Galileo Galilei

Lincean Academician

Chief Philosopher and Mathematician to the
Most Serene Grand Duke of Tuscany

Discourses & Mathematical Demonstrations Concerning

Two New Sciences

Pertaining to Mechanics & Local Motions

With an Appendix
On Centers of Gravity of Solids

Leyden
At the Elzevirs, 1638

* * *

To which is added a further dialogue
On the force of Percussion

Title pages and
The Added
Day
5

Galileo Galilei

Two New Sciences

Including Centers of Gravity & Force of Percussion

Translated, with
Introduction and Notes, by

Stillman Drake

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Added Day

On The Force of Percussion[1]

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Interlocutors: Salviati, Sagredo and Aproino

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SAGR. Your absence during this past fortnight, Salviati, has given me an opportunity to look at the propositions concerning centers of gravity in solids, as well as to read carefully the demonstrations of those many new propositions on natural and violent motions, and since there are among these not a few that are difficult to apprehend, it has been a great help to me to confer with this gentleman whom you see here.

SALV. I was about to ask you concerning the gentleman's presence, and about the absence of our good Simplicio.

SAGR. I imagine—indeed, I think it certain—that the reason for Simplicio's absence is the obscurity to him of some demonstrations of various problems relating to motion, and still more, that of those about centers of gravity. I speak of those [demonstrations] which, through their long chains of assorted propositions of [Euclid's] *Elements of Geometry*, become incomprehensible to people who do not have those elements thoroughly in hand.

1. Although the word *percussio* is literally translated in the title above, it is rendered by "impact" in the text as the more usual English term. Galileo first wrote on these problems in 1594 as a brief appendix to his *Mechanics.* The composition of this dialogue probably began about March, 1635, shortly after his old pupil, Aproino, saw at Venice the manuscript of the First Day.

The gentleman you see is Signor Paolo Aproino, a nobleman of Treviso, who was a pupil of our Academician when he taught at Padua, and not only his pupil, but his very close friend, with whom he held long and continual conversations, together with others [of like interests]. Outstanding among {282} these was the most noble Signer Danielle Antonini[2] of Udine, a man of surpassing intellect and superhuman worth who died gloriously in defence of his country and its serene ruler, receiving honors worthy of his merit from the great Venetian Republic. [With him, Aproino] took part in a large number of experiments that were made at the house of our Academician, concerning a variety of problems. Now, about ten days ago this gentleman came to Venice, and he visited me, as is his custom, and learning that I had here these treatises by our friend, he wanted to look them over with me. Hearing about our appointment to meet and talk over the mysterious problem of impact, he told me that he had discussed this many times with the Academician, though always questioningly and inconclusively, and he told me that he was present at the performance of divers experiments relating to various problems, of which some were made with regard to the force of impact and its explanation. He was just now on the point of mentioning, among others, one which he says is most ingenious and subtle.

SALV. I consider it my great good fortune to meet Signer Aproino and to know him personally, as I already knew him by reputation and the many reports of our Academician. It will be a great pleasure for me to be able to hear at least a part of these various experiments made at our friend's house on different propositions, and in the presence of minds as acute as those of Aproino and Antonini, gentlemen of whom I have heard our friend speak on many occasions with praise and admiration. Now since we are here to reason specifically about impact, you, my dear Aproino, may tell us what was drawn from the experiments, in this matter, promising, however, to speak on some other occasion about others made concerning other problems. For I know that such are not lacking to you, from our Academician's assurance that you were always no less curious than careful as an experimentalist [*sperimentatore*].

APR. If I were to try with proper gratitude to repay the debt to which your excellency's courtesy obliges me, I should have to spend so many words that little or no time would be left in this day to speak of the matter here undertaken.

2. Antonini (1588-1616) became a correspondent of Galileo's after studying with him at Padua about 1608-10. This coupling of his name with that of Aproino suggests that the experiments described belonged to that period, as do the most precise experiments of which Galileo left any manuscript records.

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SAGR. No, no, Aproino, let us start right in with learned discussion, leaving ceremonious compliments to the courtiers. For what it is worth, I shall stand pledge between you two [323] that mutual satisfaction will be given by words that are few, but candid and sincere.

APR. I hardly expect to say anything not already known to Salviati, so the entire burden of discourse ought to be borne on his shoulders. Yet to give him a start at least, if for no other reason, I shall mention the first steps and the first experiment that our friend essayed in order to get to the heart of this admirable problem of impact.

What is sought is the means of finding and measuring its great force, and if possible simultaneously of resolving the essence [of impact] into its principles and prime causes, for this effect seems, in acquiring its great power, to proceed very differently from the manner in which multiplication of force proceeds in all other mechanical machines, I say "mechanical" to exclude the immense force of gunpowder [*fuoco*, fire].[3] In machines, it is very conclusively perceived that speed in a weak mover compensates the power [*gagliardia*] of a strong resistent [which is] moved but slowly. Now since it is seen that in the operation of impact, too, the movement of the striking body conjoined with its speed acts against the movement of the resistent and the much or little that it is required to be moved, it was the Academician's first idea to try to find out what part in the effect and operation of impact belonged, for example, to the weight of a hammer, and what [part belonged to] the greater or lesser speed with which it was moved. He wanted if possible to find one measure that would measure both of these, and would assign the energy of each;[4] and to arrive at this knowledge, he imagined what seems to me to be an ingenious experiment.

