years)	Exp. x
NEPTUNE 1	1	171.444290	171.767283		199.0050249	11
Synodic 2-1	1	171.444290	171.767283		122.9918694	10
URANUS 2	2	85.7221448	85.8836413		76.01315562	9
Synodic 3-2	4	42.8610724	42.9418207		46.97871376	8
SATURN 3	6	28.5740483	28.6278804		29.03444185	7
Synodic 4-3	9	19.0493655	19.0852536		17.94427191	6
JUPITER 4	15	11.4696193	11.4511522	11	11.09016994	5
Synodic 5-4	25	6.85777158	6.87069131	7	6.854101966	4
(MJ-Gap) 5	40	4.28610724	4.29418207	4	4.236067977	3
Synodic 6-5	64	2.67881702	2.68386379	3	2.618033988	2
MARS 6	104	1.64850278	1.65160849	(2)	1.618033988	1
Earth/*Syn* 7-6	169	1.01446325	1.01637445	1	1.000000000	0
VENUS 7	273	0.62800106	0.62918419	1	0.618033988	-1
Synodic 8-7	441	0.38876256	0.38949497	-	0.381966011	-2
MERCURY 8	714	0.24011805	0.24057042	-	0.236067977	-3

Table 5. Solar System and HR 8799 with emphasis on the *Lucas* & *Phi*-Series.

All of which is encouraging enough to precipitate a return to the Solar System and an investigation of these two fundamental constants and related variants with respect to real-time varying motions of the planets, commencing with those of Jupiter and Saturn followed by the terrestrial planets and the remaining two superior planets.

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John Nicholas Harris, M.A.
spirasolaris.ca