SINGLE EVENTS, TIME SERIES ANALYSIS, AND PLANETARY MOTION

John N. Harris

INTRODUCTION

The advent of modern computing devices and their application to time-series analyses permits the investigation of mathematical and astronomical relationships on an unprecedented scale. Since neither numerical complexity nor calculation intensity pose insuperable difficulties, it becomes feasible to treat single events sequentially and apply detailed time-series analyses to the results. The following discussion is primarily concerned with the real-time heliocentric motions of the four major superior planets (Jupiter, Saturn, Uranus and Neptune) and the four terrestrial planets (Mercury, Venus, Earth and Mars), plus their various interactions. Shown in graphical form in the second section, the final outputs were based initially on the single-event formulas provided by Bretagnon and Simon (1986) adapted to produce time-series data utilizing spreadsheet techniques.

A. THE MAJOR SUPERIOR PLANETS

The methodology and formulas applied to planetary motion in this context are provided by Pierre Bretagnon and Jean-Louis Simon in *Planetary Programs and Tables from - 4000 to +2800* (Willman-Bell, Richmond, 1986). The astronomical programs in the latter concern the determination of the positions of the planets as viewed from Earth (i.e., geocentric coordinates with corrections for aberration, nutation, and precession, etc). The first stage of the computation, however, concerns the determination of heliocentric coordinates which for Jupiter, Saturn, Uranus and Neptune are obtained from the following power series formulas:

HELIOCENTRIC LONGITUDE *(L)*

$$\int(V_L) = \alpha_0 + \alpha_1 V^1 + \alpha_2 V^2 + \alpha_3 V^3 + \alpha_4 V^4 + \alpha_5 V^5 + \alpha_6 V^6 \quad [1]$$

HELIOCENTRIC LATITUDE *(B)*

$$\int(V_B) = \beta_0 + \beta_1 V^1 + \beta_2 V^2 + \beta_3 V^3 + \beta_4 V^4 + \beta_5 V^5 + \beta_6 V^6 \quad [2]$$

HELIOCENTRIC RADIUS VECTOR *(R)*

$$\int(V_R) = r_0 + r_1 V^1 + r_2 V^2 + r_3 V^3 + r_4 V^4 + r_5 V^5 + r_6 V^6 \quad [3]$$

The parameter V is measured in units of 2000 julian days from the beginning of successive five-year intervals; units are *radians* for L and B and *astronomical units* (AU) for R. The motions and positions of Jupiter, Saturn, Uranus and Neptune are obtained from power series data provided for five-year intervals, e.g., for the period 1990 to 1995 commencing with Julian Day 2447892.5 the power series data are as follows (Bretagnon and Simon 1986:124,140):–

JUPITER

1990 2447892.5

L)	1.678682	2.956725	-0.414596	0.004826	0.299734	-0.151349	0.029332
B)	-0.005204	0.067083	-0.000759	-0.109760	0.078191	-0.029462	0.007110
R)	5.155577	0.717884	0.187303	-1.133334	0.310164	0.141854	-0.042529

SATURN

1990 2447892.5

L)	4.993758	1.054503	0.014505	0.023160	-0.000553	-0.000863	-0.000059
B)	0.005629	-0.045382	-0.003796	0.007466	0.000345	0.000362	-0.000177
R)	10.027146	-0.144092	-0.300680	0.032117	0.003847	0.022473	-0.008193

URANUS

1990 2447892.5

L)	4.808885	0.401780	-0.007396	0.001186	-0.000138	-0.000220	0.000115
B)	-0.004951	-0.000503	0.000528	-0.000054	0.000306	-0.000299	0.000108
R)	19.380045	0.357595	-0.005398	-0.008060	-0.013812	0.011760	-0.004261

NEPTUNE

1990 2447892.5

L)	4.923200	0.207762	0.000166	0.000853	-0.000671	0.000373	-0.000118
B)	0.015270	-0.005562	-0.000339	0.000013	0.000032	-0.000016	0.000004
R)	30.210400	-0.047301	0.013832	0.001610	-0.018511	0.014834	-0.005138

The first line gives the starting *year* of the five year time-span followed by the initial *julian day* (January 1) at 0^h ET. The second line gives the seven coefficients of the polynomial for the heliocentric longitude *L*, the third the coefficients for the heliocentric latitude *B*, and the forth the coefficients for the heliocentric radius vector *R*.

TIME

Ephemeris Time (ET) with the variable *V(t)* obtained from the following relation:

$$V = \frac{ET - T_0^i}{2000} \quad [4]$$

where T_0 is the beginning julian date for the five year time-span and T^i the required instant (or successive instants) for the superior planet(s) in question. *V(t)* ranges from 0 to 0.915.