He took a very sturdy rod, about three braccia long, pivoted like the beam of a balance, and he suspended at the ends of these balance-arms two equal weights, very heavy One of these consisted of copper containers, that is, of two buckets, one of which hung at the said extremity of the beam and was filled with water. From the handles of this bucket

3. Cf. p. 278 and note 14 to Fourth Day.
4. Galileo's approach related the problem to compound ratios, see Introduction and Glossary. His discussion is accordingly mainly one of momentum rather than of force in its modern sense. This concentrates attention on velocity rather than on acceleration, but see pp. 330, 332, 344, and notes 12, 14, below.

{284} hung two cords, about two braccia each in length, to which was attached by its handles another like bucket, but empty, this hung plumb beneath the bucket already described as [324] filled with water. At the end of the other balance-arm he hung a counterweight of stone or some other heavy material, which exactly balanced the weight of the whole assembly of buckets, water, and ropes. The bottom of the upper bucket had been pierced by a hole the size of an egg or a little smaller, which hole could be opened and closed.

Our first conjecture was that when the balance rested in equilibrium, the whole apparatus having been prepared as described, and then the [hole in the] upper bucket was un-stoppered and the water allowed to flow, this would go swiftly down to strike in the lower bucket, and we conceived that the adjoining of this impact must add to the [static] moment on that side, so that in order to restore equilibrium it would be necessary to add more weight to that of the counterpoise on the other arm. This addition would evidently restore and offset the new force of impact of the water, so that we could say that its momentum was equivalent to the weight of the ten or twelve pounds that it would have been necessary [as we imagined] to add to the counterweight.

SAGR. This scheme seems to me really ingenious, and I am eagerly waiting to hear how the experiment succeeded.

APR. The outcome was no less wonderful than it was unexpected by us. For the hole being suddenly opened, and the water commencing to run out, the balance did indeed tilt toward the side with the counterweight, but the water had hardly begun to strike against the bottom of the lower bucket when the counterweight ceased to descend, and commenced to rise with very tranquil motion, restoring itself to equilibrium while water was still flowing,[5] and upon reaching equilibrium it balanced and came to rest without passing a hairbreadth beyond.

SAGR. This result certainly comes as a surprise to me. The outcome differed from what I had expected, and from which I hoped to learn the amount of the force of impact. Nevertheless it seems to me that we can obtain most of the desired information. Let us say that the force and moment of impact is equivalent to the moment and weight of

5. The experimenters expected some constant effect as long as the flow of water continued, enabling them to re-establish equilibrium by adding weight to the counterpoise.

whatever amount {85} of falling water is found to be suspended in the air between the upper and lower buckets, which quantity of water does not weigh at all against either upper or lower bucket. Not against the upper, for the parts of water are not attached together, so they cannot exert force and draw down on those [325] above, as would some viscous liquid, such as pitch or lime, for example. Nor [does it weigh] against the lower [bucket], because the falling water goes with continually accelerated motion, so its upper parts cannot weigh down on or press against its lower ones. Hence it follows that all the water contained in the jet is as if it were not in the balance. Indeed, that is more than evident, for if that [intermediate] water exerted any weight against the buckets, that weight together with the impact would greatly incline the buckets downward, raising the counterweight, and this is seen not to happen. This is again exactly confirmed if we imagine all the water suddenly to freeze, for then the jet, made into solid ice, would weigh with all the rest of the structure, while cessation of the motion would remove all impact.

APR. Your reasoning conforms exactly with ours—immediately after the experiment we had witnessed. To us also, it seemed possible to conclude that the speed alone, acquired by the fall of that amount of water from a height of two braccia, without [taking into account] the weight of this water, operated to press down exactly as much as did the weight of the water, without [taking into account] the impetus of the impact. Hence if one could measure and weigh the quantity of water hanging in air between the containers, one might safely assert that the impact has the same power to act by pressing down as would be that of a weight equal to the ten or twelve pounds of falling water.

SALV. This clever contrivance much pleases me, and it appears to me that without straying from that path, in which some ambiguity is introduced by the difficulty of measuring the amount of this falling water,[6] we might by a not unlike experiment smooth the road to the complete understanding which we desire.

Imagine, for instance, one of those great weights (which I believe are called pile drivers [*berte*]) that are used to drive stout poles into the ground by allowing them to fall from some height onto such poles. Let us put the weight of such a {286} pile driver at 100

6. Notes survive in which Galileo made calculations concerning the volume of this jet of water.

pounds, and let the height from which this falls be four braccia, while the entrance of the pole into hard ground, when driven by a single [such] impact, shall be four inches. Next, suppose that we want to achieve the same pressure and entrance of four inches without using impact, and we find that this can be done by a weight of 1000 pounds, which, operating by its heaviness alone, without any preceding motion, we may call "dead weight." I ask whether, without [326] error or fallacy, we may affirm that the force and energy of a weight of 100 pounds, combined with its speed acquired in falling from a height of four braccia, is equivalent to the dead weight of 1000 pounds. That is, does the force [*virtu*] of this speed alone signify as much as the pressure of 900 pounds of dead weight, which is the remainder after subtracting from 1000 [pounds] the 100 of the pile driver?

I see that you both hesitate to reply, perhaps because I have not explained my question properly Then let us merely ask briefly whether, from the experiment described, we may assert that the pressure of this dead weight will always produce the same effect on a resistance as the weight of 100 pounds falling from a height of four braccia. To make things perfectly clear, [say that] the pile driver, falling from the same height but striking on a more resistant pole, will drive it no more than two inches. Now, can we be sure of this same effect from the pressing down alone of the dead weight of 1000 pounds? I mean, will that drive the pole two inches?

APR. I think, at least on first hearing this, that it would not be rejected by anyone.

SALV. And you, Sagredo, do you raise any question about this?

SAGR. Not at the moment, no, but my having experienced a thousand times the ease with which one is deceived prevents my being so bold as to feel no trepidation.

