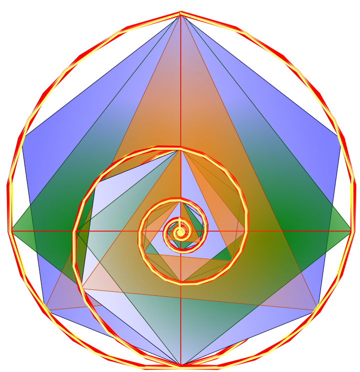
INTIMATIONS
OF
COMMONALITY
IN
PLANETARY
SYSTEMS



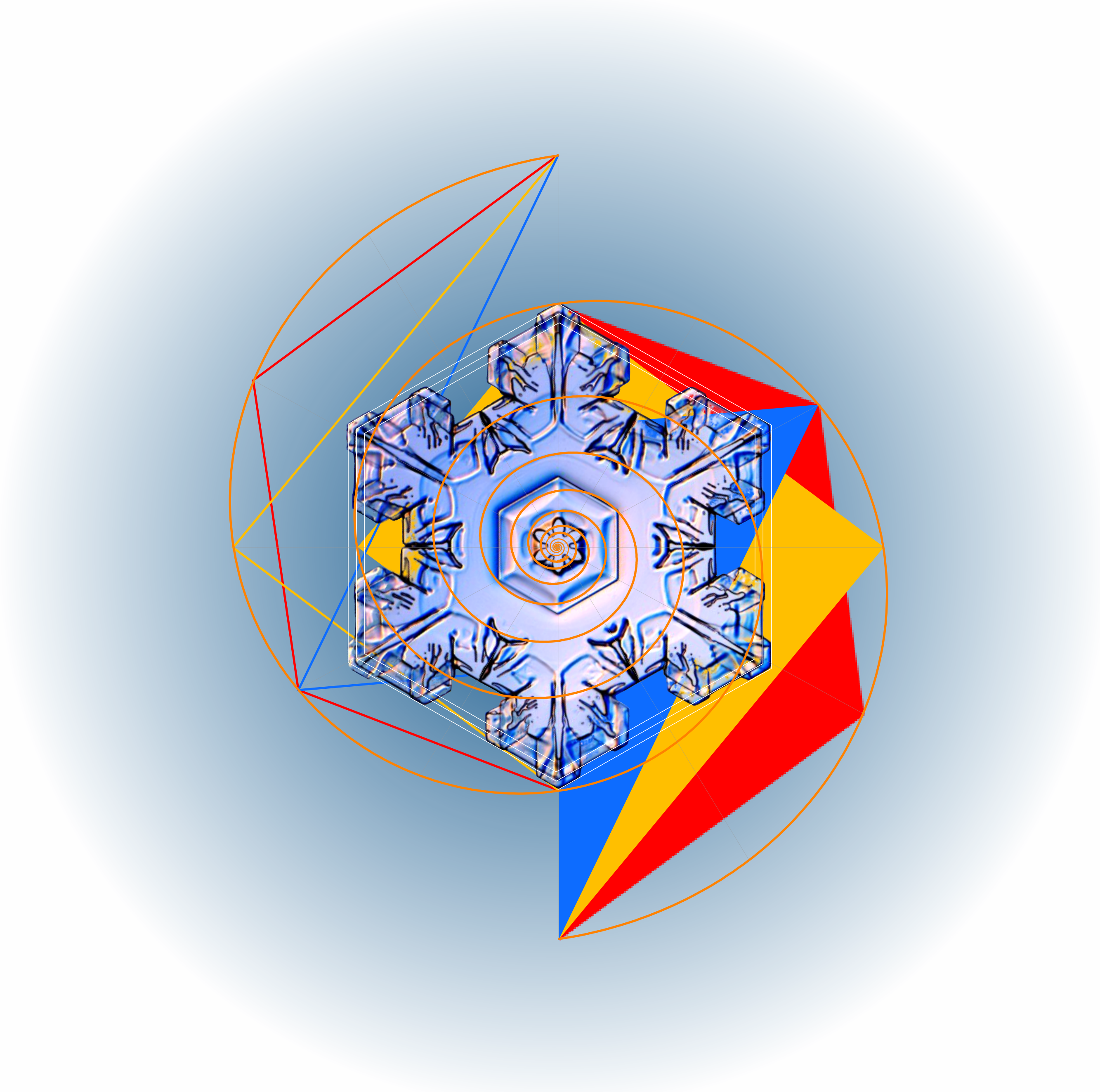
2023

PART ONE

THE PIERCE PLANETARY FRAMEWORK (1850)
REVISITED

JNH

1611[1] – *"A New Year's Gift of Hexagonal Snow"* – 2023[2]



[1] *"Strena Seu de Nive Sexangula."* (Johannes Kepler's 1611 treatise *The Six-Cornered Snowflake*)

[2] Spirasolaris.ca/2023-2024

Snow Crystal: http://www.its.caltech.edu/atomic/snowcrystals/photos2/w040123b025.jpg (*sans* background)

INTRODUCTION

In the 1850s American scientist Benjamin Peirce (1809–1880) produced a robust heliocentric planetary framework by applying Fibonacci-based reduction ratios to the mean periods of revolution of the eight Solar System planets.[1] Partially incomplete in dynamic terms and subjected to alternative viewpoints, this promising approach was oddly dismissed despite the attendant ramifications and total absence of any comparable planetary theory. Fortunately, however, a condensed version was at least preserved by Louis Agassiz in the latter's *Essay on Classification* (1859).[2]

As described in the latter work, Peirce began by assigning the outermost planet Neptune a convenient (albeit high) mean period of revolution of 62,000 days. Next, moving inwards, planetary periods rounded to the nearest day were derived from planet-to-planet reduction factors formed from Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,144, 233, etc.), specifically, successive *alternate Fibonacci ratios* of 1/2, 1/3, 2/5, 3/8, 5/13, 8/21, 13/34 and 21/55. Thus the 62,000-day period of Neptune was reduced by one-half to obtain a mean period of revolution for Uranus of 31,000 days followed by a one-third reduction of the latter to produce a 10,333-day period for Saturn, 4,133 days for Jupiter (2/5), and so on down to an 87-day period for the innermost planet Mercury from a final reduction ratio of 13/34. However, despite this encouraging end-to-end correspondence a reduction factor for Earth was entirely absent from the alternate Fibonacci sequences. In fact, the inclusion of the latter required two *additional* reduction ratios of 8/13 and 13/21. The last ratio in Pierce's original list (21/55) remained unused but was most likely included for continuity and support for the latter's contention that "There can be no planet planet exterior to Neptune, but there may be one interior to Mercury."[3]

The Fibonacci-based reduction ratios, resulting periods and comparison with 19th Century Solar System periods were published in the *Essay on Classification* in two sparse, unlabelled tables[4] based on subdivisions of the 62,000-day period for Neptune. The initial results are shown in Table 1a with title and column assignments added:

PLANETS (ca. 1850)	PERIODS actual/days	PERIODS (reductions)	RATIOS (Pierce)
Neptune,	60,129	62,000	
Uranus,	30,687	31,000	1/2
Saturn,	10,759	10,333	1/3
Jupiter,	4,333	4,133	2/5
Asteriods,	1,200 to 2,000	1,550	3/8
Mars,	687	596	5/13
Earth,	365	366	8/13 } 8/21
Venus,	225	227	13/21 } 8/21
Mercury,	88	87	13/34

Table1a. The initial planetary structure, Peirce (1852:129)

Next, the planetary framework was extended to include *twinned ratios* provided by *adjacent* Fibonacci numbers. This produced the same periods of revolution for the planets plus intermediate periods on either side with Earth in an intermediate location between Mars and Venus. Pierce included the intermediate positions for comparable 19th Century data in the fourth column, but apart from 365 days for Earth no other intermediate periods were given. The final ratios and reductions are shown in Table 1b, again with the title and column assignments added:

PLANETS —?—	RATIOS (Pierce)	PERIODS I (reductions)	PERIODS II (actual/days)
Neptune	1/1	62,000	60,129
"	1/1	62,000	——
Uranus	1/2	31,000	30,687
"	1/2	15,500	——
Saturn	2/3	10,333	10,759
"	2/3	6,889	——
Jupiter	3/5	4,133	4,333
"	3/5	2,480	——
Asteriods,	5/8	1,550	1,200
"	5/8	968	——
Mars	8/13	596	687
Earth	8/13	366	365
Venus	13/21	227	225
"	13/21	140	——
Mercury	21/34	87	88

Table1b. The Final planetary structure, Peirce (1852:129)

The final framework languished in this unfinished form despite correlations which included the Mars-Jupiter gap plus the possibility that planet Earth may, perhaps, be occupying an *intermediate* location. This troubling indicator should surely have been investigated, beginning, one might suggest, with mean synodic motion in general and mean synodic lap-cycles in particular.

Mean synodic motion and the intermediate periods

In fact, *all of the intermediate intervals* introduced by Pierce *are the mean synodic periods between adjacent planets*. In other words, lap-cycle times faster-moving inner planets require to complete 360° of direct orbital motion with respect to that of slower-moving outer planets. Adjacent or otherwise, mean synodic periods (*S*) between planets with mean periods of revolution T_1 and T_3 are derived from the lesser used *general synodic formula*:

$$\textit{Synodic period } S_2 = \frac{T_1 \bullet T_3}{T_1 - T_3} \;\; (T_1 > S_2 > T_3) \tag{1}$$

although in modern practice relation (1) is rarely applied in this form. Synodic periods in planetary tables normally pertain to either the lap-cycles of *Earth* with respect to the *slower* outer (superior) planets or the lap-cycles of the *faster* inner (inferior) planets with respect to *Earth* itself. In both cases, with the reference period of Earth exactly one year, redundant multiplications by unity are unstated, resulting in the standard synodic formulas:

$$\textit{Superior planets } S_s = \frac{T_s}{T_s - 1} \quad \text{(1s)} \qquad\qquad \textit{Inferior planets } S_i = \frac{T_i}{1 - T_i} \quad \text{(1i)}$$

Nevertheless, relation (1) is more useful in the present context, as is relation (2), where, with both the outer period T_1 and intermediate period *S* (= T_2) known, the innermost period T_3 can be obtained from:

$$\textit{Inner period } T_3 = \frac{T_1 \bullet T_2}{T_1 + T_2} \;\; (T_1 > T_2 > T_3) \tag{2}$$

Relation (1) permits the restoration of the missing intermediate periods in Table 1b, and allied with relation (2) plus period formulas employing geometric means - relations (4) and (4E) introduced later - all have roles to play in tests on external planetary systems that follow. More immediately, with missing synodic periods supplied and dynamic component incorporated, a standard planetary framework predicated on Peirce's Fibonacci-based approach can now be assembled as follows.