REAL-TIME PLANETARY ORBITS

Plan-view plots of planetary orbits require the computation of the heliocentric longitude (*L)* and the heliocentric radius vector (*R*) for successive values of *V* within a given time-span. However, none of the major superior planets have sidereal periods that are shorter than five years thus the computation of each orbit entails the use of successive five-year data sets. For one complete orbit of Jupiter, a minimum of two sets of data is required; for Saturn five, Uranus seventeen, and for Neptune thirty-three. For the interval *1600–2100 BP*, one hundred consecutive sets of power series data are therefore required for each planet.

B. THE FOUR TERRESTRIAL PLANETS

In contrast to the relatively simple power-series methodology for the major superior planets, formulas for the terrestrial planets are both cumbersome and difficult to implement in times-series format without the heavy use of computing devices. Here the formulas vary from planet-to-planet and all require tables and lengthy trigonometric summations. For example, in the case of *Mercury* the formulas and tables for the heliocentric radius vector (*R*), the heliocentric latitude (*B*) and heliocentric longitude (*L)* are as follows:

MERCURY: HELIOCENTRIC RADIUS VECTOR (*R*)

$$R = 0.3952020 + 10^{-7} x \sum_{i=1}^{14} r_i \cos(a_i + v_i U) \quad [5a]$$

TABLE 1: i = 1 to 14

i	r_i	a_i	v_i
1	780141	6.202782	260878.753962
2	78942	2.98062	521757.50830
3	12000	6.0391	782636.2640
4	9839	4.8422	260879.3808
5	2355	5.062	0.734
6	2019	2.898	1043514.987
7	1974	1.588	521758.140
8	1859	0.805	260877.716
9	426	4.601	782636.915
10	397	5.976	1304393.735
11	382	3.86	521756.47
12	306	1.87	1043515.34
13	102	0.62	782635.28
14	92	2.60	1565272.52

MERCURY: HELIOCENTRIC LATITUDE (*B*)

$$B = 10^{-7} x \sum_{i=1}^{18} \beta_i \sin(a_i + v_i U) \quad [5b]$$

TABLE 2: *i* = 1 to 18

i	b_i	a_i	v_i
1	680303	3.82625	260879.17693
2	538354	3.30009	260879.66625
3	176935	3.74070	0.40005
4	143323	0.58073	521757.92658
5	105214	0.05450	521758.44880
6	91011	3.3915	0.9954
7	47427	1.9266	260878.2610
8	41669	3.5084	782636.7624
9	19826	3.1539	782637.4813
10	12963	0.2455	1043515.6610
11	8233	4.886	521756.972
12	6399	0.358	782637.769
13	3196	3.253	1304394.380
14	1536	4.824	1043516.451
15	824	0.04	1565273.15
16	819	1.84	782635.45
17	324	1.60	1304395.53
18	201	2.92	1826151.86

MERCURY: HELIOCENTRIC LONGITUDE (*L*)

$$L = 4.4429839+260881.4701279U+10^{-6}\{(409894.2+2435U-1408U^2+114U^3+233U^4-88U^5)$$
$$\times \sin(3.053817+260878.756773U-0.001093U^2+0.00093U^3+0.00043U^4+0.00014U^5)\}$$
$$+ 10^{-7} x \sum_{i=1}^{25} l_i \sin(a_i + v_i U) \quad [5c]$$

TABLE 3: i = 1 to 25

i	l_i	a_i	v_i
1	510728	6.09670	521757.52364
2	404847	4.72189	1.62027
3	91048	2.8946	782636.2744
4	30594	4.1535	521758.6270
5	15769	5.8003	1043515.0730
6	13726	0.4656	521756.9570
7	11582	1.0266	782638.007
8	7633	3.517	521759.335
9	5247	0.418	1043516.352
10	4001	3.993	1304393.680
11	3299	2.791	1043514.724
12	3212	0.209	1304394.627
13	1690	2.067	1304395.168
14	1482	6.174	782635.409
15	1233	3.606	1043516.88
16	1152	5.856	1565272.646
17	845	2.63	1565273.50
18	654	3.40	1826151.56
19	359	2.66	11094.77
20	356	3.08	1565273.50
21	257	6.27	1826152.20
22	246	2.89	5.41
23	180	5.67	56613.61
24	159	4.57	250285.49
25	137	6.17	271973.50

HELIOCENTRIC RADIUS VECTORS: VENUS, SUN (EARTH) AND MARS

In so much as the present paper is an introduction rather than a detailed description the corresponding formulas and tables for the longitudes and latitudes of the other terrestrial planets will not be presented here *in toto*. For general information, however, a limited treatment of the remaining heliocentric radius vectors for this trio of planets is shown below; for further details refer to the descriptions and explanations provided by Bretagnon and Simon (1986).