SALV. Even you, whose great perspicacity I have known on many occasions, now show yourself as leaning toward the wrong side, hence I believe that it would be hard to find even one or two men in a thousand who would not be snared into so plausible a fallacy But what will astonish you still more will be to see this fallacy to be hidden beneath so thin a veil that the slightest breeze would serve to uncover and reveal it, though it is now concealed and hidden.

First, then, let the pile driver in question fall on the pole as before, driving this four inches down, and let it be true {287} that to accomplish this with dead weight would require exactly 1000 pounds. Next, let us raise this same pile driver to the same height, so that it falls a second time on the same pole, but drives it only two inches, by reason of the pole's having encountered harder ground. Must we suppose that it would be driven as much by the pressure of that same dead weight of 1000 pounds?

APR. So it seems to me.[327]

SAGR. Alas, Paolo, for us, this must be emphatically denied. For if in the first placement, the dead weight of 1000 pounds drove the pole only four inches and no more, why will you have it that by merely being removed and replaced, it will drive the pole two more inches? Why did it not do this before it was removed, while it was still pressing? Do you suppose that just taking it off and gently replacing it makes it do that which it could not do before?

APR. I can only blush and admit that I was in danger of drowning in a glass of water.

SALV. Do not reproach yourself, Aproino, for I can assure you that you have plenty of company in remaining fastened by knots that are in fact quite easy to untie. No doubt every fallacy would be inherently easy to discover, if people went about untangling it and resolving it into its principles, for it cannot be but that something connected with it, or close to it, would plainly reveal its falsity. Our Academician had a certain special genius in such cases for reducing with a few words to absurdity and contradiction conclusions that are palpably false, and which nevertheless have hitherto been believed to be true. I have collected many conclusions in physics that had always passed for true, which were later shown by him to be false by means of brief and quite simple reasoning.

SAGR. Truly this is one of them, and if the others are like this, it will be good that at some time you will share them with us. But meanwhile let us continue with the question we have undertaken, we are searching for a way (if there is one) in which to give a rule and assign a just and known measure to the force of impact. It seems to me that this cannot be had through the experience proposed, for as sensible experiment shows us, repeated blows of the pile driver on the pole do drive it further and further, and it is clear that each succeeding blow does act, which is not true of the dead weight. Having acted when it made its first pressure, {288} it does not go on and produce the effect of the second [blow] when replaced, that is, [it does not] again drive the pole. Indeed, it is clearly seen that for this second entrance we need a weight of more than 1000 pounds, and if we want with dead weights to equal the entrances of the third, fourth, and fifth blow, and so on, we shall need the heaviness of [328] continually greater and greater dead weights. Now, which of these can we take as a constant and secure measure of the force of that blow which, considered by itself, seems to be always the same?

SALV. This is one of the prime marvels that I believe must doubtless have held in perplexity and hesitation all speculative minds. Who, indeed, will not find it novel to hear that the measure of the force of impact must be taken not from that which strikes, but rather from that which receives the impact? As to the experiment cited, it seems to me that from this one may deduce the force of impact to be infinite—or rather, let us say indeterminate, or undeterminable, being now greater and now less, according as it is applied to a greater or lesser resistance.

SAGR. Already I seem to understand that the truth may be that the force of impact is immense, or infinite. For in the above experiment, given that the first blow will drive the pole four inches and the second, three, and continuing ever to encounter firmer ground, the third blow will drive it two inches, the fourth an inch and one-half, the ensuing ones a single inch, one-half, one-fourth, and so on, it seems that unless the resistance of the pole is to become infinite through this firming of the ground, then repeated blows will always budge the pole, but always through shorter and shorter distances. But since the distance may become as small as you please, and is always divisible and sub-divisible, entrance [of the pole] will continue, and this effect having to be made by the dead weight, each [movement] will require more weight than the preceding. Hence it may be that in order to equal the force of the latest blows, a weight immensely greater and greater will be required.

SALV. So I should certainly think.

APR. Then there cannot be any resistance so great as to remain firm and obstinate against the power of any impact, however light?[7]

7. The compact phrasing here is meant to convey the double idea that (1) no resistance exists that can withstand a blow of unlimited strength, and (2) any impact, however small, has some effect on any given resistent. Cf. note 26 to Fourth Day; pp. 337, 341, below; and Fragment 4 at end.

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SALV. I think not, unless what is struck is completely immovable, that is, unless its resistance is infinite.

SAGR. These statements seem remarkable, and so to speak, prodigious. It appears that in this effect [and in this] alone, art may overpower and defraud nature—something that at first glance it [mistakenly] appears that other mechanical instruments can do, very heavy weights being raised with small force by the power of the lever, screw, pulleys, and the [329] rest. But in this effect of impact, a few blows of a hammer weighing no more than ten or twelve pounds may flatten a cube of copper that is not broken or mashed by resting a big marble steeple or even a very high tower upon the hammer This seems to me to defeat all the physical reasoning by which one might try to remove the wonder from it. Therefore, Salviati, take the clue in your hand and lead us from this complicated maze.

SALV. From what you two have to say, it appears that the principal knot of the difficulty lies in puzzlement why the action of impact, which seems infinite, may arise in a different way from that of other machines which overcome immense resistances with very small forces. But I do not despair of explaining how in this, too, one proceeds in the same manner. I shall try to clear up the process, and though it seems to me quite complicated, perhaps, as a result of your questions and objections, my remarks may become more subtle and acute, and sufficient at least to loosen the knot, if not to untie it.