Units of time and measure

Standard years with respect to *unity* and also the Julian year of 365.25 days are applied in the present study, the first for comparison with modern periods in Julian years,[5] and the second for real-time calculations of planetary motion in Part Three utilising the methodology developed by Bretagnon and Simon (1986).[6]

Standard order, positions and titles

Following the order adopted by Pierce, the mean periods of revolution and the mean synodic intervals have been assigned standard position numbers and uniform titles commencing with the first and outermost planet. Thus for the eight-planet Solar System the relative *synodic* period (or lap-cycle) of Planet #2 (Uranus) with respect to that of outermost Planet #1 (Neptune) is *Synodic* 2-1 followed by *Synodic* 3-2 between Planet #3 (Saturn) and Planet #2 (Uranus), and so on, down to *Synodic* 8-7 between innermost planet Mercury (#8) and Planet #7 Venus. Planetary positions interior to Mercury (Intra-Mercurial-Objects, or IMOs) commence at IMO 1 followed by IMO 2, etc., with the intermediate synodic periods, Pierce reduction ratios and later divisors continuing inwards in due order. In this theoretical framework, Earth (with reservations) occupies the *Synodic* 7-6 location between #6 Mars and #7 Venus.

Divisors for the sequential periods of revolution and intermediate synodic intervals

Next, the awkward multiplications by successive reduction factors used by Pierce are replaced by a standard set of divisors applied to the base period alone, a practice already in use for exoplanets. Thus for the eight-planet Solar System the standard integer divisors for the periods of revolution of the planets beginning with the outermost (#1) are: **1, 2, 6, 15, 40, 104, 273, 714**. Divisors for the intermediate mean synodic periods (lap-cycles) are in turn: 1, 4, 9, 25, 64, 169, 441, thus the synodic divisors are all *sequential squares of the Fibonacci Series.*

The complete set of divisors with intermediate synodic divisors shown in brackets is therefore: **1,** (1) **2,** (4) **6,** (9) **15,** (25) **40,** (64) **104,** (169) **273,** (441) **714,** plus (1156) **1870,** (3025) and **4895** for ten-planet systems, etc.

Base periods B1, B2, B3, B4 and B5 for the divisors

Although the period of revolution of the outermost planet (base period B1) is of fundamental importance in Pierce's planetary model, the calculated value for *Synodic* 2-1 is in fact 62,620 days (hereafter base period B2) which exceeds the latter's initial base period of 62,000 days (hereafter, one-off base period P2). Nevertheless, when used as the base period for the divisors, *Synodic* 2-1 yields marginally superior results compared to those obtained with P2. Therefore, where *Synodic* 2-1 differs from B1 a second base period (B2) can be added for further testing. Other bases (B3s) can be approximated by applying the planetary divisors in reverse, *i.e.*, as multipliers of known periods with known locations in otherwise incomplete systems. Where advantageous, the mean value (B4) of multiple B3 products and/or a substitute B5 (yielding least errors) may also be applied at the expense of further complexity.

Resonant triples between planets [RZT]

Resonant triples between planets are included for completeness in Solar System Table 2 and elsewhere. Related to both the twinned Pierce ratios and added divisors, resonant triples are obtained from the bracketing periods of revolution of adjacent planets and the synodic periods in between. Thus, for Neptune and Uranus [1(1)2], Uranus and Saturn [1(2)3], Saturn and Jupiter [2(3)5], etc. Their immediate relevance lies in the fact that the associated divisors are sequential Fibonacci multiples with the central value of each triple providing the multiplication factor. - 1x for the first set: [1(1)2], 2x for the second, thus [2(4)6], 3x for the third [6(9)15], 5x for the fourth [15(25)40], etc.

Fibonacci Periods in days below Mercury

The resulting Pierce P2 planetary framework for a thirteen-planet extension of the Solar System is shown in Table 2a with intermediate positions for the synodic periods and division of modern periods (Base B2/Divisors) included for comparison. The paired resonances from Unity to the Major Sixes (the reverse of the Pierce *reduction* ratios) aid the analyses of exoplanetary systems in Part Two while also bringing to mind ancient methodology, *e.g.*, "Music of the Spheres," which, though not music *per se,* nevertheless appears to have a role in this complex matter. As does the presence of the Fibonacci series below Mercury expressed in days generated by the P2 and the B2 divisors also included in the Table.

PLANETS Synodic	N #	RATIOS (Pierce)	DIVISORS (added)	RESONANCES (to Major 6's)[a]	RES.TRIPLES (to IMO 5)	PERIODS1 P2/Divisors	PERIODS1 T (JYR = 365.25)	DISTANCES1 R (Ref. unity/a.u)	PERIODS2 B2/Divisors	
Neptune	1	1/1	1	1 : 1	1(1)2	62,000	169.74675	30.657329		62,620
Synodic	2-1	1/1	1	1 : 1		62,000	169.74675	30.657329		62,620
Uranus	2	1/2	2	Octave #1, 2 : 1	1(2)3	31,000	84.873374	19.312907		31,310
Synodic	3-2	1/2	4	Octave #2, 4 : 2		15,500	42.436687	12.166369		15,655
Saturn	3	2/3	6	Fifth #1 , 6 : 4	2(3)5	10,333	28.291125	9.2846772		10,437
Synodic	4-3	2/3	9	Fifth #2, 9 : 6		6,889	18.860750	7.0855348		6,958
Jupiter	4	3/5	15	Major 6 #1, 15 : 9	3(5)8	4,133	11.316450	5.0404993		4,175
Synodic	5-4	3/5	25	Major 6 #2, 25 : 15		2,480	6.7898700	3.5857029		2,505
M-J Gap	5	5/8	40		5(8)13	1,550	4.2436687	2.6211647		1,566
Synodic	6-5	5/8	64			986	2.6522930	1.9160830	Fibonacci [b]	978
Mars	6	8/13	104		8(13)21	596	1.6480267	1.3952204	610	602
Earth/*Syn*	7-6	8/13	169			366	1.0044186	1.0029436	377	371
Venus	7	13/21	273		13(21)34	227	0.6217830	0.7284938	233	229
Synodic	8-7	13/21	441			140	0.3849133	0.5291457	144	142
Mercury	8	21/34	714		21(34)55	87	0.2377405	0.3837681	89	88
Synodic	9-8	(21/34)	1156			54	0.1468397	0.2783315	55	54
IMO 1	9	(34/55)	1870		34(55)89	33	0.0907737	0.2019792	34	33
Synodic	10-9	(34/55)	3025			20	0.0561146	0.1465719	21	21
IMO 2	10	(55/89)	4895		55(89)144	13	0.0346776	0.1063406	13	13
Synodic	11-10	(55/89)	7921			8	0.0214300	0.0771521	8	8
IMO 3	11	(89/144)	12816		89(144)233	5	0.0132449	0.0559800	5	5
Synodic	12-11	(89/144)	20736			3	0.0081861	0.0406179	3	3
IMO 4	12	(144/233)	33552		144(233)377	2	0.0050592	0.0294706	2	2
Synodic	13-12	(144/233)	54289			1	0.0031267	0.0213826	1	1
IMO 5	13	(233/377)	87841			1	0.0019324	0.0155144	1	1

Table2a. The enhanced planetary structure: ratios, divisors, triples, periods in days & years; P2 distances (a.u.).
[a] Octave 2 : 1, Fifth 3 : 2, Major Six 5 : 3. [b] The extension to Planet 13 concludes at Fibonacci number 1.

The Solar System revisited

Table 2b shows the uniform assignments, the twinned Pierce ratios, added divisors, resonant triples and the results generated by Pierce base P2, followed by modern base periods B2 and B1 with the latter in both Julian years and days. Also included with the two sets of data are the calculated synodic periods, Mars-Jupiter geometric mean between the periods of the latter pair, associated synodic positions on either side, and Earth located in the synodic position between Venus and Mars. The 365.25 day period for Earth is substituted in the second set of modern data although the actual synodic period (*Synodic* 7-6) is 335 days, thus less than one year and 366-day period obtained from the P2 ratio and 62,000-day base period. Atypical Venus-Earth and Earth-Mars synodic periods and the Mars-Jupiter synodic cycle are omitted for clarity. The periods in days are rounded; red periods equal exact Fibonacci numbers.

PLANETS Synodics	N #	RATIOS (Pierce)	DIVISOR (added)	RES.TRIPLE [(RZT)]	PERIODS1 P2/Divisors	PERIODS2 Actual/days	MODERN1 B2/Divisors	MODERN2 B1Julian yrs	MODERN1 (Days)	MODERN2 (Days)
Neptune	1	1/1	1		62,000	60,129	171.44429	163.72320	62,620	59,800
Synodic	2-1	1/1	1	1(1)2	62,000	62,672	171.44429	171.44429	62,620	62,620
Uranus	2	1/2	2		31,000	30,687	85.722145	83.747407	31,310	30,589
Synodic	3-2	1/2	4	1(2)3	15,500	16,658	42.861072	45.360219	15,655	16,568
Saturn	3	2/3	6		10,333	10,759	28.574048	29.423519	10,437	10,747
Synodic	4-3	2/3	9	2(3)5	6,889	7,255	19.049366	19.858872	6,958	7,253
Jupiter	4	3/5	15		4,133	4,333	11.429619	11.856525	4,175	4,331
Synodic	5-4	3/5	25	3(5)8	2,480	2,867	6.8577717	7.8476788	2,505	2,866
M-J Gap	5	5/8	40		1,550	1,725	4.2861072	4.7221497	1,556	1,725
Synodic	6-5	5/8	64	5(8)13	986	1,142	2.6788170	3.1255291	978	1,142
Mars	6	8/13	104		596	687	1.6485028	1.8807111	602	687
Earth/*Syn*	7-6	8/13	169	8(13)21	366	335	1.0144633	1.0000000	371	365
Venus	7	13/21	273		227	225	0.6280011	0.6151826	229	225
Synodic	8-7	13/21	441	13(21)34	140	145	0.3887626	0.3958008	142	145
Mercury	8	21/34	714		87	88	0.2401186	0.2408445	88	88
Synodic	9-8	21/34	1,156	21(34)55	54	55	54.169573	54.689759	54	55
IMO 1	9	34/55	1,870		33	34	33.486645	33.723773	33	34
Synodic	10-9	34/55	3,025	34(55)89	20	21	20.700835	20.860438	21	21
IMO 2	10	55/89	4,895		13	13	12.792651	12.888208	13	13
Synodic	11-10	55/89	7,921	55(89)144	8	8	7.9055709	7.9663542	8	8
IMO 3	11	89/144	12,816		5	5	4.8860820	4.9232407	5	5
Synodic	12-11	89/144	20,736	89(144)233	3	3	3.0198701	3.0427860	3	3
IMO 4	12	144/233	33,552		2	2	1.8663570	1.8805320	2	2
Synodic	13-12	144/233	54,289	144(233)377	1	1	1.1534570	1.1622358	1	1
IMO 5	13	233/377	87,841		1	1	0.7128793	0.7183005	1	1

Table 2b. The complete framework and the Solar System. Positions, ratios, divisors and Base periods P2, B1, B2.