VENUS: HELIOCENTRIC RADIUS VECTOR (*R*)

R = 0.723 5481+10^{-7} {(48982-34549U+7096$U^2$3360U^3+890U^4-210U^{5})
x cos(4.02152+102132.84695U+0.2420U^2+0.0994U^3+0.0351U^4-0.0013U^5-0.015U^6)}
+ 10^{-7}{(166-234U+131U^2) x cos(4.90+204265.69U+0.48U^2+0.20U^3)}

$$+10^{-7} x \sum_{i=1}^{5} r_i \cos(a_i + v_i U) \quad [6]$$

TABLE 4: i = 1 to 5

i	r_i	a_i	v_i
1	72101	2.828	0.361
2	163	2.85	78604.20
3	138	1.13	117906.29
4	50	2.59	96835.94
5	37	1.42	39302.10

SUN (EARTH): HELIOCENTRIC RADIUS VECTOR [Table 5: i = 1 to 50 omitted]

$$R = 1.0001026 + 10^{-7} x \sum_{i=1}^{50} r_i \cos(a_i + v_i U) \quad [7]$$

MARS: HELIOCENTRIC RADIUS VECTOR (*R*) [Table 6: i = 1 to 29 omitted]

R = 1.5298560 + 10^{-6}{(141 849.5 +13651.8U-1230U^2-378U^3 + 187U^4-153U^5-73U^6)
cos(3.479698+33405.349560U+0.030669U^2 -0.00909U^3+0.00223U^4+0.00083U^5-0.00048U^6)}
+ 10^{-6}{(6607.8 +1272.8U-53U^2-46U^3+14U^4-12U^5+99U^6)x
cos(3.81781+ 66810.6991U+ 0.0613U^2-0.0182U^3+0.0044U^4+0.0012U^5+0.002U^6)}

$$+10^{-7} x \sum_{i=1}^{29} r_i \cos(a_i + v_i U) \quad [8]$$

TIME

Ephemeris Time (ET) with the variable *(U)t* obtained from the following relation:

$$U = \frac{ET - 2451545}{3652500} \quad [9]$$

C. HELIOCENTRIC RADIUS VECTORS

Although relations [9] and [4] require additional corrections for historical research, for present purposes it is more useful to remain with *julian dates* throughout since the latter lend themselves readily to looping and incrementation in a variety of complex applications. Moreover, although it still remains feasible to calculate the planetary positions by applying related formulas for the heliocentric distances, longitudes and latitudes in standard manner, it is the *heliocentric distances* that are by far the most useful.

The exact sequential value for the radius vector of a planet moving in an elliptical orbit carries with it both corresponding orbital velocities and orbital "periods" for the exact position and time in question. In other words, the variable radius vector that moves between the limits established by the points of perihelion and aphelion provides two further related time-series functions. The first describes the manner in which the radius vector *changes*, the second the orbital velocity itself, and the third—though not immediately apparent—the corresponding "range" of the period of revolution. To put the latter in a clearer light, the mean synodic time (*Ts)* between a pair of co-orbital planets—essentially the time a faster moving inner planet (mean orbital period *T1)* takes to lap a slower outer planet (mean orbital period *T2)* may be obtained from the general synodic formula:

$$Synodic\,Period\,(T_s) = \frac{T_2 \cdot T_1}{T_2 - T_1} \quad (T_2 > T_1) \qquad [10]$$

In practice, however, adjacent pairs of planets are rarely both precisely at the particular points in their orbits that correspond to their respective mean value radius vectors. Thus the *mean* synodic period remains basically a theoretical parameter. From a more practical viewpoint, however, for every value of the radius vector between perihelion and aphelion there are corresponding "periods" of revolution, and as a consequence, real-time synodic functions may be determined directly from the resulting radius vectors by the application of the *Harmonic Law* ($T^2 = R^3$). For the superior planets this poses no great problem since true radius vectors may be obtained from power series data and associated tables in a relatively straightforward manner. For the terrestrial planets the same basic approach holds, except that the more complex formulas and tables are involved. Both methods, however, lend themselves readily to looping and incrementation and all provide the means for investigating interactive relationships. Examples of the latter include visualization of the well known *2 : 1* Earth-Mars and *2 : 5* Jupiter-Saturn resonances, relationship between differences in inverse orbital velocities of the latter pair and the orbital velocity of Mars. The latter and further complexities associated with Venus-Earth-Mars resonances are examined briefly in Part Two.

SOURCE

Part C and Relation 10 excepted, the above formulas, tables, power series data and general methodology are from Bretagnon and Simon(1986):–

TABLES FOR THE MOTION OF THE SUN AND THE FIVE PLANETS FROM - 4000 TO + 2800
TABLES FOR THE MOTION OF URANUS AND NEPTUNE FROM + 1600 TO + 2800
Pierre Bretagnon and Jean-louis Simon.
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77, avenue Denfert-Rochereau, 75014 Paris, France.
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John N. Harris. Last updated February14, 2012.