It is evident that the property [*facoltà*] of force in the mover and [that] of resistance in the moved is not single and simple, but is compounded from two actions, by which their energy must be measured. One of these is the weight, of the mover as well as of the resistent, the other is the speed with which the one must move and the other be moved. Thus, if the moved must be moved with the speed of the mover—that is, if the spaces traversed by both in a given time are equal—it will be impossible for the heaviness of the mover to be less than that of the moved, but rather it must be somewhat greater, for in exact equality [of weight] resides equilibrium and rest, as seen in the balance of equal arms. But if with a lesser weight we wish to raise a greater, it will be necessary {290} to arrange the machine in such a way that the smaller moving weight goes in the same time through a greater space than does the other weight, that is to say, the former is moved more swiftly than the latter. And thus we are taught by experience that in the steelyard, for example, in order for the counterweight to raise a weight ten or fifteen times as heavy, the distance along the beam from the center round which it turns must be ten or fifteen times as great as the distance [330] between that same center and the point of suspension of the other weight, and this is the same as to say that the speed of the mover is ten or fifteen times as great as that of the moved. Since this is found to happen in all the other instruments, we may take it as established that the weights and speeds are inversely proportional. Let us say in general, then, that the momentum of the less heavy body balances the momentum of the more heavy when the speed of the lesser has the same ratio to the speed of the greater as the heaviness of the greater has to that of the lesser—to which, any small advantage being allowed, equilibrium is overcome and motion is introduced.

This settled, I say that not only in impact does the action [*operazione*] seem infinite as to the overcoming of whatever great resistance, but that this also shows itself in every other mechanical device. For it is clear that a tiny weight of one pound, descending, will raise a weight of 100 or 1000 or as much more as you please, if we place it 100 or 1000 times as far from the center on the arm of the steelyard as the other, great, weight, that is, if we make the space through which the former shall descend to be 100 or 1000 or more times as great as the space through which the other is to rise, so that the speed of the former is 100 or 1000 times the speed of the latter. Yet I wish, by means of a more striking example, to make it palpable to you that any little weight, descending, makes any immense or very heavy bulk ascend.

Suppose a vast weight to be attached to a rope fastened to a firm high place, around which as center you are to imagine to be described the circumference of a circle that passes through the center of gravity of the suspended bulk. This center of gravity, you know, will be vertically beneath the suspending rope, or, to put it better, will be in that straight line which goes from the point of suspension to the common center of all heavy things, that is, the center of the earth. Next, imagine a fine thread to which any weight, as small {291} as you please, is attached in such a way that its center of gravity always remains in the previously mentioned circumference, and suppose that this little weight just touches and rests against the vast bulk. Do you not believe that this new weight, added at the side, will push the greater one somewhat, separating its center of gravity from the previously mentioned vertical line in which it originally lay? Yet it will unquestionably move along the circumference mentioned, and being moved, it will separate from the horizontal line tangent [331] to the lowest point of the circumference in which the center of gravity of this vast bulk was situated. As to the space, the arc passed through by the heavy weight will be the same as that passed through by the tiny weight which was supported against the vast one. Yet the rise of the center of the great weight will not thereby equal the descent of the center of the tiny weight, because the latter descends through a place or space much more tilted than that of the ascent of the other center, which is made in a certain way from the tangent of the circle along an angle less than the most acute [rectilineal] angle.[8] Here, if I were dealing with people less versed in geometry than you are, I should demonstrate how a move-able leaving [along a circle] from the lowest point of tangency [with the horizontal], its [vertical] rise from [*della*] the horizontal line to some point in the circumference outside [*separato dal*] the tangent may be smaller in any desired ratio than its [vertical] drop along an equal arc [*asse*] taken at any other place not containing the point of tangency, but surely you have no doubt as to this.[9]

Now, if the simple touching of the tiny weight against the great bulk can move and raise that, what will it do when, drawn back and allowed to run along the circumference, it comes to strike there?

APR. Truly, it seems to me that there is no room left for doubt that the force of impact is infinite, from what the experiment adduced explains about it. But this information does not suffice my mind for the clearing away of many dark shadows which hold it so obscured that I do not see how this business of impacts proceeds, at least, not so that I could reply to every question that might be asked of me.

8. The "mixed angle" of note 33 to Second Day; cf. Euclid, *Elements* III.16.

9. *Asse* (axis) was probably a scribal error for *area;* the idea is that the vertical drop for any arc not touching the lowest point is greater than the drop for an equal arc touching that point, while the latter may be made as small as one pleases by shortening the arc. Many dubious readings are found in this posthumously printed work, cf. note 10, below.

SALV. Before going further, I want to reveal to you a certain equivocation that is lurking in ambush. This lets us believe that, in the previous example, all blows on the pole were equal (or the same), being made by the same pile driver raised always to the same height. But this does not follow. To understand this, imagine striking with your hand against a ball that comes falling from above, and tell me if, when this arrived upon your hand, you were to have your hand sinking along the same line and with the same speed as the ball, what shock would you feel? Surely none. But if, upon the arrival of the ball, you yielded only in part, by dropping your hand with less speed than that of the ball, [332] you would indeed receive an impact—not as with the whole speed of the ball, but only as with the excess of its speed over that of the dropping of your hand. Thus if the ball should descend with ten degrees of speed, and your hand yielded with eight, the blow would be made as by two degrees of speed of the ball. The hand yielding with four [degrees], the blow would be as six, and the yielding being as one, the blow would be as nine, the entire impact of the speed often degrees would be [only] that which struck the hand that did not yield.