Solar System Periods, Pierce Ratios and Divisors below Mercury

Originally the inner region was limited to *Synodic* 9-8 and Planet 9 (IMO 1) to accommodate Pierce's unused inner reduction ratio of 21/55. Accordingly, relation (2) was applied twice, firstly to the mean periods of *Synodic* 8-7 and Mercury resulting in 54.689759 days for *Synodic* 9-8, and then once again to the latter period and that of Mercury to obtain 33.723773 days for Planet #9. However, the last two rounded periods are clearly sequential Fibonacci numbers 55 and 34, an occurrence that allied with the previous sequential pair of periods (145 and 88 days versus Fibonacci 144 and 89) provided the impetus to extend the range as far as Planet 13 (IMO 5) in Tables 2a and 2b.

Regarding the present location of Earth *near* the Mars - Venus *synodic* position, the calculated synodic period, *i.e.*, *Synodic* 7-6 = 335 days represents an enigma since it is neither 366-days as required by the divisors, nor it is close to the actual 365.25 days (Julian) and other variants for the year. Although perhaps masked by a possible outward shift by Mars, this still does little to explain the obvious Fibonacci/*Phi* ratio exhibited by the Venus-Earth periods of revolution expressed in years. In more detail, using modern values for these two adjacent planets the mean periods are 0.61518257 : 1, whereas the reciprocals of *Phi* and Earth (Unity) are 0.61803398875 : 1. Furthermore, there is also the well-known 5 : 8 ratio between the two planets and associated 5 : 8 : 13 *Fibonacci* resonant triple, *i.e.,* 5 synodic periods of Venus in 8 years with 13 corresponding periods of revolution for this planet. All of which, in addition to the above Fibonacci data from Mercury through IMO5, leads logically enough to the following major expansions.

The Pierce planetary framework, the *Phi*-series, and the structure of the Solar System
It is abundantly clear from Table 1b that the final Pierce reduction ratios are successive twinned members of the Fibonacci series, albeit one position removed between the numerators and denominators. Nevertheless, despite the title of Pierce's original publication[1] and obvious nature of the ratios applied by the latter, the Fibonacci series and related Golden Ratio *Phi* (ϕ) :

$$Phi\ (\phi) = \sqrt{(5/4)} + \tfrac{1}{2} = 1.618033988749895 \qquad (3)$$

– are nowhere stressed by Peirce or Agassiz, although this constant clearly plays a major role in the proposed model. This is all the more apparent when it is recalled that the golden *section* is defined as the division of a line such that the proportion of the smaller section to the larger is identical to the proportion of the larger section to the whole. Whereas the golden *ratio* can be defined as the limiting value of the ratios of adjacent Fibonacci numbers. It is also clear in the present astronomical context that moving inwards, the limiting value of the inverse alternate Fibonacci ratios applied by Peirce will be ϕ^{-2} (0.38196601125) with reciprocal limit the outward multiplier ϕ^{2} (2.61803398875). Furthermore, after the inclusion of the ratios for the intermediate periods between planets the limiting value is ϕ^{-1} (0.61803398875) with a reciprocal limit and a corresponding multiplier of ϕ^{1} (1.61803398875), which is *Phi* itself.

***The Phi*-series in astronomical context** *(Periods T, S years, Distance R, Velocity Vi and Vr relative to unity)*
As it so happens, apart from filling the intermediate gaps introduced by Pierce, relation (1) – the general synodic formula – is already present with one central exception among the four constants just mentioned, *i.e.*, ϕ^{-2}, ϕ^{-1}, ϕ^{1} and ϕ^{2}. In short, combined with the calibration and the unification provided by the mean period of Earth (ϕ^{0} = 1 year) the latter become sequential mean periods *in years* generated by the *Phi*-series ϕ^{x} for successive integer exponents x = -2, -1, 0, 1, 2 in the present context. Moreover, with the addition of the next lower integer and also continued outward extensions, integer exponents -3 through 7 generate a complete planetary framework from Mercury to Saturn with all synodic periods included. Beyond this outer region correlation with the solar system parameters begins to diminish, but nevertheless, for the stipulated range the inter-related parameters are as shown in Table 3. Here, following ancient practice it is helpful to include the *inverse velocity* Vi, *e.g.*, $Vi^{2} = R$, $Vi^{3} = T$, $Vi^{-1} = Vr$ (best remembered by the Triple interval [$3^0 3^1 3^2 3^3$ = 1, 3, 9, 27] [7] which also pertains to Saturn at perihelion) with the frame of reference (unity) provided by the mean heliocentric distance (*R*) in a.u, mean period of revolution (*T*) in years and mean orbital velocity (*Vr*) of Earth. Thus in the same sense [1,1,1,1], hence the assignment of the cube to planet Earth and *tetradic* point-line-area-volume analogy applied to planetary motion. The last three modern periods in days (in red) owe their origins to relation (2) and a 33.0225-day period for IMO 1 by Leverrier (1875).[8]

PLANETS N Synodics #	MODERN *T* (Julian Years)	ϕ^{x} = x	*Phi*-series *T* (Years)	*Phi*-series (*R*) Distance (a.u.)	*Phi*-series (*Vi*) Inverse Velocity	*Phi*-series (*Vr*) Velocity (Ref.1)	MODERN *T* (Days)
Saturn 3	29.4235194	7	29.03444185	9.446602789	3.073532624	0.325358512	10746.9404
Synodic 4-3	19.8588721	6	17.94427191	6.854101966	2.618033989	0.381966011	7253.45303
Jupiter 4	11.8565250	5	11.09016994	4.973080251	2.230040414	0.448422366	4330.59576
Synodic 5-4	7.84767877	4	6.854101966	3.608281187	1.899547627	0.526441130	2866.36470
M-J Gap 5	4.72214968	3	4.236067977	2.618033989	1.618033989	0.618033989	1724.76517
Synodic 6-5	3.12552908	2	2.618033989	1.899547627	1.378240772	0.725562630	1141.59949
Mars 6	1.88071105	1	1.618033989	1.378240772	1.173984997	0.851799642	686.929711
Earth/*Syn* 7-6	0.91422728	0	1.000000000	1.000000000	1.000000000	1.000000000	365.25(JYR)
Venus 7	0.61518257	-1	0.618033989	0.725562630	0.851799642	1.173984997	224.695433
Synodic 8-7	0.39580075	-2	0.381966011	0.526441130	0.725562630	1.378240772	144.566223
Mercury 8	0.24084445	-3	0.236067978	0.381966011	0.618033989	1.618033989	87.9684354
Synodic 9-8	0.14474748	-4	0.145898034	0.277140264	0.526441130	1.899547626	54.6897591
IMO 1 9	0.09041068	-5	0.076806725	0.201082619	0.448422366	2.230040414	33.0225000
Synodic 10-9	(0.0556507)	-6	0.055728090	0.145898034	0.381966011	2.618033989	20.3264209
IMO2 10	(0.0344447)	-7	0.040434219	0.105858161	0.325358512	3.073532624	12.5818709

Table 3. Modern periods *T, S*, *Phi*-series, exponents (x), *T, R*, Velocity *Vi* (Inverse) and *Vr* (relative to unity).

Returning to the present, notwithstanding the Mars-Jupiter Gap and anomalous location of Earth *between* Mars and Venus, the *Phi*-series planetary framework outlined above includes the following properties and relations:

Heliocentric properties of the *Phi*-series with respect to unity in the Solar System

1. For any three successive *Phi*-series periods, the middle period is the *product* of the periods on either side *divided* by their *difference*. Thus, in the same astronomical context, the general synodic formula, relation (1)

2. If two upper adjacent *Phi*-series periods are known, the third and lower period can be obtained from the *product of the two adjacent periods divided by their sum*. Thus (in addition to relation 1), synodic relation (2)

3. The underlying constant of the *Phi*-series planetary model is ***Phi*** ($\phi^1 = ½ \sqrt{5} + ½ = 1.618033988749895$), the limiting value of successive ratios of the Fibonacci series: 1,1,2,3,5,8,13,21,34,55,89,144,233,377, 610,... (3)

4. For any three successive *Phi*-series periods, the middle period is the *geometric mean* of the two periods on either side, as are the means from positions ± 2, ± 3, etc. Extended geometric means, relations (4), (4E) & (4F)

5. For every *Phi*-series period *except that of Earth* there exists a corresponding Lucas series integer period (_ 3, 4, 7,11,18,29,47,76,123,199,... years) generated by the *alternating Phi*-Lucas relation: $(T, S) = |\phi^x \pm \phi^{-x}|$ Periods of revolution: $T = |\phi^x - \phi^{-x}|$. Intermediate Synodic Periods: $S = |\phi^x + \phi^{-x}|$ *Phi*-Lucas relation (5)

6. Pertaining *to planet EARTH*, the product of the parameters of the planets on either side (Mars and Venus) is *UNITY*, as are *all such Phi-series products*, *i.e.*, periods ± 2, ± 3, etc., both inwards and outwards. Relation (6E)

The limiting *Phi*-series constants in the present astronomical context are:

A: PLANETS: Mean sidereal periods of revolution, mean heliocentric distances, mean orbital velocities:

Phi-series mean *periods* of revolution (*T*) decrease	ϕ^{-2}	(0.38196601125), Inwards (the Pierce limit)	(7)
Phi-series mean *periods* of revolution (*T*) increase	ϕ^{2}	(2.61803398875), Outwards	(8)
Phi-series mean heliocentric *distance* (*R*) increase	$\phi^{4/3}$	(1.89954762695), Planets, Outwards	(9)
Phi-series *Planet-to-Planet Velocities* (*Vr*) decrease	$\phi^{-2/3}$	(0.725562630246), Planets, Outwards	(10)

B: SYNODICS: Mean synodic periods, corresponding heliocentric "distances," mean "orbital" velocities:

Phi-series mean *synodic* (*S*) to *Planet* (*T*) decrease	ϕ^{-1}	(0.61803398875), Inwards	(11)
Phi-series mean *synodic* (*S*) to *Planet* (*T*) increase	ϕ^{1}	(1.61803398875), Outwards	(12)
Phi-series mean *synodic* (*R*) to *Planet* (*R*) increase	$\phi^{2/3}$	(1.37824077249), Outwards	(13)
Phi-series mean *synodic* (*Vi*) to *Planet* (*Vi*) increase	$\phi^{1/3}$	(1.17398499671), Outwards	(14)
Phi-series mean *synodic* (*Vr*) to *Planet* (*Vr*) decrease	$\phi^{-1/3}$	(0.85179964208), Outwards	(15)

C: GENERATION:

The *mean periods* of revolution (*T*) and the *mean synodic periods* (*S*) in years from Mercury to Neptune are generated by the *Phi*-Series (ϕ^x) and integer exponents x = -3,-2,-1,0,1,2,3,4,5,6,7,8,9,10 and 11. (16)

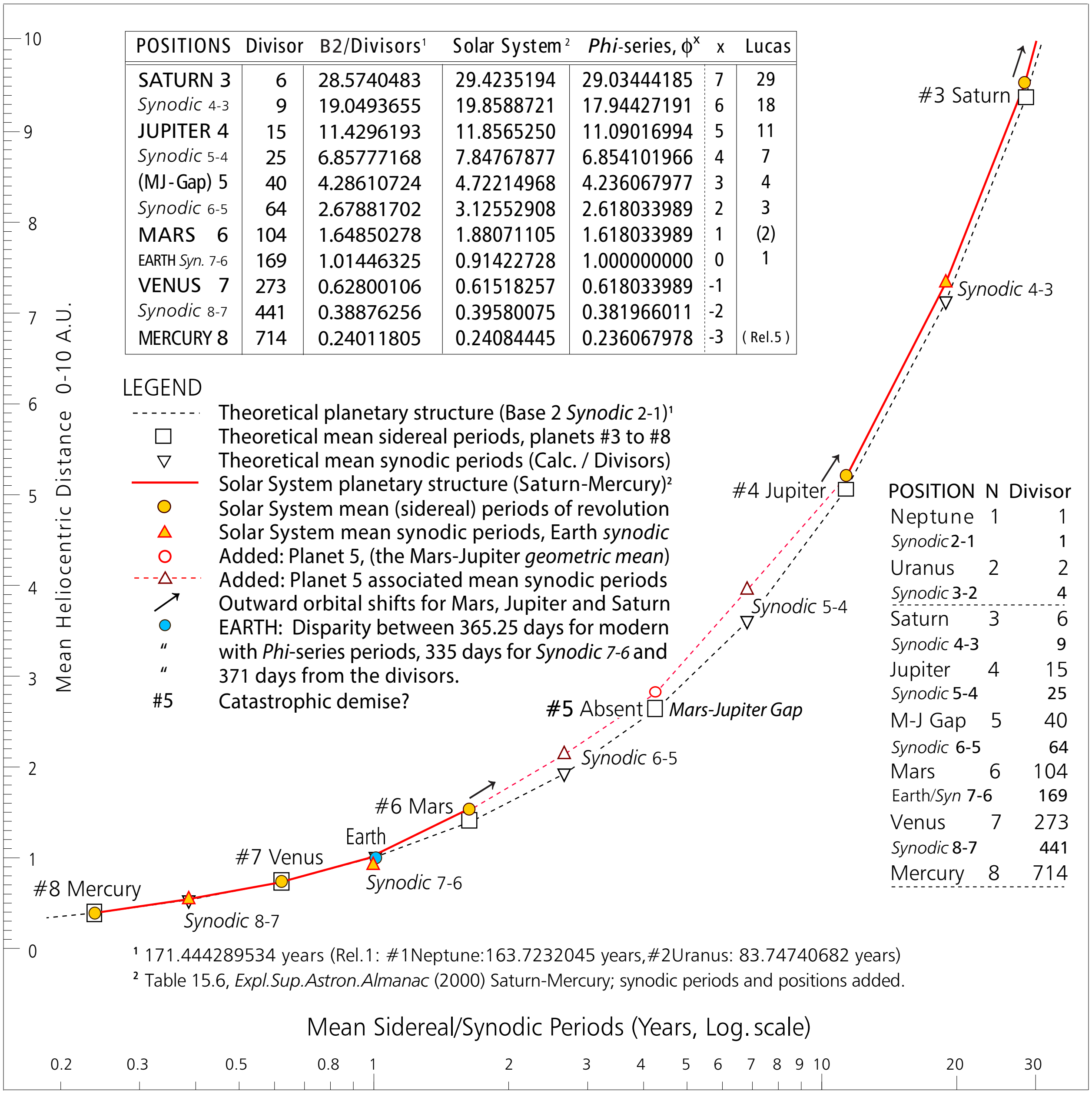
The mean *periods of revolution* are generated by *ODD* exponents, *mean synodic periods* by the *EVEN*. (17)

D: OVERALL PLANETARY FRAMEWORK with increasing departures beyond Saturn (periods *T*, *S* in years). Period divisors, modern values, exponents, Lucas series and *Phi*-series framework are shown in Table 3s. *n.b.*, the *Phi*-series also includes each key *Pheidian* constant as a mean period (*T*,*S*), a mean distance (*R*) and both velocities (*Vr* & *Vi*) with the latter (0.381966011) at *Synodic* 10-9) not shown. (18)

DIV. (syn)	PLANETS N Synodics #	MODERN *T,S* (Julian years)	ϕ^x exp.	*LUCAS* (years)	*Phi*-series *T,S* (ϕ^x = years)	*Phi*-series (*R*) Distance (a.u.)	*Phi*-series (*Vr*) Velocity (ref.1)	*Phi*-series (*Vi*) Velocity (Inv.)
1	Neptune 1	163.7232045	11	199	199.0050294	34.08599912	0.171282103	5.838321602
(1)	*Synodic* 2-1	171.4442895	10	123	122.9918694	24.73152718	0.201082619	4.973080251
2	Uranus 2	83.7474068	9	76	76.01315562	17.94427191	0.236067977	4.236067978
(4)	*Synodic* 3-2	45.3598213	8	47	46.97871376	13.01969312	0.277140264	3.608281187
6	Saturn 3	29.4235194	7	29	29.03444185	9.446602789	0.325358512	3.073532624
(9)	*Synodic* 4-3	19.8588721	6	18	17.94427191	6.854101966	0.381966011	2.618033989
15	Jupiter 4	11.8565250	5	11	11.09016994	4.973080251	0.448422366	2.230040414
(25)	*Synodic* 5-4	7.84767877	4	7	6.854101966	3.608281187	0.526441130	1.899547627
40	M-J Gap 5	4.72214968	3	4	4.236067977	2.618033989	0.618033989	1.618033989
(64)	*Synodic* 6-5	3.12552908	2	3	2.618033989	1.899547627	0.725562630	1.378240772
104	Mars 6	1.88071105	1	(2)	1.618033989	1.378240772	0.851799642	1.173984997
(169)	Earth/*Syn* 7-6	0.91422728	0	1	1.000000000	1.000000000	1.000000000	1.000000000
273	Venus 7	0.61518257	-1	-	0.618033989	0.725562630	1.173984997	0.851799642
(441)	*Synodic* 8-7	0.39580075	-2	-	0.381966011	0.526441130	1.378240772	0.725562630
714	Mercury 8	0.24084445	-3	(Rel. 5)	0.236067978	0.381966011	1.618033989	0.618033989

Table 3s. Divisors, modern periods (*T* & *S*), *Phi*-series (x) *T, S*, Lucas *T, S*, *Phi*-series *R*, *Velocity Vr* & *Inverse Vi*.

Disparities in the modern Solar System from Mercury through Saturn with emphasis on the Pierce Divisors and insights from the above are shown in Figure 1.



POSITIONS	Divisor	B2/Divisors¹	Solar System²	*Phi*-series, ϕ^x	x	Lucas
SATURN 3	6	28.5740483	29.4235194	29.03444185	7	29
Synodic 4-3	9	19.0493655	19.8588721	17.94427191	6	18
JUPITER 4	15	11.4296193	11.8565250	11.09016994	5	11
Synodic 5-4	25	6.85777168	7.84767877	6.854101966	4	7
(MJ-Gap) 5	40	4.28610724	4.72214968	4.236067977	3	4
Synodic 6-5	64	2.67881702	3.12552908	2.618033989	2	3
MARS 6	104	1.64850278	1.88071105	1.618033989	1	(2)
EARTH *Syn.* 7-6	169	1.01446325	0.91422728	1.000000000	0	1
VENUS 7	273	0.62800106	0.61518257	0.618033989	-1	
Synodic 8-7	441	0.38876256	0.39580075	0.381966011	-2	
MERCURY 8	714	0.24011805	0.24084445	0.236067978	-3	(Rel.5)

POSITION	N	Divisor
Neptune	1	1
Synodic 2-1		1
Uranus	2	2
Synodic 3-2		4
Saturn	3	6
Synodic 4-3		9
Jupiter	4	15
Synodic 5-4		25
M-J Gap	5	40
Synodic 6-5		64
Mars	6	104
Earth/*Syn* 7-6		169
Venus	7	273
Synodic 8-7		441
Mercury	8	714

Fig. 1. The Pierce planetary framework, Solar System Mercury–Saturn, Mars-Jupiter Gap and the location of Earth.