Now apply this reasoning to the pile driver, when the pole yields to the impetus of the pile driver four inches the first time, and two [inches] the second, and a single inch the third. These impacts come out unequal, the first being weaker than the second, and the second than the third, according as the yielding of four inches retires[10] more from the [initial] speed of the first blow than the second [yielding of only two inches], and the second [impact] is weaker than the third, which takes away twice as much as the second from the same [initial] speed. Hence, if the great yielding of the pole to the first shock, and its lesser yielding to the second and still less to the third, and so on continually, is the reason that the first blow is less effective [*valido*] than the {293} second, and this than the

10. Reading *retrae* for *deirae* of the printed text. In pursuance of his previous argument. Galileo reasons that even though the terminal speed of fall (initial speed of impact) is the same in each case, we should call the effective blow, or impact, weaker in the earlier strokes, because the pole offers less resistance. A quite different adumbration of Newton's third law of motion was already present in Galileo's first work, cf. *On Motion*, pp. 64, 109 *(Opera*, I, 297, 336).

third, what wonder is it if a lesser quantity of dead weight is needed for the first driving of four inches, and more is needed for the second, of two inches, and still more for the third, and always more and more continually, in proportion as the drivings go diminishing with diminutions of the yielding of the pole, which amounts to saying with the increase of the resistance?

From what I have said, it seems to me that one may easily gather how difficult it is to determine anything about the force of impact made upon a resistent that varies its yielding, such as this pole that becomes indeterminately more and more resisting. Hence I think it necessary to give thought to something that receives the impacts and always opposes them with the same resistance. Now, to establish such a resistent, I want you to imagine a solid weight of, say, 1000 pounds, placed on a plane that sustains it. Next, I want you to think of a rope tied to this weight and led over a pulley fixed high above. Here it is evident that when force is applied by pulling down on the end of the rope, it will always meet with quite equal resistance in raising the weight, that is, the [333] opposition of 1000 pounds of weight. For if from the end of the rope there were suspended another weight, equal to the first, equilibrium would be established, and being raised up without support from anything below, they would remain still, nor would this second weight descend and raise the first unless given some excess of weight. And if we rest the first weight on the said plane that sustains it, we can use other weights of varying heaviness (though each of them less than the weight sustained at rest) to test what the forces of different impacts are. [This is done] by tying such weights to the end of the rope and then letting them fall from a given height, observing what happens at the other end to that great solid that feels the pull of the falling weight, which pull will be to that large weight as a blow that would drive it upward.

Here, in the first place, it seems to me to follow that however small the falling weight, it should undoubtedly overcome the resistance of the heavy weight and lift it up. This consequence seems to me to be conclusively drawn from our certainty that a smaller weight will prevail over another, however much greater, whenever the speed of the lesser shall have, to the speed of the greater, a greater ratio than the weight of the greater has to the weight of the smaller, and this [always] happens in the present instance, since the speed of {294} the falling weight infinitely surpasses the speed of the other, whose speed is nil when it is sustained at rest. But the heaviness of the falling solid is not nil in relation to that of the other, since we did not assume the latter to be infinite, or the former to be nil, hence the force of this percussent will overcome the resistance of that on which it makes its impact.

Next we shall seek to find out how great is the space through which the impact received will raise it, and whether perhaps this [distance] will correspond to that of other mechanical instruments. Thus it is seen in the steelyard, for example, that the rise of the heavy weight will be that part of the fall of the counterweight, which the weight of the counterweight is of the greater weight. So in our case we should have to see, supposing the weight of the big resting solid to be 1000 times that of the falling weight—which falls, let us say, from a height of one braccio—whether this raises the other [weight] one one-hundredth of a braccio, if so, it would appear to be following the rule for the other mechanical instruments. Let us imagine making the first experiment by dropping from [334] some height, say one braccio, a weight equal to the other, which we have placed on a [supporting] plane, these weights being tied to the opposite ends of the same rope. What shall we believe to be the effect of the pull of the falling weight, with regard to the moving and raising of the other, which was at rest? I should be glad to hear your opinion.

APR. Since you look at me, as if you were waiting for my reply, it appears to me that the two weights being equally heavy, and the one which falls having in addition the impetus of its speed, the other must be raised by it far beyond equilibration, inasmuch as the mere weight of the other was sufficient to hold it in balance. Hence, in my opinion, it will rise through much more than a space of one braccio, which is the measure of the descent of the falling weight.

SALV. And what do you say, Sagredo?

SAGR. The reasoning seems conclusive to me at first glance, but, as I said a while ago, many experiences have taught me how easily one may be deceived, and accordingly how necessary it is to go circumspectly before boldly pronouncing and affirming anything.

Hence I shall say, still dubiously, that it is true that the weight of 100 pounds of the falling heavy body will suffice to raise the other, which also weighs 100 pounds, as far as to equilibrium, even without its being {295} endowed and supplied with speed, [to do this,] the excess of a mere half-ounce will suffice. But I also think that that equilibration will be made very slowly, and hence that when the falling body acts with great speed, it will necessarily raise its companion on high with like speed. Now, there seems to me no doubt that greater force is needed to drive a heavy body upward with great speed than to push it very slowly,[11] so it might happen that the advantage of the speed acquired by the falling body in free fall through one braccio would be consumed, and so to speak spent, in driving the other with equal speed to a like height. Hence I am inclined to believe that these two movements, upward and downward, would end in rest immediately after the rising weight had gone up one braccio, which would mean two braccia of [335] fall for the other, counting the first braccio of free fall as executed by that one alone.

SALV. I truly lean toward the same belief. For though the falling weight is an aggregate of heaviness and speed, the operation of its heaviness in raising the other [weight] is nil, this being opposed by the resistance of equal heaviness in that other, which clearly would not be moved without the addition of some small weight. Therefore the operation is entirely that of the speed, which can confer nothing but speed.[12] Being unable to confer other [speed] than what it has, and having nothing other than that which it acquired in the descent of one braccio after leaving from rest, it will drive the other upward through a like space and with a like speed, in agreement with what can be discerned in various experiences, namely, that the falling weight, leaving from rest, is everywhere found to have that impetus which suffices to restore it to the original height.