Similarities and Disparities

Applied to the present Solar System, the *Phi*-series from x = -3 to 7 yields a planetary framework which includes all the intermediate (synodic) periods from Mercury through Saturn plus periods for theoretical planet #5 and both adjacent intermediate synodic intervals. Beyond Saturn correlation diminishes with distance, while the ratios of the of the integral Lucas series increasingly approach *Phi* itself. Whereas, moving inwards, ratios of *the period divisors* also begin to approach the same fundamental constant. Nevertheless, two identical disparities in the Solar System are indicated by all three sequences: (1) the absence of a planet between Jupiter and Mars, and (2), the unexpected presence of a planet between adjacent Mars and Venus, namely Earth. Moreover, in addition to this location there is also a marked difference between the calculated intermediate period for Earth of 335 days and the 365-day year.

In so much as Venus and Earth have the lowest eccentricities among the planets and their periods of revolution are also closest to their *Phi*-series equivalents - with *zero* error for Earth - the position of the latter can be examined in terms of residual effects of the *Phi*-series with relation 6E a possible factor. This, however, is difficult to investigate because of the missing periods between Mars and Jupiter, and also the accepted absence of planets below Mercury.

About the only option remaining pertains to the periods of Jupiter and IMO 1, *i.e.,* the periods corresponding to *Phi*-series exponents +5 and -5 which yield *a product* of exactly 1 year. Whereas in the Solar System the mean period of Jupiter of 11.85652502 Julian years and that of IMO 1 (0.09035592 years) yields a product of 1.071307238, with the replacement of IMO 1 by the mean synodic month resulting in 0.9586044 years. *Unity* does, however result from a period of 30.8058220 days from *the reciprocal* of Jupiter's mean period, a concept which owes its origins to Friberg's approach to AO 6484, a Babylonian mathematical text concerned with the number 0;59,15,33,20 and its reciprocal 1; 00,45.[9] The product is necessarily *unity* with a *sum* of 2; 0,0,33,20 and 1; 0,0,16,40 for the half.[9] Which, albeit radical shifts in both time and place, can be considered in terms of *elliptical parameters for the orbit of Earth*. This is an unexpected bi-product of a reappraisal of the 1964 analyses by A. Aaboe[10] of a possible *daily increment* of 0; 0,1,32,42,13,20^{o} (0.000480109739369) for the velocity of the "Sun" in BM 37089, a Babylonian lunar fragment.

The relevance of the latter is that the value 0;59,15,33,20$^{O/Day}$ can be shown to be inherent in data in Aaboe's study which corresponds to a period of exactly 364.5 years. This value is shown below in the last column of Table 1A from an expansion of Aaboe's analyses incorporating a Babylonian System B *varying velocity* function for planet *Earth:*

> ". . . Although Aaboe surmises that the original table may have supplied daily longitudes for a complete year, he gives a partial restoration since neither the maximum nor the minimum values are present. It is, however, sufficient to give Aaboe's daily longitudes and differences for lines 5 through - 5 plus added corresponding lengths of the year in days to show that line 0 is the closest to the Sidereal year:

Line #	Col. II (Longitude^{o})	Δ Col. II (Daily velocity^{o})	*T* (years, added)
Line -5	[8;51,51,51,6,40]	0;59,17,17,2,13,20	364.32289859
Line -4	[9;51,6,40]	0;59,15,33,20	364.5
Line -3	[10;50,20,29,37,46,40]	0;59,13,49,37,46,40	364.67727367
Line -2	[11;49,32,35,33,20]	0;59,12,5,55,33,20	364.85471987
Line -1	[12;48,42,57,46,40]	0;59,10,22,13,20	365.03233883
Line 0	[13;47,51,36,17,46,40]	0;59,8,38,31,6,40	365.21013081
Line 1	[14];46,58,[31,6,40]	0;59,6,54,48,53,20	365.38809607
Line 2	15;46,[3, 42,13,20]	0;59,5,11,6,40	365.56623485
Line 3	16;45,7,[9,37,46,40]	0;59,3,27,24,26,40	365.74454742
Line 4	17;44,8,[53,20]	0;59,1,43,42,13,20	365.92303402
Line 5	18;43,[8,53,20]	0;59.	366.10169492

Table 1A. Daily solar positions and velocities with periods *T* added to Aaboe (1964:32).

> This demonstrates that from a modern perspective the mean daily velocity from line 0 of 0; 59, 8,38,31, 6,40$^{O/d}$ and the 365.21013081-day year are optimum for (*u*) and (*T*) respectively. But not quite. In order to restore the longitudes and velocities for the entire table, the period *T* turns out to be exactly 364 days. Thereafter, with (*d*) given, (*u*) from Line 0, and *T = 364* days, relation (X) is reduced to: (*M, m*) = 0;59,8,38,31,6,40 ± 0;2,37,17,2,13,20 which produces the following six-sexagesimal place values for the apsidal velocities and the daily velocities in between.
>
> Minimum daily velocity (*m*) = 0;56,31,21,28,53,20$^{O/day}$ (abbrev. 0;56,30)
> Mean daily velocity (*u*) = 0;59, 8,38,31, 6, 40$^{O/day}$ (abbrev. 0;59, 9)
> Maximum daily velocity (*M*) = 1; 1,45,55,33,20,00$^{O/day}$ (abbrev. 1; 1,46)
> eccentricity (*e*) = 0.0295589.
>
> The occurrence of 0;59^{o} in line 5 of column 3 suggests choice rather than coincidence and there are other matters of interest in addition." [Excerpt from "Aaboe64 Revisited"].

The above dialogue concludes with an associated ellipse and additional variants which are beyond the scope of the present study. Except to note that Friberg's analysis mentioned earlier is accompanied by a two-part figure for the Babylonian mathematical procedure known as "Completing the Square." The latter, however, in consort with the calculation of the heliocentric distances *R* (by a procedure provisionally named here "Completing the Cube" inherent in Old Babylonian mathematical text VAT 8547) suggests that these procedures ultimately concern the derivation of the parameters of *ellipses* for Earth and the major superior *Planets*. In so much as the eccentricities (*e*) are small (*e.g.*, that of Earth is 0.01670862) the orbits appear to be *almost* circular, which provides an impetus to revisit Babylonian mathematical texts with accompanying "circles" and non-integer numerical values *close to unity or 2*. Thus possible semi-major (*a*) and major axes (2*a*) for Earth/*Sun ellipses, e.g.,* although conceivably with alternate meanings:

> "Fig. 3. 1. 12. MS 3050. An OB round hand tablet with square inscribed in a circle." Friberg (2005:135).[11]
> "Fig. 16. 7. 3. *UET*/67 2 222 *rev*. A square side algorithm using elimination of square factors." Friberg (2006:401) [12]
> "Fig. 16. 7. 4. 1st. Si. 428. Computation of the square side 2;02,02,02,05,05,04." Friberg (2006 :403).[13]

where the first figure appears to be a rough *rectangl*e with diagonals inscribed in an equally rough *ellipse*.

Seeking further enlightenment the inquiry leads to Babylonian planetary and luni-solar parameters, but before this it is necessary to caution the casual reader about prevailing nihilistic views concerning Babylonian astronomy, especially ill-founded claims that the Babylonians had neither a fictive approach to orbital motion nor any planetary model whatsoever. Long overdue additional research shows that nothing could be further from the truth.

But before proceeding, the notation, conventions and additional data in this context are introduced for those who may be unfamiliar with this relatively obscure material, along with standard definitions of astronomical terms, and in particular, luni-solar and planetary parameters in both modern and Babylonian contexts.

Sexagesimal notation, Units, Time, and Motion

Sexagesimal numbers 1 to 59 are separated by commas with equivalent decimal place locations indicated by semi-colons, thus in addition to hours; minutes and seconds, the thirds, fourths, fifths, sixths, sevenths, etc. For example, the Old Babylonian estimate for the square root of 2 rounded at the *third* place is 1;24,51,10[13] with the *exact* value for the Babylonian mean synodic arc of Saturn[13] 12;39,22,30° (12.65625°) with a corresponding mean synodic time of 1;2,6,33,45 years (1.03515625 versus the modern mean synodic period for this planet of 1.035182135… years). Days, degrees, months and "tithis" (thirtieths) are denoted by the superscripts n^d, n^o, n^m, n^r with the predominant Babylonian mean synodic month (MSM) of 29;31,50,8,20 ^{d} (29.5305941358...^{d}) represented by superscript ^{M}.

Next, expanded later, definitions and tools for the present study include the following luni-solar constants:

(1) DAY: *Daily axial rotation* and *daily sidereal motion of Earth* with subdivisions of the 24-hour day for time & angular motion which far exceed modern usage, extending from 360° per day through Large Hours (30°), Hours (15°), Minutes and Seconds, etc., down to 50 seconds of arc (0;00,50°).

(2) MONTH: MEAN SYNODIC MONTH of 29;31,50,8,20 days = 29.5305941358^d with last base-60 pair rounded for convenience. Even so it is still quite accurate; the modern estimate is 29;31,50,*7*,30 days.

(3) YEAR: SIDEREAL YEAR of 12;22,8 Mean synodic months = 365;15,38,17,44,26,40 days (365.2606376886). Although the latter is high compared to the modern estimate of 365.2564^d it is almost certainly selected for convenience. A better estimate for the sidereal year is also available from the accurate Babylonian mean sidereal month of 27;19,18^d and above mean synodic month which generate a year of 365.2564698 days.

(4) METHODOLOGY: Explanations of the fundamental motions involved according to the methods laid out in the Babylonian procedure texts and related data determined from the Babylonian end products, *i.e.*, the Ephemerides. And in addition, the implications of the Earth/Sun duality in the Babylonian context.

(5) Closely associated to (4), the underlying formulas required to assess Babylonian results and procedures. In this case, since Babylonian planetary theory deals to a considerable extent with *synodic* motion, and the latter understanding is also applicable to the lunar component, the computation of synodic cycles, synodic periods and synodic arcs also play a role in the current investigation, the following especially:

Synodic periods and synodic formulas

The synodic period (S) or lap-cycle between two Solar System planets with mean periods of revolution T_1 and T_2 is given by the *general synodic formula for co-orbital bodies* applied earlier to the Pierce data :

$$\text{Synodic period } S = \frac{T_1 \bullet T_2}{T_1 - T_2} \; (T_1 > T_2) \qquad (1)$$

along with the simplified standard synodic formulas for the Superior and Inferior planets:

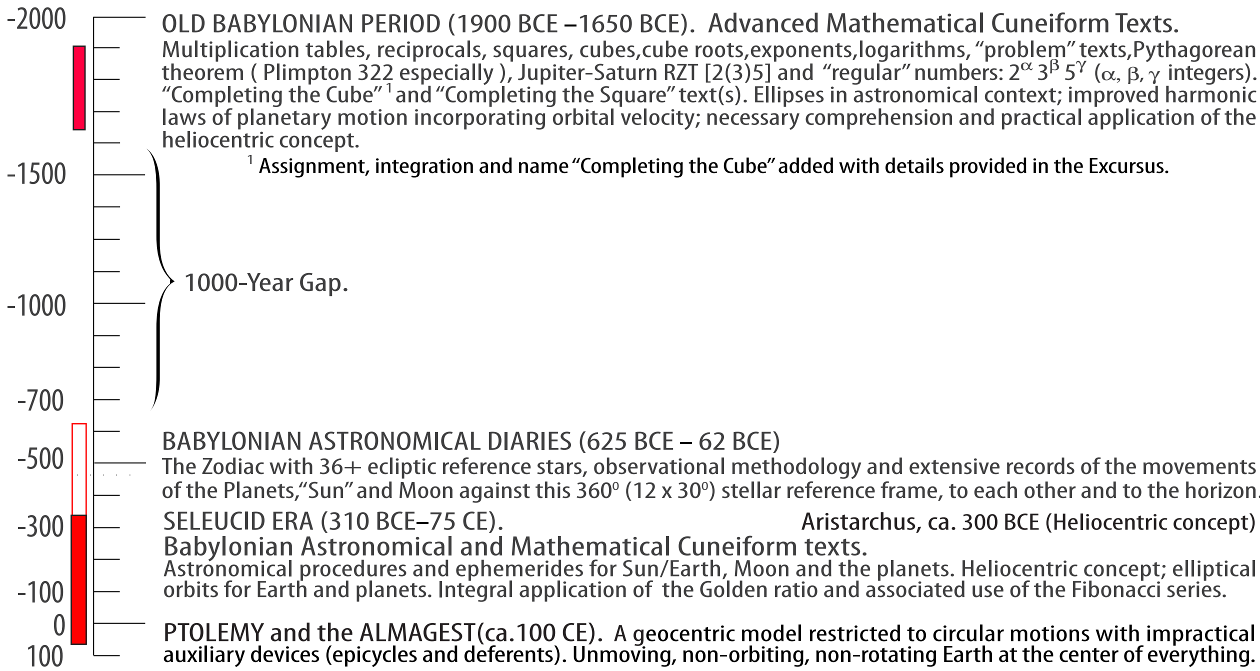
$$\text{Superior planets, } S_s = \frac{T_s}{T_s - 1} \quad (1s) \qquad\qquad \text{Inferior planets, } S_i = \frac{T_i}{1 - T_i} \quad (1i)$$

augmented, if required, by synodic relation (2) where periods T_1 and S are known and period T_2 is of interest:

$$\text{Period } T_2 = \frac{S \bullet T_1}{S + T_1} \; (S > T_2) \qquad (2)$$

Synodic relations (1), (2) and modern equivalents all have roles to play in what follows, but relation (1) in full has a further application arising from the inclusion of the mean synodic month in Babylonian planetary theory beyond calendaric considerations. Although obvious, this was either missed or - for whatever reasons - ignored by noted authority Otto Neugebauer. More on this matter later.

As for the relevance of Babylonian astronomy in the presence context, further examination the mathematical cuneiform texts from the Old Babylonian Period (1900 BCE - 1650 BCE),[14] the Babylonian astronomical diaries from 652–62 BCE,[15] details in the Babylonian astronomical "procedure" texts and the resulting *Ephemerides* of the Seleucid Era (310 BCE–75 CE)[16] represent an extensive source of largely misrepresented and/or misunderstood information. Included here are specific parameters with descriptions in the procedure texts concerning their determination, sufficient details, in fact, for *the heliocentric concept* and *refined laws of planetary motion* to be added to the already complex mathematics of the Old Babylonian Period. The acceptance of which is adversely influenced by the time line between the sources and lack of connectivity with the earliest in terms of known astronomical concepts.



Text-Fig 1. Old Babylonian Mathematics, Babylonian Astronomical Diaries and Seleucid Era Astronomy.

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Introduction

Although the roles of synodic relations (1) and (2) with respect to the Solar System periods and synodic cycles are not entirely surprising the two relations are nevertheless both underlying elements of the structure of the *Phi*-series planetary framework. Just how well the Babylonian luni-solar material reflects this is another matter, but assuredly the subject is worthy of further investigation, especially with respect to attested Babylonian periods and velocities for the five planets known in Antiquity. But then again, the *Fibonacci, Lucas* and the *Phi*-series are all considered to be relatively recent in both origins and understanding, hence the following introduction to the historical side of the matter.

I. The Fibonacci and the Lucas series in early times.

Although the first of these two elementary series is still credited to Fibonacci (*ca.*1175–1240 CE) and likewise the second to Francois Lucas (1842–1891), as Thompson pointed out over a century ago,[17] it is unlikely that the former would have escaped the attention of Greek philosophers or even earlier inquiring minds. Furthermore, this same argument applies equally (if not more so) to the latter series (1,3,4,7,11,18,29,47) since it is simply the *next additive sequence* after the *Fibonacci, i.e.*, *1,1,**2,3**,5,...* is followed by: *1,**3,4**,7,...* (the *Lucas),* then: *1,**4,5**,9,..* and *1,**5,6**,11,...* etc., all with the same limiting ratio (ϕ) between adjacent pairs. The last mentioned (provisionally the *Penta* series 1,***5, 6***,11, 17,**28,**..) also includes the first two perfect numbers **6** and **28** (numbers equal to the sum of their own parts). And eventually, the convenient approximation for the Golden ratio of 809/500 = 1618/1000, thus **1.618** (1;37,4,48).

II. Babylonian Jupiter/Saturn mean synodic arcs; the *Phi*-series and the Golden ratio

Both historically and in astronomical terms, the ratio **5 : 6** is known to play an underlying role in the location of the extremal synodic arcs for Jupiter[18] and Saturn[19] in the Babylonian astronomical cuneiform texts of the Seleucid Era (310 BCE–75 CE). Furthermore, despite current dismissive views on this subject, another point of relevance is found in the Babylonian estimates for the *sidereal* periods of revolution for Jupiter (11;51,40 = 11.86111* years) and Saturn (29;26,40 = 29.444* years) which provide the basis for the mean synodic arcs (*u*) according to Babylonian System B. In particular, it is *the ratios* of these synodic arcs - 33;8,45° (33.1458333*) for Jupiter[20] and 12;39,22,30° (12.65625) for Saturn[21] - which are of immediate interest, since:

$$\frac{\text{Saturn }(u)}{\text{Jupiter }(u)} = \frac{12.65625}{33.14583333^*} = 0.38183534 \textit{ versus } \phi^{-2} = 0.38196601125\text{, the Pierce Limit, } \textit{Phi}\text{-series relation} \qquad (7)_{S/J}$$

$$\frac{\text{Jupiter }(u)}{\text{Saturn }(u)} = \frac{33.14583333^*}{12.65625} = 2.61893004 \textit{ versus } \phi^{2} = 2.61803398875\text{, Planet-to-Planet } \textit{Phi}\text{-series relation} \qquad (8)_{J/S}$$

whereas the difference between the two mean synodic arcs, *i.e.*, Jupiter (*u*) -Saturn (*u*) = 20;29,22,30° (20.48958333*) not only provides the arc for the difference cycle *SD*1 between the two planets (Synodic 4-3 in the Peirce framework) but also two further inter-related ratios of similar interest:

$$\frac{\text{Jupiter }(u)}{SD1(u)} = \frac{33.14583333^*}{20.48958333^*} = 1.61769192 \textit{ versus } \phi = 1.61803398875\text{, Planet-to-Synodic } \textit{Phi}\text{-series relation} \qquad (12)_{J/D}$$

(1.61764706 = Fibonacci 55/34)

$$\frac{SD1(u)}{\text{Saturn }(u)} = \frac{20.48968333^*}{12.65625} = 1.61893004 \textit{ versus } \phi = 1.61803398875\text{, Synodic-to-Planet } \textit{Phi}\text{-series relation} \qquad (12)_{D/S}$$

III. Babylonian Jupiter/Saturn mean synodic arcs and the Fibonacci series

In addition to this pair of mean synodic arcs, System A′ for Jupiter[22] features an intermediate arc of 33;45° (33.75° = u_2) as opposed to (*u*), the mean synodic arc of 33;8,45.° Retaining Saturn's mean synodic arc of 12;39,22,30°. but using 33.75° for Jupiter and new difference arc *SD*1′= 21;5,37,30° (21.09375) the divisions for the new arcs now yield the following familiar Fibonacci ratios which suggests the previous relations are *unlikely to be coincidental or unknown*;

$$\frac{\text{Jupiter }(u_2)}{SD1(u)'} = \frac{33.75}{21.09375} = 1.6 \text{ (1;36). Fibonacci ratio (8/5)} \qquad (12)_{J/DF}$$

$$\frac{SD1(u)'}{\text{Saturn }(u)} = \frac{21.09375}{12.65625} = 1.666^* \text{ (1;40). Fibonacci ratio (5/3)} \qquad (12)_{D/SF}$$

$$\frac{\text{Jupiter }(u_2)}{\text{Saturn(u)}} = \frac{33.75}{12.65625} = 2.666^* \text{ (2;40). Fibonacci ratio (8/3)} \qquad (8)_{J/SF}$$

IV. Jupiter and Saturn mean value ratios for Babylonian Systems A and B

Remaining with Jupiter and Saturn, there is a major difference between the two primary methods for dealing with *varying* synodic motion (Systems A and B) with the two-velocity configurations of System A using a minimum arc (w) and a maximum arc (W) sensibly understood to be *apsidal* velocities with *pheidian* elements in an associated ***5 : 6*** ratio. For Jupiter the minimum and maximum synodic velocities (or apsidal arcs) are 30° and 36°,[23] whereas for Saturn the minimum (w) and the maximum (W) have a marked difference in the number of sexagesimal places, *i.e.*, (w) = 11;43,7,30° (11.71875), and (W) = 14;3,45° (14.0625).[24]

On further examination, however, it seems possible that the latter set may have originated from the former since the seemingly more accurate apsidal synodic arcs for Saturn can be derived from the Jupiter data by *simple division*, *i.e.,* 36°/2.56 = 14;3,45° (14.0625) and 30°/2.56 = 11;43,7,30° (11.71875). Plus one further point; the common divisor is also the *square* of Fibonacci ratio 8/5 ($1.6^2 = 2.56$), thus a practical reduction factor for the periods of revolution of these two adjacent major planets in keeping with the *Fifths* and the *Sixths* of the final Pierce framework.