SAGR. I recall that this is clearly shown by a weight hanging from a thread fixed above. Removed from the vertical by any arc less than a quadrant, and set free, this

11. Galileo's emphasis on speed as such underlay the essential difference between his mechanics and that of medieval, as that of Cartesian, writers; cf. note 32 to Fourth Day. But see also note 15, below.
12. This inference was probably suggested by the use of compound ratios in physics (note 4, above). The Aristotelian position was very different, defining greater "force" or "power" in terms of the imparting of greater speed; cf. *Physica* vii, § 5, especially at line 250a. Medieval physicists followed that lead, thus the theory of proportions in motion developed by Thomas Bradwardine (1297-1349) was intended to justify precisely this passage in Aristotle.

weight descends {296} and passes beyond the vertical, rising through an arc equal to that of its descent. From this it is evident that the ascent derives entirely from the speed acquired in descent, inasmuch as in [any] rising upward, the weight of the moving body can have played no part. Indeed, that weight, resisting ascent, goes despoiling the moveable of the speed with which it was endowed by the descent.

SALV. If the example of what is done by the heavy solid on the thread, of which I remember that we spoke in our [336] discussions of days past, squared and fitted as well with the case we are now dealing with as it fits with the facts [*alia verita*], your reasoning would be very cogent. But I find no trifling discrepancy between these two operations, I mean between that of the heavy solid hanging from the thread, which released from a height and descending along the circumference of a circle, acquires impetus to transport itself to another equal height, and this other operation of the falling body tied to the end of a rope in order to lift another one equal to itself in weight. For that which descends along the circle continues to acquire speed as far as the vertical [position], favored by its own weight, which impedes its ascent as soon as the vertical is passed, ascent being a motion contrary to its heaviness. Thus [in return] for the impetus acquired in natural descent, it is no small repayment to be carried along by violent motion or through a height. But in the other case, the falling weight comes upon its equal placed at rest, not only with its acquired speed but with its heaviness as well, and this [heaviness], being maintained, by itself alone removes all resistance on the part of its companion [weight] to being lifted.[13] Hence the [previously] acquired speed meets with no opposition from any weight that resists rising, and just as impetus conferred downward on a heavy body would encounter no cause in that [body] for annihilation or retardation [of that impetus], so none is encountered in that rising weight whose [effective] heaviness remains nil, being counterpoised by the other, descending, weight.

13. Here Galileo begins to speak of an inertial motion in the modern sense, using balanced weights for the study of impact rather than the usual and intuitive analysis in terms of frictionless bodies striking while supported on a hard flat surface. It was on the latter basis that Descartes deduced, in contradiction with the ensuing discussion, that a smaller body could never budge a larger one, however great the speed of the smaller.

Here, it seems to me, precisely the same thing takes place {297} which happens to a heavy and perfectly round moveable placed on a very smooth plane, somewhat inclined, this will descend naturally by itself, acquiring ever greater speed. But if, on the other hand, anyone should wish to drive it upward from the lower part [of the plane], he would have to confer impetus on it, and this would be ever diminished and finally annihilated [in the rise]. If the plane were not inclined, but horizontal, then this round solid placed on it would do whatever we wish, that is, if we place it at rest, it will remain at rest, and given an impetus in any direction, it will move in that direction, maintaining always the same speed that it shall have received from our hand and having no action [by which] to increase or diminish this, there being neither rise nor drop in that plane. And in this same way the two equal weights, hanging from the ends of the rope, will be at rest when placed in balance, and if impetus downward shall [337] be given to one, it will always conserve this equably Here it is to be noted that all these things would follow if there were removed all external and accidental impediments, as of roughness and heaviness of rope or pulleys, of friction in the turning of these about the axle, and whatever others there may be of these.

But since we are considering the speed acquired by one of these weights in descent from some height while the other remains at rest, it will be good to determine what and how much must be the speed with which both would be moved after the [initial] fall of the one, this descending and the other ascending. From what is already demonstrated, we know that a heavy body which falls freely on departing from rest perpetually acquires a greater and greater degree of speed, hence in our case, the greatest degree of speed of the heavy body, while it descends freely, is that which it is found to have at the point at which it commences to lift its companion. Now it is evident that this degree of speed will not go on increasing when its cause of increase is taken away, this being the weight of the descending body itself; for its weight no longer acts when its propensity to descend is taken away by the repugnance to rising of its companion of equal weight. Hence the maximum degree of speed will be conserved, and the motion will be converted from one of acceleration to uniform motion.[14]

14. Reduction of accelerated motion to some equivalent uniform motion was essential before development of the calculus, cf. Third Day, Bk. II,

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What the future speed will then be is manifest from the things demonstrated and seen in the [discussions of the] past days. That is, the future speed will be such that, in another time equal to that of the [initial free] descent, double the space of [free] fall would be passed.

SAGR. Then Apronio has philosophized better than I have. Thus far I am well satisfied by your reasoning, and admit what you have told me as most true. But I still do not feel that I have learned enough to remove the great wonder I feel at seeing very great resistances overcome by the force of impact of the striking body when its weight is not great and its speed not excessive. It increases my bafflement to hear you affirm that there is no resistance short of the infinite that will resist a blow without yielding, and moreover that there is no way of assigning a definite measure to [the force of] such a blow So it is our wish that you attempt to shed light in this darkness.