It is, however, more complicated than this, for even though Jupiter's new value from System A′ (u_2 = 33;45° as used in relations $(12)_{J/DF}$ though $(8)_{J/SF}$) resulted in three Fibonacci ratios using this constant, it is not the actual mean value for Jupiter, which is ½ ($W + w$) = (u_3) = 33°. Whereas the mean value from Saturn's System A is in turn found to be ½ (14;3,45°+11;43,7,30°) = (u_4) = 12;53,26,15° (12.890625) with the ratio between the two new mean values now:

$$\frac{\text{Jupiter }(u_3)}{\text{Saturn}(u_4)} = \frac{33}{12.890625} = 2.56 = 1.6^2.\ \text{Fibonacci ratio } (8/5)^2 \qquad (8)_{J/SF2}$$

while the ratio between Jupiter (u_2) and Saturn (u_4) is:

$$\frac{\text{Jupiter }(u_2)}{\text{Saturn}(u_4)} = \frac{33.75}{12.890625} = 2.6181818^* = \text{Fibonacci ratio } 144/55 \qquad (8)_{J/SF3}$$

with a reciprocal of:

$$\frac{\text{Saturn }(u_4)}{\text{Jupiter}(u_2)} = \frac{12.890625}{33.75} = 0.3819444^* = \text{Fibonacci ratio } 55/144 \qquad (7)_{S/JF2}$$

and a corresponding ideal growth angle (360°• 0.3819444*) of 137.5°.

Lastly, with a new difference arc *SD*1 of (33.75-12.890625) = 20.859375 (u)″, relation $(12)_{J/DF}$ now becomes:

$$\frac{\text{Jupiter }(u_2)}{SD1(u)''} = \frac{33.75}{20.859375} = 1.6179775281 = \text{Fibonacci ratio } (144/89) \qquad (12)_{J/DF2}$$

At which point Babylonian astronomy in general and the origins of these mean synodic arcs in particular begin to assume an unexpected level of importance despite almost universal dismissal of Babylonian methodology at the present time. For this reason the Babylonian observational reference frames and resulting luni-solar parameters in particular offer a minor introduction to the optional excursus at the end of Part 1.

V. Babylonian luni-solar parameters and *Phi*-series/synodic relations (1) and (2)

The inclusion of luni-solar parameters in the present context gives rise to the following *added* abbreviations, names, descriptions and periods in base-60 with decimal equivalents. All *bar* the tropical month and the tropical year were gleaned from leading authority O. Neugebauer's barely readable sexagesimal analyses rendered less understandable by the latter's non-model approach to Babylonian planetary theory. Because of these problems the following tables are largely *prior analytics* initially limited to mean values for synodic relation (1) subroutines applied in Table AP2

Abbr.	Astronomical Names and Standard Descriptions	Babylonian periods	Decimal days
MSM:	Mean Synodic month (new moon to new moon).	29;31,50,8,20 (rounded)	29.530594136^d
MSID:	Mean Sidereal month (fixed star to fixed star).	27;19,18 (rounded)	27.321666667^d
MTROP:	Tropical month (equinox to equinox; text, calc., added).	27;19,17,45 (rounded)	27.321574074^d
MAN:	Anomalistic month (perigee to perigee).	27;33,20 (unrounded)	27.555555555^d
MDRA:	Draconic month (node to node). ACT.	27;12,44 (rounded)	27.212222222^d
MDRA2:	Draconic month (node to node), ACT. calc.	27;12,43,59,40 (rounded)	27.212220679^d
SYR:	Sidereal year (fixed star to fixed star). calc. 12;22,8•MSM	365;15,38,17,44,26,40.	365.26063769^d
SYR2	Sidereal year (fixed star to fixed star). calc. (MSM : MSID).	365;15,23,17,30.	365.25646991^d
TYRB:	Tropical year (equinox to equinox; (Bab. ACT 210, Sect.3)	365;14,44,51	365.24579167^d
AYR:	Anomalous year (perihelion to perihelion (calc., added).	365;15,34,18,22,58,51,	365.25952955^d
EYC:	Eclipse cycle (lunar node to lunar node) (text/mult/calc.).	(5458/465)•MSM.	346.61931784^d
AYC:	Anomalistic cycle (text/mult/calc; added)	(251/18)•MSM.	411.78772933^d

Table AP1. Astronomical terms, Babylonian mean luni-solar periods and decimal equivalents I

Next, the role played by the two primary *Phi*-series/synodic relations in the present context should also be noted:

$$\textit{Synodic period } S_2 = \frac{T_1 \bullet T_3}{T_1 - T_3} \ (T_1 > S_2 > T_3) \quad (1) \qquad \textit{Inner period } T_3 = \frac{T_1 \bullet T_2}{T_1 + T_2} \ (T_1 > T_2 > T_3) \quad (2)$$

Significantly, the mean synodic month (MSM = T_1) and the tropical month (MTROP = T_1 *and* T_2) also play a role in the comparisons between other Babylonian luni-solar cycles, mean luni-solar periods and the modern values with variants of *Phi*-series/synodic relation (1) predominating in Table AP2.

#	Cycles and/or Periods	Subroutine ($T_1>T_2>T_3$)	Mean periods	Modern equivalents/; (sources)	Relations (1x)
[1]	Eclipse cycle (EYC)	(MSM : MDRA)	= 346.619576 days	(Modern: 346.620107 days; (calc.)	(1e)
[2]	Anomalistic cycle (AYC)	(MSM : MAN)	= 411.780405 days	(Modern: 411.783870 days; (calc.)	(1a)
[3]	Nodal cycle (days)	(MTROP : MDRA)	= 6797.54400 days	(Modern: 6798.26051 days; Tables: 6798)	(1n)
--	Nodal cycle (years)	(MTROP : MDRA)	= 18.6108756 years	(Modern: 18.6128373 years:(calc.,)	(1n)
[4]	Lunar perigee	(MAN : MTROP)	= 3231.88186 days	(Modern: 3231.56072 days; Tables: 3232)	(1p)
[5]	Sidereal year (SYR. MYR)	(MSM •12;22,8)	= 365.260637 days	(Expressed in mean synodic months = MYR)	--
[6]	Sidereal year (SYR2,calc.)	(MSM : MSID)	= 365.256469 days	(Modern: 365.256365 days; Tables)	(1s)
[7]	Tropical year (TYR, calc.)	(MSM : MTROP)	= 365.244059 days	(Modern: 365.242189 days; Tables)	(1t)
[8]	Tropical year (TYR, text)	(TYRB: 18-yr pd	= 365.245792 days	365;14,44,51 days (ACT 210, Sec. 3)	--
[9]	Anomalistic year (AYR)	MSM•(360°/(u°)	= 365.259529 days	(Modern: 365.259641 days; Tables)	u = 29; 6,19,20°
[10]	SAROS, 19 EYC or	(223 MSM, calc.)	= 6585.32249 days	(Modern: 6585.32163 days; Tables)	--

Table AP2. Astronomical terms, Babylonian luni-solar cycles and decimal equivalents II.

For example, although the slightly too large yet practical sidereal year SYR (#[5]) is12;22,8 mean synodic months or 365.260637 days, the more accurate value (SYR 2 = 365.256469 days) is readily available by way of synodic relation (1) utilizing the Babylonian mean synodic month (MSM = 29;31,50,8,20 days) and the mean sidereal month (MSID = 27;19,18 days):

$$\text{SYR2} = \frac{\text{MSM} \bullet \text{MSID}}{\text{MSM} + \text{MSID}} = 365.256469811878 \ (365;15,23,17,28,45,43 \text{ days}) \qquad (1\text{Syr2})$$

VI. The Tropical month from Babylonian luni-solar parameters

Also noteworthy are the extended Babylonian luni-solar cycles, especially those stated in eight lines of lunar text No. ACT 210, Section 3.[23] Although rarely recognized as such, they include one of the more contentious issues likely to arise in this context, *i.e.*, *presence of the Tropical month and the Tropical year* in Babylonian astronomy. The latter ([8] in Table AP2) occurs as "1,49,45,19,20 days of 18 years of the moon," [25] yielding 354;14,44,51 days, which is superior to that used by Claudius Ptolemy (365;14,48 = 365.24666* days). More helpful, however, the presence of a tropical year supplies the means for determining a theoretical length for the *Tropical month* in Babylonian astronomy.

Applying a value for the Tropical year (TYRB) of 365;14,44,51days and mean synodic month (MSM) of 29;31,50,8,20 days, an estimate for the tropical month (MTROP) is available from synodic relation (2) *i.e.*, subroutine TYRB : MSM:

$$\text{MTROP} = \frac{\text{TYRB} \bullet \text{MSM}}{\text{TYRB} + \text{MSM}} = 27.32160692 \ (27;19,17,47,5, \ldots \text{ days}) \qquad (2\text{tr})$$

which rounds conveniently to 27;19,17,45 days and the even more convenient Babylonian estimate of 29;19,17,40 days for (perhaps) ACT 210 Section 3. The assignment of 365;14,44,51[d] for a Babylonian tropical year was previously proposed by Hartner in an erudite discussion concerning *the tropical year and precession* which ended as follows[26]

> The inevitable conclusion to be drawn from the preceding demonstrations is, that in Babylonia under Achaernenian rule at the latest in 503 B.C., a clear distinction is made between the length of the tropical year: A = 365;14,48,33,37[d] (possibly already then found exchangeable in practice with Ar = *365;14,44.51*[d]) and that of the sidereal year as underlying System B: PB ′ = 365;15:34, 18,1... [d] (italics supplied)
>
> Willi Hartner, "The Young Avestan and Babylonian Calendars and the Antecedents of Precession." *JHA*, X,1979:1–22.