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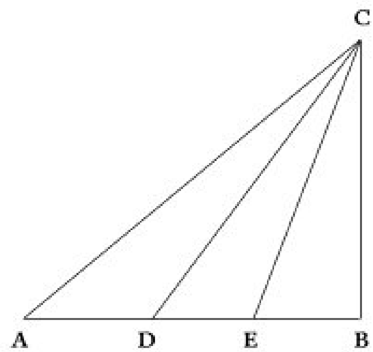
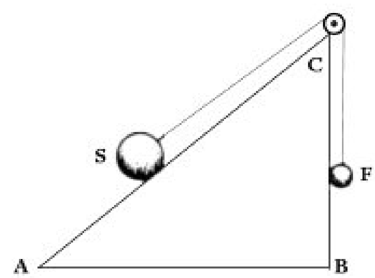
SALV. No demonstration can be applied to a proposition unless what is given is one and certain, and since we wish to philosophize about the force of a striking body and the resistance of one which receives the impact, we must choose a percussent whose force shall be always the same, such as that of the same heavy body falling always from the same height, and likewise let us establish a recipient of the blow that will always offer the same resistance. To have this, and keep to the above example of the two heavy bodies hanging from the ends of the same rope, I shall have the percussent be the small weight that is allowed to fall, and the other shall be a weight as much greater [than this] as you please, in the raising of which the impetus of the small falling weight is to be exercised. It is manifest that the resistance of the larger body is the same at all times and all places, as would not be the case with the resistance of a nail, or of the pole, in which resistance increases continually with penetration, but in some unknown ratio because of the various accidental events involved, such as hardness of wood or ground, and so on, even though the nail and the pole remain always the same. It is further necessary to remember some true conclusions of which we spoke in past days in the treatise on motion. The {299} first

Theorem 1. No clear physics of force was likely to emerge under the theory of proportion alone, in which force can appear only as some kind of relation rather than as an entity. As will be seen in Salviati's next speech, "force" is simply made to cancel out.

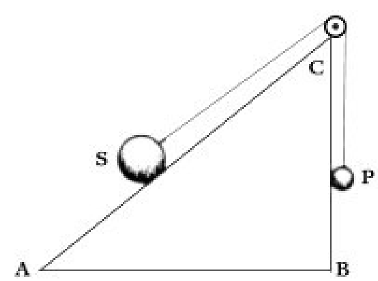
of these is that heavy bodies, in falling from a high point to a horizontal plane beneath, acquire equal degrees of speed whether their descent is made vertically or upon any of diversely inclined planes.



For example, *AB* being a horizontal plane upon which the vertical *CB* is dropped from point C, and other planes, diversely inclined, *CA, CD,* and *CE,* fall from the same C, we must understand that the degrees of speed of bodies falling from the high point *C* along any of the lines going from *C* to end at the horizontal are all equal. In the second place, it is assumed that the impetus acquired at *A* by the body falling from the point *C* is such that it is exactly needed to drive the same falling body (or another one equal to it) up to the same height, from which we may understand that such force is required to raise that same heavy body from the horizontal to height *C,* whether it is driven from point *A, D, E,* or *B.* Let us recall in the third place that the times of descent along the designated planes have the same ratio as the lengths [339] of these planes, so that if the plane *AC,* for example, were double the length of *CE* and quadruple that of *CB,* the time of descent along *CA* would be double the time of descent along *CE* and four times that along *CB.* Further, let us recall that in order to pull the same weight over diverse inclined planes, lesser force will always suffice to move it over one which is more inclined [to the vertical] than over one less inclined, according as the length of the latter is less than the length of the former.



Now, these truths being supposed, let us take the plane *AC* to be, say, ten times as long as the vertical *CB,* and let there be placed on *AC* a solid, *S,* weighing 100 pounds. It is manifest that if a cord is attached to this solid, riding over a pulley placed above the point *C,* and to the other end of this cord a weight of ten pounds is attached, which shall be the weight of *P,* then that weight *P* will descend with any small addition of force, drawing the weight *S* along the plane *AC*. Here one must note that the space through which the greater weight moves over the plane beneath it is equal to the space through which the small descending weight is moved, from this, someone might question the general truth applying to all mechanical propositions, which is that a small force does not overcome and move a great resistance unless the motion of the former exceeds the motion of the latter in inverse ratio of their weights. But in the present instance {300} the descent of the [340] the descent of the small weight, which is vertical, must be compared [only] with the vertical rise of the great solid *S,* observing how much this is lifted vertically from the horizontal, that is, one must consider how much *S* rises in the vertical *BC.*



Having made various meditations, gentlemen, about the setting forth of that which remains to be said by me, which is the crux of the present matter, I affirm the following conclusion, which will then be explained and demonstrated.

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PROPOSITION

If the effect made by an impact of the same weight falling from the same height shall be to drive a resistent of constant resistance through some space, and [if] to produce a similar effect there is needed a determined quantity of dead weight [merely] pressing, without impact, I say that if the original percussent, [acting] upon some greater resistent, with the given impact shall drive it (for example) through one-half the space that the other was driven, then in order to accomplish this second driving, the pressure of the said dead weight will not suffice, but there will be required another one, twice as heavy And similarly in all other ratios, when a shorter [constantly resisted] drive is made by the same percussent, then inversely by that much there will be required, to do the same, a greater pressing quantity of dead weight.