VII. Precession and the Babylonian Sidereal/Tropical years

Thus once again *Phi*-series/General synodic relation (2) is indicated, albeit with respect to mean values, whereas although the standard sidereal year [5] and tropical year [8] are both on the high side, their difference nevertheless yields a Seleucid Era value (perhaps known, perhaps not) for annual precession of 0;0,52,40,41,...° and 24,602 years for the full cycle.

VIII The Anomalistic year

Unlike the derivations based on synodic relations *the anomalistic year* can be obtained from the *mean sidereal arc of Earth* (29;6,19,20°) *per mean synodic month* of 29;31,50,8,20^d. This ratio yields a daily velocity of 0;59,8,9,43,22,7, ...°

$$\text{Mean daily velocity of Earth} = \frac{29;6,19,20^{\circ}}{29;31,50,8,20^{d}} = 0;59,8,9,43,\mathit{22,7}, \quad (0;59,8,9,43,20 \text{ rounded} = u') \qquad (3m)$$

for a corresponding year of 365;15,34,18,22,58,51,40^d or 365.2595295... days. The modern estimate is 365.259641. . Or more simplistically, the amount moved by Earth along its orbit from one full-moon to the next. Thus, from ratio (3u) the mean period of Earth in mean synodic months is :

$$\text{Period of revolution of Earth} = \frac{360^{\circ}}{u'} = 12;22,7,51,53,40, \ldots \text{ mean synodic months} = 365.25952955 \text{ days} \qquad (3u)$$

IX. Multiple luni-solar extensions from *Phi*-series/synodic relation (1)

The simplicity of this relation permits similar derivations for the Draconic, Anomalistic and Nodal Cycles. The first pair include the mean synodic month (MSM) whereas the last cycle uses the Draconic (MDRA) and Tropical (MTROP) months:

$$\text{Draconic Cycle} = \frac{\text{MSM} \cdot \text{MDRA}}{\text{MSM - MDRA}} = 346.6195761217 \ldots \text{days} \qquad (1dc)$$

$$\text{Anomalistic Cycle} = \frac{\text{MSM} \cdot \text{MAN}}{\text{MSM - MAN}} = 411.7805352634 \ldots \text{days} \qquad (1ac)$$

$$\text{Nodal Cycle} = \frac{\text{MTROP} \cdot \text{MDRA}}{\text{MTROP - MDRA}} = 18.6101191842.. \text{ years} \qquad (1nc)$$

Here the nodal cycle is of potential interest in view of its association with lunar standstills in the first place and the apparent trouble the ancients took to delineate this phenomenon in the second, *e.g.* Chaco Canyon in the United States, Stonehenge in England and Callanish in Scotland.[27]

At this point Babylonian astronomy in general and the origins of these mean synodic arcs in particular begin to assume an unexpected level of importance despite almost universal dismissal of Babylonian methodology at the present time. For this reason it appears necessary to to offer an optional **excursus** after the Bibliography for Part V to explain the statements in Text-Fig 1concerning advanced knowledge of astronomy in the Old Babylonian period and other matters of concern.

PART ONE: CLOSING REMARKS

Rejections: (1) Expansions of the Laws of planetary motion; (2) Benjamin Pierce's planetary framework

Starting with Galileo and the velocity expansions of the laws of planetary motion described in the Excursus, the concern here is that while the present writer was merely a tertiary restorer, and as such did not expect much in the way of applause, it seemed a reasonable assumption that the extended version $T^2 = R^3 = Vi$,[6] $R = \text{Vi}^2$ would at least take its place next to Kepler's twin parameter format $R^3 = T$.[2] And further, that variants of the former would simplify routine tasks, *e.g.,* the calculation of angular momentum *L*, Table 1 mean velocities and the like. But this did not come to pass, and so it has remained ever since. On the other hand, modern science appears to have been able to function without such expansions, though not necessarily as well, it is suggested, had these velocity components also been incorporated.

But the real problem is not this historical item *per se*, but rather, that the same process and rapid dismissal was also applied to Benjamin Peirce's Fibonacci-based planetary framework with no replacement or improved version to take its place. And oddly, because of this situation which has remained unaddressed, humankind is now avidly searching for external planetary systems without any *overall dynamic understanding* of our own. Think not? Simply stated, no current model appears to exist which would, for example, provide the *precise* information and the *theoretical basis* for the possible existence of another planet interior to Mercury. Whereas, even in its initial form (*sans* intermediate intervals) this possibility was expressly incorporated in Pierce's initial approach, while in light of present concerns with Global Warming the possible *intermediate* location of Earth becomes more than a mere historical asterisk.

Weakened by special interests, discouraged by behavioural deficiencies and also impeded by disbelief, even the most fundamental question concerning whether Climate Change originates *primarily* from *within*, *i.e*., confines of planet Earth, or from *without* as an integral component of a larger System cannot be tackled adequately at present. Furthermore, what can be made of the location of Earth itself in the intermediate position between Venus and Mars, and what role might this apparent anomaly have played in global warming during the past, distant or otherwise?

All of which is further exacerbated by increasing population growth, unceasing deforestation, rapidly diminishing resources with warfare and mental illness also rising on a Global scale. Truly an Age of Disillusionment and concern.

In the meantime the present inquiry turns next to the initial application of the Pierce Divisor approach to external planetary structures with or without the following suggested guidelines.

PROVISIONAL GUIDELINES FOR EXTERNAL SYSTEMS

Test Format, *Phi*-series Relations and Base Periods

Remaining with the order adopted by Peirce which commences with the outermost PLANET #1 with the greatest period of revolution, moving inwards (by way of Synodic 2-1, then PLANET #2, etc.), will generally involve three consecutive mean periods. All of which can be determined by the following *Phi*-series synodic relations if needed:

Relation (1) The Synodic mean between two bracketing periods of revolution, thus the product of the periods divided by their difference.

Relation (2). Relation (2) requires *two adjacent periods above* to generate *the next value below,* and thereafter generates all further lower periods if or as required.

Relation (4). The geometric mean of any pair of bracketing periods. Thus Relation (4 $\pm$ 1), or simply Relation (4) as used.

Relation(4E) Relation (4E $\pm$ 2), Relation (4E $\pm$ 3), Relation (4E $\pm$ 4), Relation (4E $\pm$ 5) and Relation (4E $\pm$ 6). Such applications depend on specific prior restorations (in due order) above and below the target position(s).

Relation(4F) Relation (4F+3). Special case for PLANET #2 only. Requires both the Base period and periods below #2. This application serves to synchronize the restored periods at this point with the those of the divisor framework

The above relations are provided in Table 4 with the Fibonacci and the Lucas series in vertical and inverted form to match their inclusions in Tables 2a, 2b and also the format adopted for exoplanetary structures.

Lastly, possible departures from the framework are included as variations which may be encountered among external systems. For similar systems, however

(a) Planets may occupy intermediate (synodic) locations, as in the case of Earth.
(b) Planets and adjacent synodic locations may be unfilled (*i.e.*, absent), *e.g.*, the Mars-Jupiter Gap.
(c) Departures from the theoretical framework, or (a) and (b) may indicate disrupted planetary systems.
(d) Planetary systems may possess residual Fibonacci indicators, as in the Solar System.
(e) Planetary systems may also possess residual Lucas indicators for the same reason as (d).

Planets N *Synodics #*	**Divisors** (added)
PLANET 1	**1**
Synodic 2-1	1
PLANET 2	**2**
Synodic 3-2	4
PLANET 3	**6**
Synodic 4-3	9
PLANET 4	**15**
Synodic 5-4	25
PLANET 5	**40**
Synodic 6-5	64
PLANET 6	**104**
Synodic 7-6	169
PLANET 7	**273**
Synodic 8-7	441
PLANET 8	**714**
Synodic 9-8	1156
PLANET 9	**1870**
Synodic 10-9	3025
PLANET 10	**4895**

PHI-SERIES RELATION (1), the Synodic mean: B = AC/| (A - C) |.
For any three successive *Phi*-series periods, A, B, C middle period (B) is the *product of the periods* on either side *divided by their difference*.

PHI-SERIES RELATION (2), the Half-harmonic mean: C = AB/(A + B).
If two upper adjacent periods A, B are known, the third and lower (C) is the *product* of the two adjacent periods *divided by their sum*.

PHI-SERIES RELATION (4), the extended Geometric mean: B = $\sqrt{(AC)}$.
For any three successive periods,the middle period (B) is the *geometric mean* of the periods on either side, as are the resulting periods for the positions $\pm$ 2, $\pm$ 3,.. = (4E) ; relation (4F+3) pertains to Planet #2 alone.

BASE PERIODS B1/B2.
Base period B1 is the period of the outermost planet as detected.
Base Period B2 may be applicable if *Synodic* 2-1 is marginally > B1.

BASE PERIOD B3.
Approximate base periods (B3s) result from *reversed procedures*, *i.e.* ,the *products* of known periods and their assigned divisors.

BASE PERIOD B4.
Approximate base periods (B4s) can be obtained from the averaged values of the available B3 *products.*

BASE PERIOD B5.
Working, non-standard base periods (B5s) can be applied when none of the above prove to be applicable.

Fibonacci series 1, 1, **2, 3, 5**		Lucas series 1, **3, 4, 7**	
1		-	
1	4181	1	5778
2	2584	3	3571
3	1597	4	2207
5	610	7	1364
8	377	11	843
13	233	18	521
21	144	29	322
34	89	47	199
55	55	76	123
89	34	123	76
144	21	199	47
233	13	322	29
377	8	521	18
610	5	843	11
987	3	1364	7
1597	2	2207	4
2584	1	3571	3
4181	1	5778	1
Fibonacci series		Lucas series	

Table 4. Divisor assignments, numerical series, *Phi*-series relations and conventions for base periods B1 thru B5.

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INTIMATIONS OF COMMONALITY IN PLANETARY SYSTEMS

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