In the earlier example of the pole, the resistance is to be understood to be such that it cannot be overcome by less than one hundred pounds of dead weight pressing, and [it is understood that] the weight of the percussent is only ten pounds, falling from a height of, say, four braccia, and driving the pole four inches. Here, in the first place, it is evident that the weight of ten pounds falling vertically will be sufficient to raise a weight of one hundred pounds along a plane so inclined that its length is ten times its height, according to what has been said above, and that as much force is needed to raise ten pounds of weight vertically as to raise one hundred on a plane whose length is ten times its vertical elevation. Hence if the impetus acquired by the falling body through such a vertical space is applied to raise another that is equal to it in resistance, it will raise it a like space, but the resistance of the vertically falling body of ten pounds is equal to that of the body of one hundred pounds rising along a plane of length ten times its vertical height. Therefore, {301} let the weight often pounds fall through any height vertically, and its acquired impetus, applied to the weight of one hundred pounds, will drive this through as much space on the inclined plane as corresponds to the vertical height as great as one-tenth part of this inclined space. And it is already concluded above that the force able to drive a weight on an inclined plane is sufficient to drive it through the vertical [341] corresponding to the height of this inclined plane—which vertical, in the present instance, is one-tenth the space passed along the incline, which is equal to the space of fall of the first weight, of ten pounds.

Thus it is manifest that the fall of the weight of ten pounds made vertically is sufficient to raise the weight of one hundred pounds, also vertically, but only through a space that is one-tenth the descent of the falling body of ten pounds.[15] But that force which can raise a weight of one hundred pounds is equal to the force with which the same weight of one hundred pounds presses down, and this was its power to drive the pole when placed upon it and pressing it. Behold, therefore, the explanation how the fall often pounds of weight is able to drive a resistance equivalent to that which a weight of one hundred pounds has to being raised, while the driving will be no more than one-tenth the descent of the percussent. And if we now assume the resistance of the pole to be doubled or tripled, so that to overcome it there is needed the pressure of two hundred or three hundred pounds of dead weight, then repeating the reasoning, we shall find that the impetus of the ten pounds falling vertically will be able to drive the pole the second and the third time, as it did the first time, and as [far as] the tenth part of its fall the first time, so the twentieth the second time, and the third time, the thirtieth of this descent. And thus, multiplying the resistance *in infinitum,* the same blow will always be able to overcome it, but by driving the resisting body always through less and less space, in inverse [*alterna*] proportion, from which it seems that we may reasonably assert the force of impact to be infinite.[16]

But we must also consider that in another way, the force {302} of pressing without

15. Here the neglect of time (or speed), unusual for Galileo, results in his adoption of a conservation principle in terms of vertical displacements alone, akin to the medieval and Cartesian approaches (cf. note 13, above) but not restricted to connected motions as in the simple machines.
16. Cf. notes 26 to Fourth Day, note 7, above, and Fragment 4 at end.

impact is also infinite, inasmuch as if it overcomes the resistance of the pole, it will drive it not merely through some space through which the blow will have driven it, but will continue to drive it *in infinitum.*[17]

SAGR. Truly, I perceive that your attack travels very directly to the investigation of the true cause of the present problem, but since it appears to me that impact may be created in many ways, and applied to a great variety of resistances, I believe it is necessary to go on and explain some [of these] at least, the understanding of which might open our minds to the understanding of all.

17. The seeming incongruity between an infinite force of impact and the finite force of dead weight (in Galileo's sense) is here removed by him through showing how either finite or infinite strength may be attributed to impact in one way or to steady pressure in another way; see also Fragment 1, at end.

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SALV. You say well, and I have already prepared myself to give examples. For one thing, we shall say that at times it may happen that the operation of the percussent is revealed not on the thing struck, but in the percussent itself. Thus, a blow being struck on a fixed anvil with a lead hammer, the effect will happen to the hammer, which will be flattened, rather than to the anvil, which will not descend. Not unlike this is the effect of the mallet on the sculptor's chisel, for the mallet being of soft untempered iron and striking repeatedly on the chisel of hard tempered steel, it is not the chisel that is damaged, but the mallet that becomes dented and lacerated. Again, in another way, the effect is reflected solely in the percussent, thus we see not infrequently that if one continues to drive a nail into very hard wood, the hammer [finally] rebounds without driving the nail forward at all, and we say in this case that the blow did not "take." Not very different is the bouncing of an inflated ball on a hard pavement, or of any other body so disposed, which indeed yields to the impact, but returns to its first shape as by arching, and such a rebound occurs not only when that which strikes yields and then recovers, but also when the same occurs in that upon which it strikes, and in such a manner a ball bounces when it is of very hard and unyielding material, but falls on the tightly stretched membrane of a drum.

Also perceived with great wonder is that effect produced when a blow is added to pressure without impact, making a compound of the two. We see this in mangles or olive presses and the like, when by the simple pushing of several {303} men the screw has been made to go down as far as they can manage. By drawing back a step from the bar and then striking swiftly against it, they move the screw more and more, and get it to such a point that the shock of the force of four or six men will achieve what mere pushing by a dozen or a score could not do. In this case it is required that the bar be very thick and of very hard wood, so that it bends little or not at all, for if it should give, the blow would be spent in bending it.

* * *

Galileo Galilei, *Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motions.* Translated by Stillman Drake, University of Wisconsin Press, Madison, 1974: 281–303. (ADDED DAY)

TRANSCRIBER'S NOTES (Added)

The "Added Day" included in Stillman Drake's 1974 translation of Galileo's *Two New Sciences* (1638) is presented here with a number of minor cosmetic changes to render the work more readable. Thus rather than retaining the large margins and small marginal figures of the 1974 publication larger figures have been incorporated *within the text* to essentially match the format adopted for the translation of the *Two New Sciences* by Henry Crew and Alfonso de Salvio (1954). Part of the latter is available below:

INTRODUCTION FIRST DAY *SECOND DAY* *THIRD DAY* FOURTH DAY

Galileo Galilei, *Dialogues Concerning Two New Sciences*, translated by Henry Crew & Alfonso de Salvio, with an introduction by Antonio Favaro, Dover Publications, Inc., New York, 1954.